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ONE WAVE TO EXPLAIN THEM ALL: A UNIFYING PERSPECTIVE ON POST-HOC EXPLAINABILITY

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ABSTRACT

Despite the growing use of deep neural networks in safety-critical decision-making, their inherent black-box nature hinders transparency and interpretability. Explainable AI (XAI) methods have thus emerged to understand a model’s internal workings, and notably attribution methods also called Saliency maps. Conventional attribution methods typically identify the locations – the *where* – of significant regions within an input. However, because they overlook the inherent structure of the input data, these methods often fail to interpret what these regions represent in terms of structural components (e.g., textures in images or transients in sounds). Furthermore, existing methods are usually tailored to a single data modality, limiting their generalizability. In this paper, we propose leveraging the wavelet domain as a robust mathematical foundation for attribution. Our approach, the **Wavelet Attribution Method (WAM)**, extends the existing gradient-based feature attributions into the wavelet domain, providing a unified framework for explaining classifiers across images, audio, and 3D shapes. Empirical evaluations demonstrate that WAM matches or surpasses state-of-the-art methods across faithfulness metrics and models in image, audio, and 3D explainability. Finally, we show how our method explains not only the *where* – the important parts of the input – but also the *what* – the relevant patterns in terms of structural components.

1 INTRODUCTION

Deep neural networks are increasingly being deployed in various applications, such as medicine, transportation, robotics, or finance (Pooch et al., 2020; Sun et al., 2022; Redmon et al., 2016; Thimonier et al., 2024). These networks often make critical decisions, such as detecting tumors in medical images or identifying obstacles in autonomous driving, yet the underlying decision-making process is difficult to interpret due to the black-box nature of the models.

This opacity has motivated the rise of explainable AI (XAI) techniques to provide human-understandable explanations for model decisions. While XAI has been predominantly applied in image classification, it is also extending into other fields, such as audio and 3D shape classification (Parekh, 2023; Paissan et al., 2024; Chen et al., 2021; Zheng et al., 2019).

Among these techniques, feature attribution methods – specifically gradient-based methods for generating saliency maps (heatmaps that highlight important input features, Zeiler & Fergus, 2014) are prevalent. These gradient-based methods (Shrikumar et al., 2017; Sundararajan et al., 2017; Smilkov et al., 2017) considered as efficient and reliable for interpreting model behavior (Crabbé & van der Schaar, 2023; Wang & Wang, 2021; Xue et al., 2023).

Feature attribution involves decomposing a model’s decision within a specific “explanation” domain. Traditionally, saliency mapping relied on the pixel domain as this domain. However, pixel-based explanations flatten the hierarchical and spatial relationships inherent in images, effectively collapsing their structural properties. In addition, the pixel domain is only relevant when the input modality

* Alphabetical ordering.

is an image. Instead, decomposing the model’s decision in the wavelet domain, which preserves the inter-scale dependencies of an input modality, could enable saliency-based methods to account for the image structure in the explanation. Besides, the wavelet domain is defined for any input dimension (images being an input of dimension 2), thus enabling a natural generalization of saliency mapping to modalities such as audio or 3D volumes.

This work introduces the **Wavelet Attribution Method (WAM)**, a universal feature attribution method. By unifying and extending existing methods, notably SmoothGrad (Smilkov et al., 2017) and Integrated Gradients (Sundararajan et al., 2017) within the wavelet domain, we enable their application to any modality defined over a continuous space, moving beyond the limitations of the pixel domain. As illustrated in Figure 1, our approach involves computing the gradient of a classification model’s prediction with respect to the *wavelet decomposition* of the input signal. We then produce smooth explanations by either averaging over noisy inputs or integrating along the prediction path.

Operating in the wavelet domain, WAM isolates the contribution of the different scales within the input signals to the model’s prediction, providing deeper insights into a model’s decision-making process. We illustrate these insights by revisiting the meaningful perturbation framework with WAM, or by carrying out noise and overlapping experiments on audio samples. Quantitative evaluations demonstrate that WAM outperforms existing attribution methods across a range of topologies, modalities, and metrics, underscoring its utility in addressing critical challenges in image, audio, and 3D shape classification.

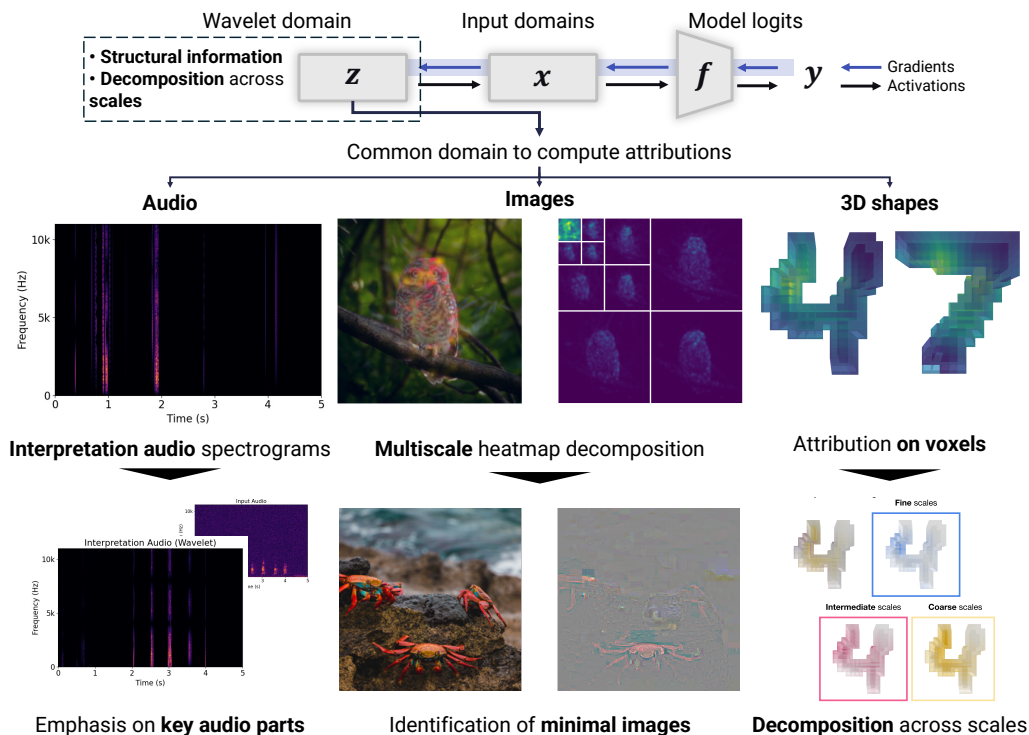


Figure 1: **Explaining any modality by decomposing the model’s decision in the wavelet domain.** By computing the gradient of the model’s prediction in the wavelet domain, we provide a unified approach to explain the decision of audio, image, and shape classifiers that preserves the structural properties of the input modalities.

2 RELATED WORKS

Images explainability. Computer vision has supported the development of numerous post-hoc explainability methods (Baehrens et al., 2010), with attribution methods being the most popular. Post-hoc methods are applied on trained model and aim to rank, i.e., estimate an importance for each

pixel or region of an image, reflecting its contribution to the score predicted by the model. Many methods have been proposed, which can be classified into two categories: White-box methods, which require access to gradients, and Black-box methods, which use perturbations on the input space.

White-box approaches leverage internal model information, such as gradients, to explain predictions. saliency maps were first introduced by Baehrens et al. (2010) and later refined in Simonyan et al. (2014); Zeiler & Fergus (2014); Springenberg et al. (2014); Sundararajan et al. (2017); Smilkov et al. (2017); or Muzellec et al. (2023). These methods calculate the gradient of the classification score with respect to the input pixels using backpropagation. However, gradients can be noisy in large vision models (Smilkov et al., 2017) and may yield misleading importance estimates due to their focus on infinitesimal input variations (Ghalebikesabi et al., 2021). On the other hand, Black-box methods rely on input perturbations without requiring access to internal model parameters. Techniques like Occlusion (Zeiler & Fergus, 2014), LIME (Ribeiro et al., 2016), RISE (Petsiuk et al., 2018), Sobol (Fel et al., 2021), and HSIC (Novello et al., 2022) generate importance maps by observing changes in the classification score when the input image is altered. For example, Occlusion uses binary masks to systematically occlude regions of the image, while RISE and HSIC apply random masks to perturb multiple regions simultaneously.

However, traditional attribution methods focus on *where* important regions in the image are. However, they fail to address *what* these regions represent in terms of meaningful, higher-level concepts – a gap that more recent research is attempting to fill (Kim et al., 2018; Ghorbani et al., 2019; Fel et al., 2023b; Zhang et al., 2021; Graziani et al., 2023; Fel et al., 2023b;a). Moreover, these methods often under-utilize the inherent structure of images, (recent work are beginning to explore attribution in the frequency domain, e.g. Muzellec et al., 2023).

Audio explainability. Previous work on post-hoc audio explainability methods has mainly expanded in three directions. The first explored the use of saliency methods to highlight key features for audio classifiers processing spectrograms (Becker et al., 2024; Won et al., 2019) or 1D waveforms (Muckenhirn et al., 2019). Moreover, while the use of time-frequency representations for classification and explanations is frequent in this regard, wavelet representations have not been explored previously for explanations. The second direction involves variants of LIME (Ribeiro et al., 2016) algorithm, proposed for different types of audio classification tasks (Mishra et al., 2017; 2020; Haunschmid et al., 2020; Chowdhury et al., 2021; Wullenweber et al., 2022). The third has pursued the development of methods to generate listenable interpretations for audio classifiers by leveraging the hidden representations (Parekh et al., 2022; Paissan et al., 2024). LIME-based methods suffer from the issue of high computational costs of explanations due to a large number of forward passes per sample. Recent methods for listenable interpretations require access to hidden layers and train separate modules and are thus unsuitable as post-hoc explainers.

3D explainability. 3D data generally comes in two main formats: point clouds and voxels. Point clouds offer an exact representation of the data but are unstructured. Voxels, conversely, are a discretized but structured representation of the data, making them suitable for processing with techniques such as 3D convolutions. Most explainability techniques for 3D data focused on explaining point clouds. Chen et al. (2021); Schinagl et al. (2022); Gupta et al. (2020), and Zheng et al. (2019) introduced techniques to generate visual explanations for interpretability of 3D object detection and classification networks. They highlight critical features in point cloud data by adapting 2D image-based saliency techniques (Gupta et al., 2020; Zheng et al., 2019), by using a perturbation-based approach (Schinagl et al., 2022) or by proposing a 3D variant of LIME (Tan & Kotthaus, 2022). Explainability on 3D volumes remains limited. A few works (Yang et al., 2018; Mamalakis et al., 2023; Gotkowski et al., 2021) have proposed attention maps on 2D slices of 3D medical scans using 3D-GradCAM.

3D and 1D explainability techniques often reproject the model’s decision onto a 2D-pixel domain. However, this projection filters out the intrinsic properties of the original signal, such as its temporal or spatial depth, resulting in an incomplete representation. This process, therefore, constitutes an improper way of generalizing attribution methods, as it disregards essential features of the original signal’s structure. In addition, the pixel domain itself is limited for explainability. More broadly, we note that the literature has only recently started discussing the expressiveness of the “explanation” domain, e.g., through the lenses of concepts, and still overlooks the broader applicability across

modalities. This work contributes to the ongoing discussion by evaluating how the wavelet domain can simultaneously address these concerns.

3 METHODS

Notations & Background. Throughout, we let $\mathcal{X} = (\Omega, \mathcal{F}, \mu)$ be a measure space with set Ω , σ -algebra \mathcal{F} , and measure μ . We denote by $\mathcal{H} = \mathbb{L}^2(\mathcal{X}, \mu)$ the Hilbert space of square-integrable functions on \mathcal{X} . Let $\mathbf{f} \in \mathcal{H}$ represent a predictor function (e.g., a classifier), which maps an input $\mathbf{x} \in \mathcal{X}$ to an output $\mathbf{f}(\mathbf{x}) \in \mathcal{Y}$. We denote $\mathbf{g} \in \mathcal{H}$ a generic, square-integrable function.

A wavelet is an integrable function $\psi \in \mathcal{H}$ that is normalized, centered at 0, and has zero average (i.e., $\int \psi(\mathbf{x}) d\mathbf{x} = 0$). Unlike a sine wave, a wavelet is localized in both *space* and *frequency* domains. This localization allows dilations of the wavelet to analyze different frequency intervals (scales) while translations enable analysis at different spatial locations. To compute an image’s continuous wavelet transform (CWT), we first define a filter bank \mathcal{D} derived from the original wavelet ψ , using a scale factor $\lambda > 0$ and 2D translation \mathbf{b} . The filter bank \mathcal{D} is given by

$$\mathcal{D} = \left\{ \psi_{\lambda, \mathbf{b}}(\mathbf{x}) = \frac{1}{\sqrt{\lambda}} \psi \left(\frac{\mathbf{x} - \mathbf{b}}{\lambda} \right) \right\}_{\mathbf{b} \in \mathbb{R}^2, \lambda > 0}.$$

The continuous wavelet transform of a function $\mathbf{g} \in \mathcal{H}$ at scale λ and location \mathbf{x} is given by

$$\mathcal{W}(\mathbf{g})(\lambda, \mathbf{x}) = \int_{-\infty}^{+\infty} \mathbf{g}(\mathbf{b}) \frac{1}{\sqrt{\lambda}} \psi^* \left(\frac{\mathbf{b} - \mathbf{x}}{\lambda} \right) d\mathbf{b},$$

which can be rewritten as a convolution (Mallat, 2008). In the discrete dyadic case, the scale factor λ takes values in a set Λ , chosen as $\Lambda = \{2^j : 1 \leq j \leq N, N \in \mathbb{N}, N > 0\}$. Mallat (1989) showed that one can compute the dyadic wavelet transform of a signal \mathbf{g} by applying a high-pass filter H to the signal \mathbf{g} and subsampling by a factor of two to retrieve the *detail* coefficients, and applying a low-pass filter G and subsampling by a factor of two to retrieve the *approximation* coefficients. Iterating on the approximation coefficients generates a multilevel transform, where the j^{th} level extracts information at resolutions between 2^j and 2^{j-1} octaves in the frequency spectrum. When the input signal \mathbf{x} has dimensionality greater than one, its detail coefficients can be decomposed into different orientations. The common orientations for 2D signals (i.e., images) are vertical, horizontal, and diagonal.

Wavelets and multiscale decompositions. Multiscale analysis consists in decomposing an input signal into different levels of detail. The resulting decomposition is particularly interesting as it generates interesting features for signal understanding: edges in images at different orientations and scales correspond to different textures. In sounds, the multiscale decomposition isolates slowly changing patterns from transient ones. Overall, the wavelet decomposition enables the decomposition of an input signal into interpretable components. As we further discuss in section 4.2, the properties of multiscale decompositions translate into several insightful properties for XAI.

3.1 GRADIENT-BASED FEATURE ATTRIBUTION IN THE WAVELET DOMAIN

Problem formalization. Let \mathbf{f} be a classifier and \mathbf{x} an input (e.g., an image, an audio, or a 3D shape). The classifier \mathbf{f} maps the input to a class c as $\mathbf{y}_c = \arg \max_{c \in \mathcal{C}} \mathbf{f}(\mathbf{x}) \equiv \mathbf{f}_c(\mathbf{x})$ with a slight abuse of notation. We recall that the original saliency map of the classifier \mathbf{f} for class c is then given by $\gamma_{\text{sa}}(\mathbf{x}) = |\nabla_{\mathbf{x}} \mathbf{f}_c(\mathbf{x})|$ where c denotes the class of interest. The saliency map is defined provided that the \mathbf{f}_c ’s are piecewise differentiable (Simonyan et al., 2014). The saliency map highlights the most influential (in terms of the absolute value of the gradient) components in the input \mathbf{x} for determining the model’s \mathbf{f} decision. The higher the value, the greater the importance of the corresponding region.

However, varying pixel values provide no information to what is changing on the image. Therefore, we argue that the pixel domain is not well suited for explaining *what* the model is seeing on the image. On the other hand, the wavelet decomposition of an image – and more broadly of any differentiable modality – provides information on the structural components of the modality. Therefore, computing the gradient of \mathbf{f} with respect to the wavelet transform of \mathbf{x} will enable us to understand

the model’s reliance on features such as textures, edges, or shapes in the case of images, transients, or harmonics in sounds or corners or small details in 3D shapes.

Denoting $z = \mathcal{W}(x)$ the wavelet transform of x , since \mathcal{W} is invertible, we can define the saliency map of in the wavelet domain as

$$\gamma_{\text{sa}}(z) = \left| \frac{\partial f_c(x)}{\partial z} \right| = \left| \frac{\partial f_c(x)}{\partial x} \cdot \frac{\partial \mathcal{W}^{-1}(z)}{\partial z} \right|, \quad (1)$$

using the fact that $x = \mathcal{W}^{-1}(z)$ and where $\frac{\partial f_c(x)}{\partial x}$ denotes the gradient of the classifier output with respect to the input image and $\frac{\partial \mathcal{W}^{-1}(z)}{\partial z}$ is the Jacobian matrix of the inverse wavelet transform. In practice, to retrieve Equation 1, we require the gradients on $\mathcal{W}(x)$ and directly evaluate $\partial f_c(\mathcal{W}^{-1}(z))/\partial z$. A remarkable property of this framework is that it **accommodates any input dimension**, and thus it is **modality-agnostic**. Therefore, we can apply it – and leverage its properties – to numerical signals such as audio (1D signals), images (2D signals), or 3D shapes (3D signals). In this paper, we demonstrate the superiority of this method in the 1D and 2D settings compared to other domain-specific methods and illustrate examples for 3D classification.

Smoothing. Smilkov et al. (2017) highlighted the fact that the saliency maps computed following Equation 1 can fluctuate sharply at small scales as f_c is not continuously differentiable. To yield smoother explanations, Smilkov et al. (2017) perturb the input image with Gaussian noise. Analogously, we propose to calculate

$$\gamma_{\text{sg}}(z) = \frac{1}{n} \sum_{i=1}^n \nabla_{\tilde{z}} f(\mathcal{W}^{-1}(\tilde{z})) \quad \text{with } \tilde{z} = \mathcal{W}(x + \delta) \text{ and } \delta \sim \mathcal{N}(0, I\sigma^2). \quad (2)$$

The number of samples n needed to compute the approximation of the smoothed gradient and the standard deviation σ^2 are hyperparameters for their method. To transpose this method to the wavelet domain, we add noise to the input before computing its wavelet transform. We refer to this method as WAM_{SG} throughout the rest of the paper. In appendix A.1, we illustrate the enhancement of the quality of the explanation after applying the smoothing to the gradients as described in equation 2.

Path integration. Another approach to derive smooth explanations from the model’s gradients consists in averaging the gradient values along the path from a baseline state to the current value. The baseline state is often set to zero, representing the complete absence of features. This technique, introduced by Sundararajan et al. (2017), satisfies two axioms, *sensitivity* and *implementation invariance*. Sensitivity states that “for every input and baseline that differ in one feature but have different predictions, then the differing feature should be given a non-zero attribution” and Implementation Invariance that “the attributions are always identical for two functionally equivalent networks”. Following Sundararajan et al. (2017), we adapt the Integrated Gradient method from the image domain to the wavelet domain. Denoting $z = \mathcal{W}(x)$, we evaluate

$$\gamma_{\text{ig}} = (z - z_0) \cdot \int_0^1 \frac{\partial f_c(\mathcal{W}^{-1}(z_0 + \alpha(z - z_0)))}{\partial z} d\alpha, \quad (3)$$

where z_0 denotes the baseline state of the wavelet decomposition of x . We refer to this implementation of WAM as WAM_{IG} . In appendix A.1, we illustrate the enhancement of the quality of the explanation after applying the smoothing to the gradients as described in Equation 2 and after integrating the gradients, as described in Equation 3. We also discuss the visualization properties that emerge when using either method.

3.2 MODALITY-SPECIFIC DECLINATIONS

Images. For images, the computation of WAM following Equation 2 can be visualized on the dyadic wavelet transform of the image (see plot (b) of Figure 2. From the wavelet transform of the image, we can derive several interesting properties regarding *what* the model sees on the input image. In particular, we can decompose the important coefficients at each scale (d), and illustrate as a reconstructed image (c) what is important for the model’s prediction. In section 4.2, we discuss

how WAM enables us to revisit the meaningful perturbation framework of Fong & Vedaldi (2017) and introduce *minimal images*, and in appendix C, we show the connections between our method and the frequency-centric perspectives on model robustness of Zhang et al. (2022) or Yin et al. (2019).

Audio. Following Equation 2 and Equation 3, we obtain the sensitivity of the model’s prediction with respect to the wavelet coefficients of the waveform. As in practice, audio classifiers take mel-spectrograms as inputs; we also retrieve the gradients with respect to the mel-spectrogram of the waveform. Therefore, our approach bridges the gap between methods that focus on explanations of the waveforms and those that return explanations on the mel-spectrogram.

3D Shapes. We apply WAM on voxels, as the wavelet transform is defined on structured data formats. Voxels represent 3D space as a grid of uniformly spaced cubes, where each voxel stores information about the object at that position. We consider the 3D wavelet transform, which generalizes the wavelet transform to upper dimensions. Implementations of WAM_{SG} and WAM_{IG} follow Equation 2 and Equation 3 analogously to images and audios.

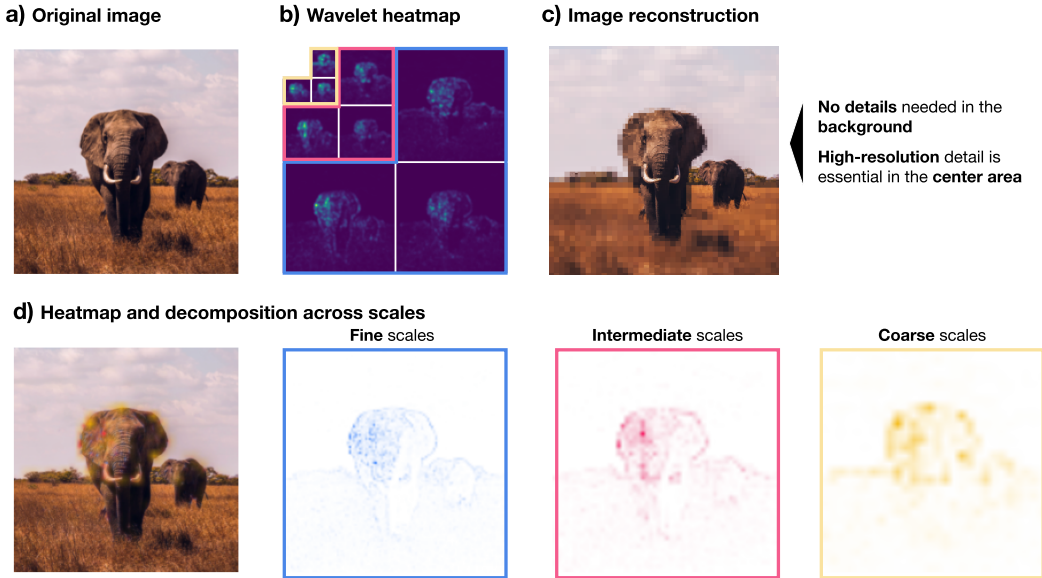


Figure 2: **WAM for images.** Our method decomposes the important components at different scales (i.e., different levels of details) and enables us to see *what* is seen on the image by the model.

3.3 EVALUATION

We evaluate WAM in two distinct settings: images and audio. Evaluation is carried out on usual benchmarks for both modalities. We do not consider the 3D setting for quantitative evaluation due to the lack of comparable baselines.

Common evaluation metrics. We quantitatively assess the accuracy of our method by leveraging the **Faithfulness** (Muzellec et al., 2023), defined as the difference between the **Insertion** and the **Deletion** scores, introduced by Petsiuk et al. (2018). Insertion and Deletion have been widely used in XAI to evaluate the quality of feature attribution methods (Fong & Vedaldi, 2017). The Deletion measures the evolution of the prediction probability when one incrementally removes features by replacing them with a baseline value according to their attribution score. Insertion consists in gradually inserting features into a baseline input. Samek et al. (2016) and Li et al. (2022) have shown that Faithfulness is effective in evaluating attribution methods. Given a model f and an explanation functional γ , the Faithfulness F is given by

$$F(f, \gamma) = \text{Ins}(f, \gamma) - \text{Del}(f, \gamma). \tag{4}$$

We provide a detailed derivation of the Insertion and the Deletion scores in appendix B.1. Insertion and Deletion were initially defined in the context of images, but we propose a definition that expands them to audio.

Modality-specific metrics. In addition to the Faithfulness, we compare WAM using the μ -Fidelity (Bhatt et al., 2021) for images and the **Faithfulness on Spectra (FF)** (Parekh et al., 2022) and **Input Fidelity (Fid-In)** (Paissan et al., 2023) for audio. We refer the reader to the appendix B.2 for a thorough definition of these metrics and a discussion of the results.

Evaluation setting for images. For images, we evaluate our method on a subset of the validation set of ImageNet (Russakovsky et al., 2015). Our subset contains 1,000 images randomly sampled from the 50,000 images of the validation set of ImageNet. We consider four model architectures representative of the popular topologies currently used. We consider the following models: the ResNet (He et al., 2016), the ConvNext (Liu et al., 2022), the EfficientNet (Tan & Le, 2019) and the Data efficient transformer (DeiT, Touvron et al., 2021). We refer the reader to the appendix A.2 for more details on the model’s parametrizations that we used. This evaluation framework is based on the frameworks of Fel et al. (2021).

We compare our method with alternative gradient-based methods, namely Saliency (Simonyan et al., 2014), Integrated Gradients (Sundararajan et al., 2017), GradCAM and GradCAM++ (Selvaraju et al., 2017), and SmoothGrad Smilkov et al. (2017). We focus only on gradient-based methods, as they are more faithful and faster to generate than alternative approaches (Crabbé & van der Schaar, 2023; Wang & Wang, 2021; Xue et al., 2023).

Evaluation setting for audio. We evaluate our method on the dataset for Environmental Sound Classification (ESC-50, Piczak, 2015). We pick the 400 samples of the first fold of ESC-50, as our backbone model has been trained on the remainder of the dataset, and evaluate the CNN classification model of Kumar et al. (2018) as our black-box model to explain. We consider a single model as alternative models (Huang & Leanos, 2018; Wilkinghoff, 2021; Lopez-Meyer et al., 2021) are only variations around the same topology. We consider two variants of the ESC-50 dataset: the original (unaltered samples) and noisy samples, for which we add 0 dB white noise to the input samples whose prediction we seek to explain. We include three baseline methods, which return explanations on the mel-spectrogram of the input samples: the GradCAM (Selvaraju et al., 2017), Integrated Gradients (Sundararajan et al., 2017), SmoothGrad (Smilkov et al., 2017) and Saliency (Simonyan et al., 2014).

4 RESULTS

4.1 QUANTITATIVE EVALUATION RESULTS

Images. As displayed on Table 1, we can see that WAM for 2D signals outperforms competing baselines according to the Faithfulness metric. In appendix B.2, we present additional results using the Insertion, Deletion, and the μ -Fidelity. The good results are mostly driven by the fact that WAM performs well in terms of Insertion. WAM also passes the randomization test (Adebayo et al., 2018). We refer the reader to appendix B.3 for more details on this test.

Audio. Table 2 presents the evaluation results for audio. For WAM, we generate the explanations from the wavelet coefficients. We can see that for audios, WAM also achieves state-of-the-art results and outperforms the competing metrics in terms of Faithfulness of spectra, Input fidelity and Insertion. The results of the other metrics are in line with those of competing approaches. In appendix B.2, we provide additional results where explanations are computed from the mel-spectrogram of the waveform. In this case, we report that WAM’s performance is more in line with competing approaches, thus showing the added value brought by explaining the model’s decision through the wavelet domain.

Table 1: **Faithfulness** (Muzellec et al., 2023) score obtained on 1,000 images from the validation set of ImageNet and for different model architectures. Higher is better. **Bolded** results are the best and underlined values are the second best.

Method	<i>ResNet</i>	<i>ConvNext</i>	<i>EfficientNet</i>	<i>DeiT</i>	Mean
Saliency	0.025	0.032	0.008	0.038	0.025
Integrated Gradients	0.000	0.001	0.000	0.003	0.001
GradCAM	0.134	0.072	0.061	0.162	0.107
GradCAM++	0.184	0.055	0.050	0.044	0.083
SmoothGrad	0.023	0.000	0.010	0.004	0.009
WAM _{SG} (ours)	0.438	0.334	0.350	0.423	0.386
WAM _{IG} (ours)	<u>0.344</u>	<u>0.359</u>	<u>0.370</u>	<u>0.420</u>	<u>0.373</u>

Table 2: **Evaluation scores** of WAM and comparison with baselines on 400 audio samples from ESC-50 (fold 1). The column "ESC" indicates that the samples are unaltered. The column "+WN" indicates that the samples have 0 dB Gaussian white noise. We report the results with explanations generated from the wavelet coefficients of the waveform. **Bolded** results are the best and underlined values are the second best.

Method	Faithfulness (↑)		Insertion (↑)		Deletion (↓)		FF (↑)		Fid-In (↑)	
	ESC50	+WN	ESC50	+WN	ESC50	+WN	ESC50	+WN	ESC50	+WN
IntegratedGradients	0.264	0.310	0.267	0.312	0.047	0.045	0.207	0.207	0.220	0.225
GradCAM	0.072	0.073	0.274	0.274	0.201	0.201	0.137	0.135	0.517	0.542
Saliency	0.066	0.065	0.220	0.221	0.154	0.156	0.166	0.168	0.253	0.245
SmoothGrad	0.184	0.184	0.251	0.251	<u>0.067</u>	<u>0.067</u>	<u>0.193</u>	<u>0.194</u>	0.177	0.175
WAM _{SG} (ours)	<u>0.197</u>	<u>0.205</u>	0.449	0.452	0.252	0.246	0.132	0.130	0.718	0.690
WAM _{IG} (ours)	0.176	0.182	<u>0.436</u>	<u>0.442</u>	0.260	0.261	0.118	0.124	<u>0.652</u>	<u>0.657</u>

4.2 XAI PROPERTIES OF THE WAVELET ATTRIBUTION METHOD

Revisiting Meaningful Perturbation. In the seminal works by Fong & Vedaldi (2017) and Fong et al. (2019), a method is introduced to explain the most important parts of an image by optimizing a mask \mathbf{m} that partially occludes certain regions. The objective is twofold: (i) preserve the classification score, while (ii) remove as much of the image content as possible to isolate the most relevant features. However, optimizing in the pixel domain presents challenges in producing smooth masks (Fong et al., 2019), necessitating various regularization techniques, smoothing operations, and data augmentations to mitigate these issues. We revisit this framework by proposing to recast the problem in the wavelet domain as a more suitable space for optimization. The wavelet domain inherently captures spatial and spectral information, providing a natural structure for producing meaningful and interpretable solutions. Specifically, we solve the following optimization problem:

$$\mathbf{m}^* = \arg \min_{\mathbf{m} \in [0,1]^{|\mathcal{X}|}} \mathbf{f}_c(\mathcal{W}^{-1}(\mathbf{z} \odot \mathbf{m})) + \alpha \|\mathbf{m}\|_1,$$

where \mathbf{f}_c represents the classification score, \mathcal{W}^{-1} is the inverse wavelet transform, \mathbf{z} is the wavelet transform of the input signal, \odot denotes element-wise multiplication, and α controls the sparsity of the mask \mathbf{m} . To solve this problem, we initialize the mask as $\mathbf{m}_0 = \mathbf{1}$ (i.e., a mask that retains all coefficients) and iteratively update it using gradient descent:

$$\mathbf{m}_{i+1} = \mathbf{m}_i - \eta \nabla_{\mathbf{m}_i} (\mathbf{f}_c(\mathcal{W}^{-1}(\mathbf{z} \odot \mathbf{m}_i)) + \alpha \|\mathbf{m}_i\|_1),$$

where η represents the step size and the gradient is taken with respect to \mathbf{m}_i . The optimization process continues until convergence is achieved. We call the resulting image the *minimal* image. In practice, we employ the Nadam (Dozat, 2016) optimizer, which combines the benefits of Nesterov

acceleration and Adam optimization. Our approach consistently produces masks with controllable sparsity levels up to 90%, meaning that 90% of the wavelet coefficients are zeroed out, *while* maintaining a classification score comparable to or better than the original prediction. This high sparsity level suggests that the model’s decision may rely on a minimal subset of wavelet coefficients.

From an interpretability perspective, our method offers significant and novel insights. Traditional meaningful perturbation methods (Fong & Vedaldi, 2017) focus on spatial localization, identifying clusters of pixels that answer the question of *where* the important features are located. However, this spatial emphasis alone provides a limited understanding of the underlying data structure. In contrast, by operating in the wavelet domain, our method captures both the *what* – the relevant scales – and the *where* – their spatial locations. This dual information enriches the explanation by revealing the location and the nature of the features influencing the model’s decision.

Figure 3 illustrates that minimal images derived using WAM recover the texture bias of the vanilla ResNet models trained on ImageNet, highlighted by Geirhos et al. (2019). The examples demonstrate how the model relies heavily on texture information, which is effectively isolated through our wavelet-domain optimization.

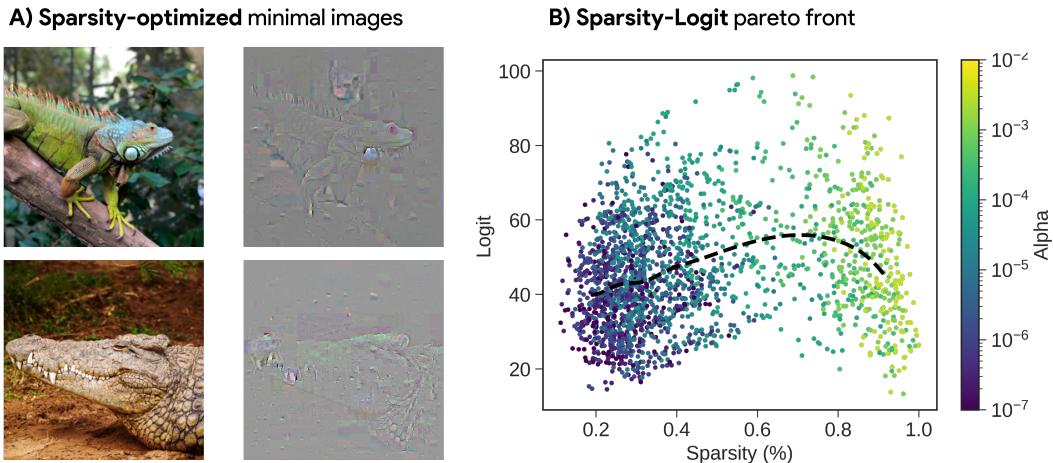


Figure 3: **A) Sparsity-optimized minimal images.** We revisit meaningful perturbation by optimizing the sparsity of the wavelet transform using masking, instead of optimizing the mask in pixel space. The displayed examples show that the resulting minimal images reveal the model’s reliance on textures. **B) Sparsity Pareto front.** As α increases, the sparsity of the wavelet coefficients increases (x-axis), but beyond a certain point, too much information is lost and the logit score drops to zero. However, we observe that many components can be removed before adversely affecting the model. Results are averaged across 1,000 images optimized for 500 steps, and for α ranging in $[0, 100]$ for each image.

By leveraging the wavelet domain, our approach addresses the challenges associated with pixel-space optimization and provides a more comprehensive understanding of the model’s behavior. This method generalizes across different data modalities and can be a valuable tool for interpreting complex neural networks in various applications.

Audio: post-hoc identification of relevant parts of the input audio. Figure 4 qualitatively illustrates an application of WAM for audio signals. Herein, we perform a noise experiment to add 0 dB white noise to a target audio to form the input audio. The model’s prediction does not alter after introducing the noise and thus the model is expected to still rely on parts of input audio coming from target audio for its decision. The interpretation audio in Figure 14 generated using top wavelet coefficients provides insights into the decision process and supports this hypothesis. In particular, it almost entirely filters out corruption audio and without requiring any training it also clearly emphasizes key parts of target audio. Similarly, we discuss in appendix C how WAM also retrieves the key parts of an audio signal that has been corrupted with another source (overlap experiment).

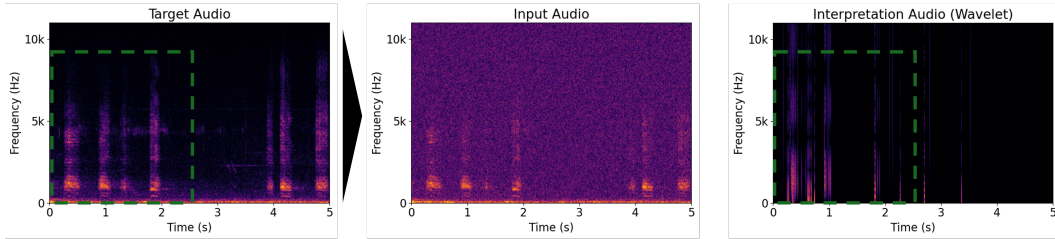


Figure 4: Qualitative illustration of WAM for audio via a Noise experiment. We add 0 dB white noise on the audio of the target class ('Crow') to form the input to the classifier. Interpretation audio reconstructed with important wavelet coefficients virtually eliminate noise, and also clearly emphasize parts of the target class audio (indicated with green box).

Explanations on voxels. We retrieve on voxels the same decomposition as for images or audios. Figure 5 highlights the significance of the edges at larger scales, whereas at smaller scales, the importance becomes increasingly concentrated at the center of the digit. To the best of our knowledge, WAM is the first method to show such decomposition on shapes.

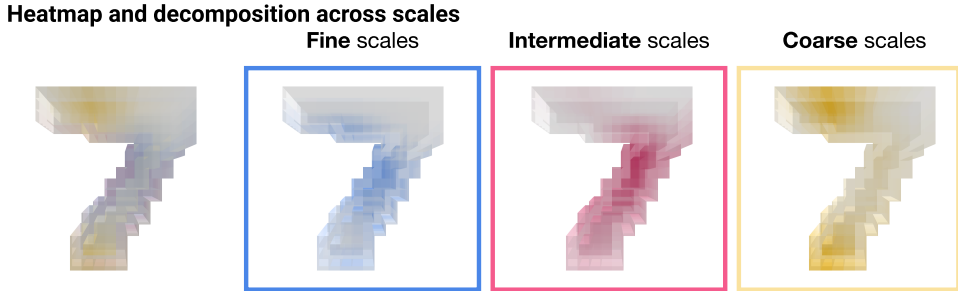


Figure 5: Decomposition of the different important scales on a voxel with the GradWCAM.

5 DISCUSSION

Conclusion. We have introduced a novel approach for feature attribution by computing explanations in the wavelet domain rather than the input domain, providing a framework applicable to audio, images, and shapes. This method shifts away from traditional pixel-based decompositions used in saliency mapping, offering more precise insights into model decisions by leveraging the wavelet domain’s ability to preserve inter-scale dependencies. This ensures that critical aspects like frequency and spatial structures are maintained, resulting in richer explanations compared to traditional feature attribution methods.

Our method, WAM, shows a strong ability to highlight essential audio components in noisy samples, isolate necessary shape and texture features for accurate predictions, and offer richer explanations for shape classification. Quantitatively, it achieves state-of-the-art results across both audio and image benchmarks.

Limitations & future works. Despite its advantages, the current method does not extend to 3D point cloud data, and for audio, the greedy extraction of important coefficients is unsuitable for generating listenable explanations. Future work could explore alternative wavelet decompositions, such as continuous or complex wavelets for audio explanations and graph wavelet transforms to handle unstructured point clouds. Additionally, our method could be applied to videos mathematically similar to the voxel data used in this work. We hope this approach will inspire further research into the properties of explanation domains, the wavelet domain being one such domain.

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A IMPLEMENTATIONAL DETAILS

A.1 EFFECT OF SMOOTHING AND INTEGRATION ON THE EXPLANATIONS

Figure 6 illustrates the improvement of the quality of the explanations by smoothing (third row) or integrating (fourth row) the gradients, compared to their raw values (second row). Smoothing follows Equation 2 and integration follows Equation 3.

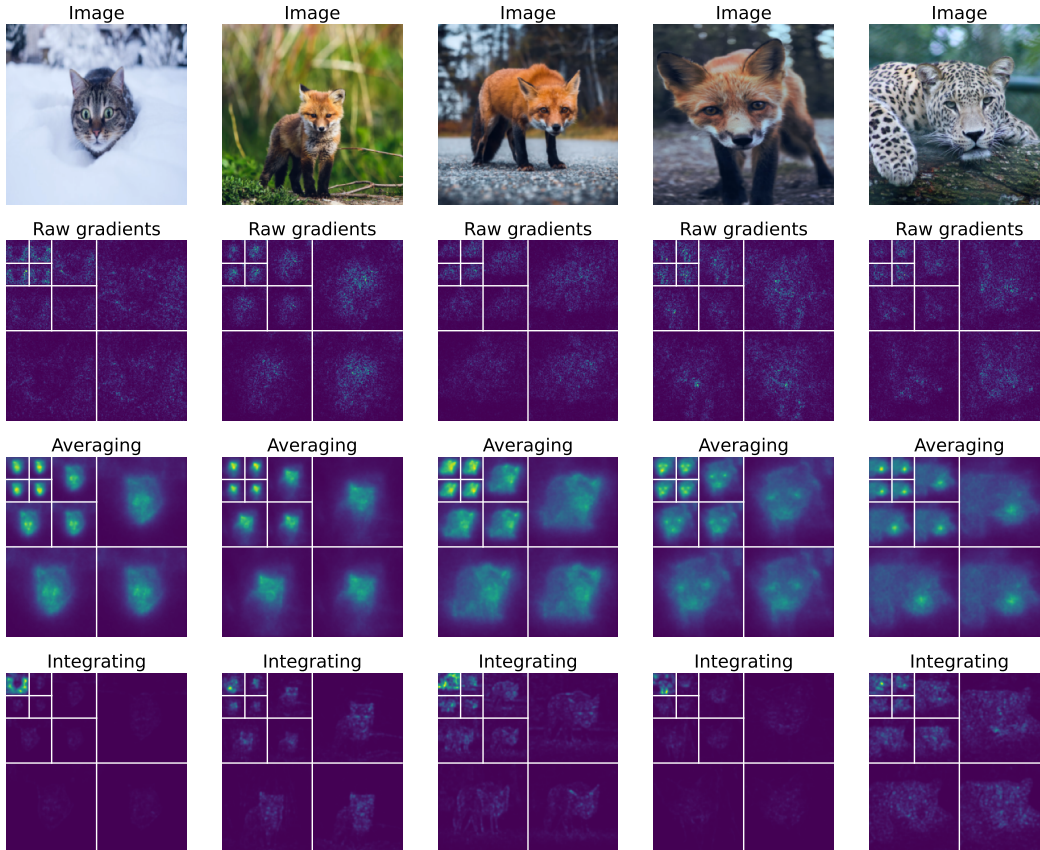


Figure 6: Effect of smoothing (3rd row) and integration (4th row) compared to the raw gradients (2nd row) when computing the WAM of the images (1st row). The explanations are depicted in the wavelet domain.

We can see that both methods display complementary properties regarding the explanation. WAM_{SG} enables to visualize highlights the important locations within scales, while WAM_{IG} emphasizes on see the relative importance of each scale.

A.2 BENCHMARK CONSTRUCTION

All benchmarks reported in this work were carried out on a server running Ubuntu 20.04.6 and on a single NVIDIA TITAN Xp GPU with 12 GB of VRAM with CUDA 12.5. The data to replicate the experiments can be downloaded in the repository accessible at this URL <https://doi.org/10.5281/zenodo.13873810>, and the source code is accessible from the Git repository.

Images. Our models’ parameterizations for benchmarking WAM on images are the following:

- ResNet: we consider the `resnet18` variant,
- EfficientNet: we consider the `tf_efficientnet_b0.ns_jft_in1k` variant,
- ConvNext: we consider the `convnext_small.fb_in22k_ft_in1k_384` variant,

- DeiT: we consider the `deit_tiny_patch16_224.fb.in1k`

All models are retrieved from the PyTorch Image Models (Wightman, 2019) repository. We load the model with the pre-trained weights and directly evaluate them on the validation set of ImageNet. We implement the SmoothGrad, GradCAM, and GradCAM Plus Plus methods ourselves and use the Captum library (Kokhlikyan et al., 2020) for implementing the Intergrated Gradients and the Saliency methods.

Audio. For audio, we use the same technical infrastructure to evaluate our method. We use the CNN classification model of Kumar et al. (2018) as our black-box model to explain. We consider a single model as alternative models (Huang & Leanos, 2018; Wilkinghoff, 2021; Lopez-Meyer et al., 2021) are only variations around the same topology. We add 0 dB white noise to the ESC-50 samples using the pseudocode displayed in Figure 7.

```

1 # Input: audio (input audio signal, array of int16)
2 # Output: noisy_audio (audio with added Gaussian noise, array of int16)
3
4 def add_gaussian_noise(audio):
5     # Convert the audio to float32 for safe computation
6     audio_float = convert_to_float32(audio)
7
8     # Calculate RMS (Root Mean Square) of the audio signal
9     rms_signal = sqrt(mean(audio_float ** 2))
10
11    # Generate Gaussian noise
12    noise = random_normal_distribution(mean=0,
13                                     std=1,
14                                     shape=audio_float.shape)
15
16    # Calculate RMS of the generated noise
17    rms_noise = sqrt(mean(noise ** 2))
18
19    # Scale noise to have the same RMS as the audio signal
20    noise = noise * (rms_signal / rms_noise)
21
22    # Add noise to the audio signal
23    noisy_audio_float = audio_float + noise
24
25    # Clip the noisy audio to ensure it stays within the int16 range
26    noisy_audio_clipped = clip(noisy_audio_float, -32768, 32767)
27
28    # Convert the clipped noisy audio back to int16
29    noisy_audio = convert_to_int16(noisy_audio_clipped)
30
31    return noisy_audio

```

Figure 7: Pseudo-code for adding Gaussian noise to audio

B COMPLEMENTS ON THE QUANTITATIVE EVALUATION

B.1 INSERTION AND DELETION

Insertion and deletion are two evaluation metrics proposed by Petsiuk et al. (2018). These metrics are "area-under-curve" (AUC) metrics, which report the change of in the predicted probability for the image class when inserting (resp. removing) meaningful information highlighted by the attribution method. Petsiuk et al. (2018) initially defined this metric for images, where the important features

correspond to pixels. We expand this metric to wavelet coefficients, thus enabling a computation of the Insertion and the Deletion for any modality.

Both metrics consider an input in a baseline state. Insertion consists in adding the most important features identified by the attribution method. Formally, at step k with a subset u_k of important features (which correspond in our case in wavelet coefficients) at step k ,

$$\text{Insertion}^{(k)} = \mathbf{f}(\mathbf{x}_{[\mathbf{x}_{-u_k}=\mathbf{x}_0]}), \quad (5)$$

where $\mathbf{f}(\cdot)$ is the predicted probability and $-u$ denotes the complementary set of u . We add features by decreasing order of importance and for $k_1 \leq k_2, u_{k_1} \subseteq u_{k_2}$, i.e., we gradually add more and more features until we eventually recover the full input \mathbf{x} .

The deletion performs the opposite operation where we start from a plain input with all variables and we gradually set features in the baseline state \mathbf{x}_0 , from the most important to the less important. We have

$$\text{Deletion}^{(k)} = \mathbf{f}(\mathbf{x}_{[\mathbf{x}_{u_k}=\mathbf{x}_0]}). \quad (6)$$

Finally, for the insertion and the deletion, we measure the AUC, which is comprised between 0 and 1. Given K steps, the Insertion score of the feature attribution γ for the model \mathbf{f} is

$$\text{Ins}(\mathbf{f}, \gamma) = \sum_{k=1}^K \text{Insertion}^{(k)} \Delta_k = \sum_{k=1}^K \mathbf{f}(\mathbf{x}_{[\mathbf{x}_{-u_k}=\mathbf{x}_0]}) \Delta_k, \quad (7)$$

where Δ_k is the width between two subintervals. The computation is analogous for $\text{Del}(\mathbf{f}, \gamma)$.

If the attribution method picks relevant features, then only including them (resp. only removing them) should result in a large increase (resp. large decrease) in the predicted probability. Therefore, the AUC should be close to 1 for the insertion and close to 0 for the deletion. We set the baseline to $\mathbf{x}_0 = 0$.

B.2 COMPLEMENTARY RESULTS

B.2.1 DEFINITIONS

μ -Fidelity. The μ -Fidelity is a correlation metric. It measures the correlation between the decrease of the predicted probabilities when features are in a baseline state and the importance of these features. We have

$$\mu\text{-Fidelity} = \text{Corr}_{\substack{u \subseteq \{1, \dots, K\}, \\ |u|=d}} \left(\sum_{i \in u} \mathbf{g}(\mathbf{x}_i), \mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{x}_{\mathbf{x}_u=\mathbf{x}_0}) \right), \quad (8)$$

where \mathbf{g} is the explanation function (i.e., the explanation method), which quantifies the importance of the set of features u .

Faithfulness on Spectra. The Faithfulness on Spectra (FF, Parekh et al., 2022) measures how important is the generated interpretation for a classifier. The metric is calculated by measuring the drop in class-specific logit value $\mathbf{f}(\mathbf{x})_c$, when the masked out portion of the interpretation mask \mathbf{m}_γ is input to the classifier. This amounts to calculating,

$$\text{FF}_x = \mathbf{f}(\mathbf{x})_c - \mathbf{f}(\mathbf{x} \odot (\mathbf{1} - \mathbf{m}_\gamma))_c. \quad (9)$$

It should be noted that this strategy to simulate removal may introduce artifacts in the input that can affect the classifier's output unpredictably. Also, interpretations on samples with poor fidelity can lead to negative FF_x . These observations point to this metric's potential instability and outlying values. Thus, we report the final faithfulness of the system as the median of FF_x over the test set, denoted by FF. A positive FF would signify that interpretations are faithful to the classifier.

Input Fidelity. The Input Fidelity (**Fid-In**, Paissan et al., 2023) measures if the classifier outputs the same class prediction on the masked-in portion of the input image. It is defined as,

$$\text{Fid-In} = \frac{1}{n} \sum_{i=1}^n \mathbb{I} \left[\arg \max_c \mathbf{f}(x_i)_c = \arg \max_c \mathbf{f}_c(x_i \odot \mathbf{m}_\gamma) \right], \quad (10)$$

where \mathbb{I} denotes the indicator function and, again, larger values are better.

B.2.2 RESULTS

Images. Table 3 reports the results using the μ -Fidelity (Bhatt et al., 2021) compared to other methods. We can see that the performance measured by the μ -Fidelity is more in line between our method and the existing approaches.

Table 3: μ -Fidelity (Bhatt et al., 2021) score obtained on 1,000 images from the validation set of ImageNet and for different model architectures. Higher is better. **Bolded** results are the best and underlined values are the second best.

Method	<i>ResNet-18</i>	<i>ConvNext</i>	<i>EfficientNet</i>	<i>DeiT</i>	Mean
Saliency	0.154	0.186	0.180	0.195	0.179
Integrated Gradients	0.228	0.223	0.219	0.241	0.228
GradCAM	0.141	<u>0.216</u>	0.149	0.151	0.164
GradCAM ++	0.135	0.212	0.141	0.222	0.178
SmoothGrad	<u>0.220</u>	0.227	0.211	<u>0.230</u>	<u>0.222</u>
WAM _{SG} (ours)	0.215	0.208	<u>0.213</u>	0.216	0.213
WAM _{IG} (ours)	0.170	0.166	0.165	0.182	0.171

Table 4 reports the Insertion and Deletion (Petsiuk et al., 2018) scores compared to other attribution methods. We can see that the good results reported in Table 1 are mostly driven by our method’s very good results in Insertion. We can see that WAM systematically outperforms the other methods in both Insertion and Deletion.

Table 4: **Insertion** and **Deletion** (Petsiuk et al., 2018) scores obtained on 1,000 images from the validation set of ImageNet and for different model architectures. For insertion, higher is better, and for deletion, lower is better. **Bolded** results are the best and underlined values are the second best.

	Method	<i>ResNet</i>	<i>ConvNext</i>	<i>EfficientNet</i>	<i>DeiT</i>	Mean
Insertion (\uparrow)	Saliency	0.134	0.381	0.148	0.194	0.214
	Integrated Gradients	0.087	0.305	0.113	0.095	0.150
	GradCAM	0.413	0.495	0.364	0.352	0.406
	GradCAM ++	<u>0.452</u>	0.562	0.350	0.313	0.419
	SmoothGrad	0.106	0.253	0.129	0.108	0.149
	WAM _{SG} (ours)	0.557	0.606	0.447	0.546	0.539
	WAM _{IG} (ours)	0.422	<u>0.557</u>	<u>0.419</u>	<u>0.492</u>	<u>0.473</u>
Deletion (\downarrow)	Saliency	0.109	0.349	0.140	0.156	0.189
	Integrated Gradients	0.087	0.304	0.113	<u>0.092</u>	<u>0.149</u>
	GradCAM	0.279	0.423	0.303	0.190	0.299
	GradCAM ++	0.268	0.507	0.300	0.269	0.336
	SmoothGrad	<u>0.083</u>	<u>0.253</u>	0.119	0.104	0.140
	WAM _{SG} (ours)	0.119	0.272	<u>0.097</u>	0.123	0.153
	WAM _{IG} (ours)	0.078	0.198	0.049	0.072	0.099

Audio. Table 5 displays the results of the quantitative evaluation of WAM when the metrics are computed from the mel-spectrogram. We can see that we still achieve state-of-the-art performance for the Faithfulness of spectra and are in line with other metrics for the Input Fidelity. These results highlight the added value of computing the metrics from the wavelet coefficients rather than from the mel-spectrogram.

Table 5: **Evaluation scores** of WAM and comparison with baselines on 400 audios from the first fold of ESC-50. The column "ESC" indicates that the samples are unaltered. The column "+WN" indicates that the samples have 0 dB Gaussian white noise. We report the results with explanations generated from the mel-spectrogram of the waveform. **Bolded** results are the best and underlined values are the second best.

Method	Faithfulness (\uparrow)		Insertion (\uparrow)		Deletion (\downarrow)		FF (\uparrow)		Fid-In (\uparrow)	
	ESC50	+WN	ESC50	+WN	ESC50	+WN	ESC50	+WN	ESC50	+WN
IntegratedGradients	0.264	0.310	0.267	0.312	0.047	0.045	0.207	0.207	0.220	0.225
GradCAM	0.072	0.073	0.274	<u>0.274</u>	0.201	0.201	0.137	0.135	0.517	0.542
Saliency	0.066	0.065	0.220	<u>0.221</u>	0.154	0.156	0.166	0.168	<u>0.253</u>	<u>0.245</u>
SmoothGrad	<u>0.184</u>	<u>0.184</u>	0.251	0.251	<u>0.067</u>	<u>0.067</u>	<u>0.193</u>	<u>0.194</u>	<u>0.177</u>	<u>0.175</u>
WAM _{SG} (ours)	0.009	0.007	0.169	0.166	0.159	0.161	0.152	0.149	0.117	0.122
WAM _{IG} (ours)	0.000	0.004	0.168	0.171	0.168	0.167	0.149	0.149	0.105	0.128

B.3 RANDOMIZATION CHECK

Movitation. The sanity checks introduced by Adebayo et al. (2018) aim at assessing whether an explanation depends on the model’s parameters and the input labels. These tests aim to assess the faithfulness of an explanation beyond visual evaluation. The randomization test evaluates whether an explanation depends on the model’s parameters. Parameters have a strong effect on a model’s performance. Therefore, for a saliency method to be useful for debugging or analyzing a model, it should be sensitive to its parameters. Adebayo et al. (2018) proposed different methods to randomize the model parameters. One particularly interesting implementation is the “cascading” randomization, in which the weights are randomized from the top to the bottom layers.

Method and results. We compute WAM for our 1,000 ImageNet validation samples for a set of increasingly randomized models. A randomized layer is a layer that we reset at its initial value. We consider a ResNet-18 and randomize its layers from the shallowest `conv1` to the deepest `fc`. We then compute the rank correlation (or Pearson correlation coefficient, Pearson, 1896) between the WAM of the original, fully-trained model (labeled `orig`) and the randomized models (labeled by the name of the layer until which they are randomized).

Figure 8 presents the results. The dotted line represents the average rank correlation across the 1,000 images, and the intervals represent the 95% confidence intervals. We can see that the correlation between WAMs significantly decreases as the randomization increases, thereby showing that WAM is sensitive to the model’s parameters. The lower decrease that we observe for WAM_{IG} compared to WAM_{SG} comes from the fact that WAM_{IG} reflects more the inter-scales distribution of the importance than WAM_{SG} does. As pointed out by Rahaman et al. (2019) and Yin et al. (2019), even random models exhibit a spectral bias, i.e., a natural tendency to favor lower frequencies over higher ones, which translates here into the fact of naturally putting more importance on coarser scales rather than finer scales, no matter the depth of the randomization.

Illustrations. Figure 9, Figure 21, Figure 22, Figure 23 display qualitative examples to further assess how WAM varies when the randomization depth of the model increases. We can see that the explanation is no longer informative when the model is randomized.

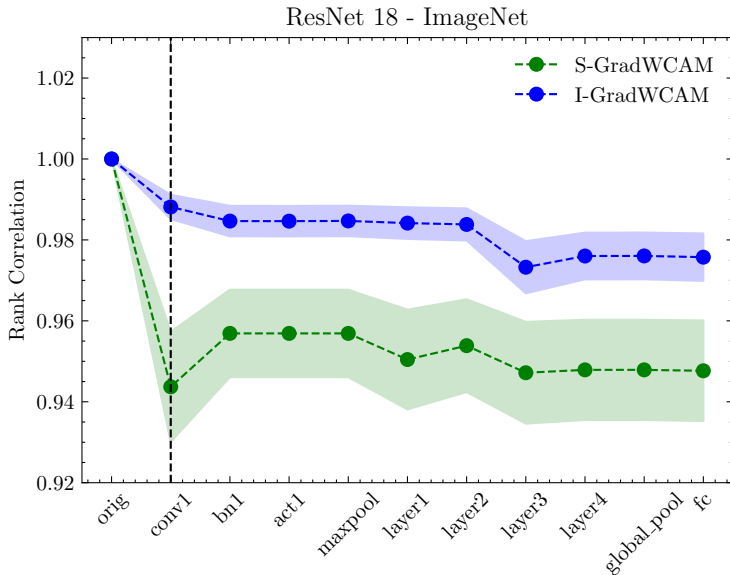


Figure 8: **Cascading randomization** of WAM for explaining a ResNet-18 on ImageNet. The y axis indicates the rank correlation between the original explanation and the explanation derived for randomization up that layer. The rank correlation is averaged over 1,000 ImageNet validation images.

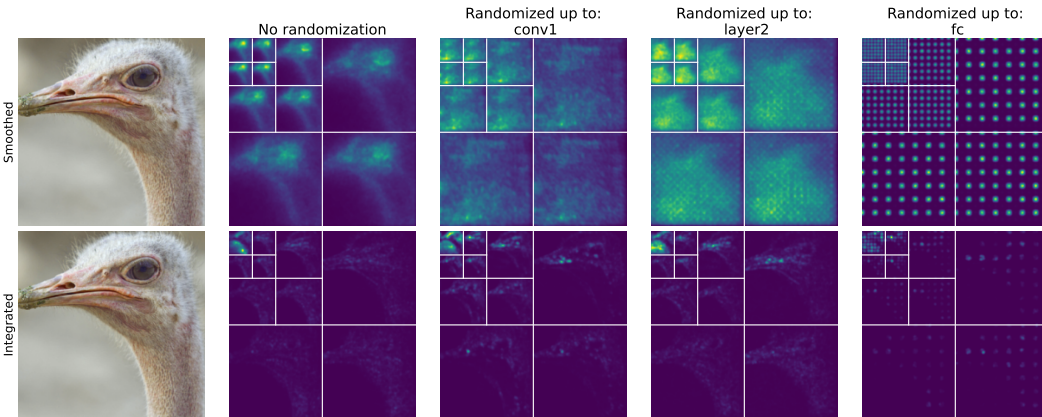


Figure 9: Illustration of the randomization test as a cascade of randomizations of the layers of the classifier. From left to right shows the explanation from WAM for an increasingly randomized ResNet-18.

C ADDITIONAL RESULTS

C.1 FREQUENCY-CENTRIC PERSPECTIVES ON MODEL ROBUSTNESS

Scales in the wavelet domain correspond to dyadic frequency ranges in the Fourier domain. Several works documented a correlation between the reliance on low frequency to make predictions and the robustness of the model (Zhang et al., 2022; Chen et al., 2022; Wang et al., 2020). We can leverage WAM to characterize a model’s robustness, thus connecting feature attribution and robustness. On Figure 10, we evaluate the reliance on the different scales by summing the importance of each component within each scale. We average this importance over 1,000 images and thus obtain the average importance of each scale for a model’s prediction. We compare a vanilla ResNet-50 (He et al., 2016) with three adversarially robust models : ADV (Madry et al., 2018), ADV-Fast (Wong et al., 2020) and ADV-Free (Shafahi et al., 2019). We can see that adversarially robust models rely

more on the coarsest scale (leftmost bars on Figure 10) than the vanilla ResNet-50. On the other hand, they rely less on the finest scales (i.e., the highest frequencies, corresponding to the rightmost bars on Figure 10), thus backing the existing results established in the Fourier domain. Therefore, WAM can be used to characterize the robustness of a model.

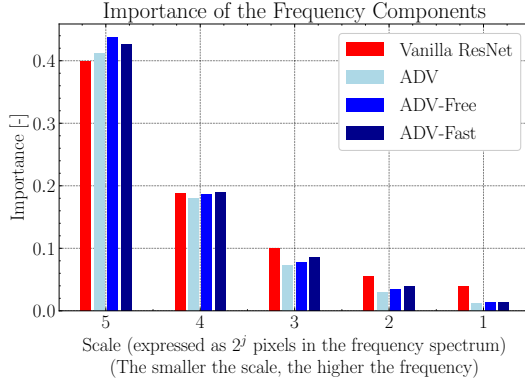


Figure 10: Assessment of model’s robustness with WAM.

C.2 ADDITIONAL RESULTS ON THE MINIMAL IMAGES

Complements on “optimal” minimal images. Figure 11 illustrates the effect of the parameter α on the sparsity of the minimal images. We can see that the stronger α , the sparser the image, but at the expense of a higher logit value. We can see that the first components of the image that disappear are the background, then the colors and eventually the shape of the target class.



Figure 11: Effect of varying values of α on the sparsity of the minimal images.

Static minimal images. Another method used to derive minimal images is to directly sort the gradient coefficients obtained with WAM and reconstruct the original image using decreasingly important coefficients.

We referred to the “insertion” quantile the threshold value after which the prediction of the model was correct, meaning that the partially reconstructed image contained enough information. We also considered the original image for completeness and removed gradually important coefficients. It turned out that both quantiles coincided, thus highlighting the fact that WAM identifies the necessary and sufficient amount of information for a correct prediction.

Figure 12 displays an example of a sufficient image extracted using WAM. We can see that the background information is unnecessary, contrary to details around the fox’s ear and eyes. Reconstruction artifacts are caused by the fact that we independently ranked the coefficients across the three color channels.

Figure 13 presents illustrations of minimal images obtained using WAM. We can see that the background can be filtered out, while foreground details are essential. The minimal images have been obtained using information solely from WAM.

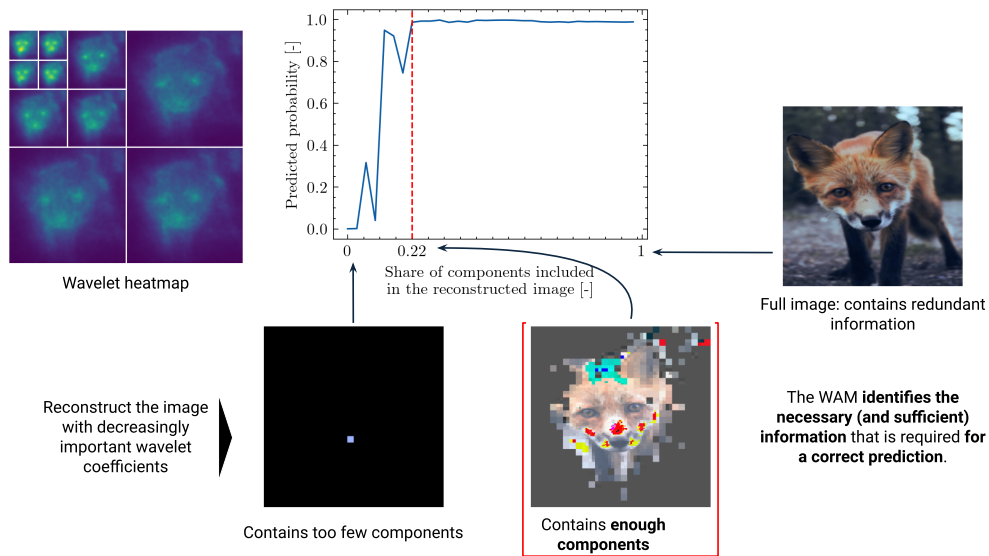


Figure 12: Extraction of the necessary and sufficient information for a correct prediction with WAM. In the example, we can see that only 22% of the components are necessary, the remaining being redundant.

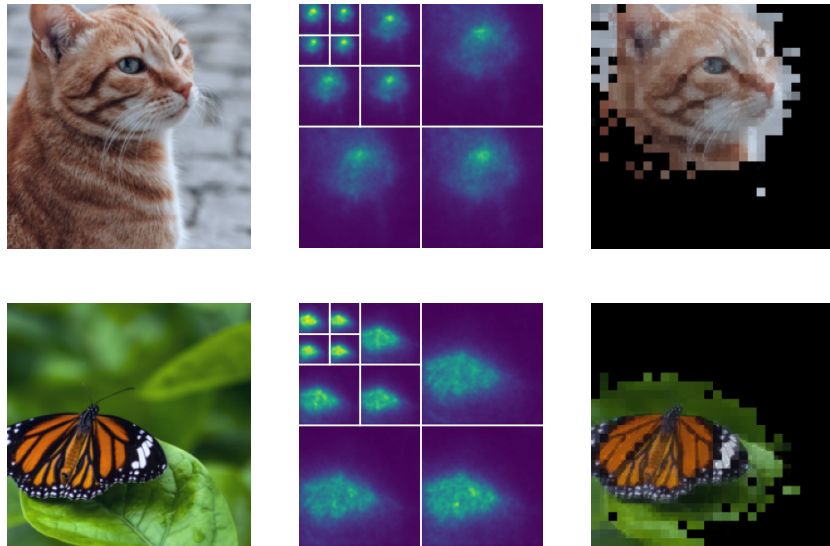


Figure 13: Example of minimal images (rightmost column), which contain enough information to correctly predict the image label. Compared to the input image (leftmost column), the background can be disregarded, but high-resolution details are necessary on the animals. The column in the center plots the wavelet heatmap obtained with WAM.

C.3 OVERLAP EXPERIMENT ON AUDIO CLASSIFICATION

Figure 14 illustrates that WAM is able to filter relevant parts corrupted or mixed audio signals. In addition, it highlights the key part of the target signal without requiring any training. Figure 14 qualitatively illustrates application of WAM for audio signals. Herein, we perform an overlap experiment to mix a corrupting audio with a target audio to form the input audio. The model's prediction does not alter after introducing the corruption, and thus, the model is expected to still rely on parts of input audio coming from the target audio for its decision. The interpretation audio in Figure 14

generated using top wavelet coefficients provides insights into the decision process and supports this hypothesis. In particular, it almost entirely filters out corruption audio, and without requiring any training, it also clearly emphasizes key parts of target audio.

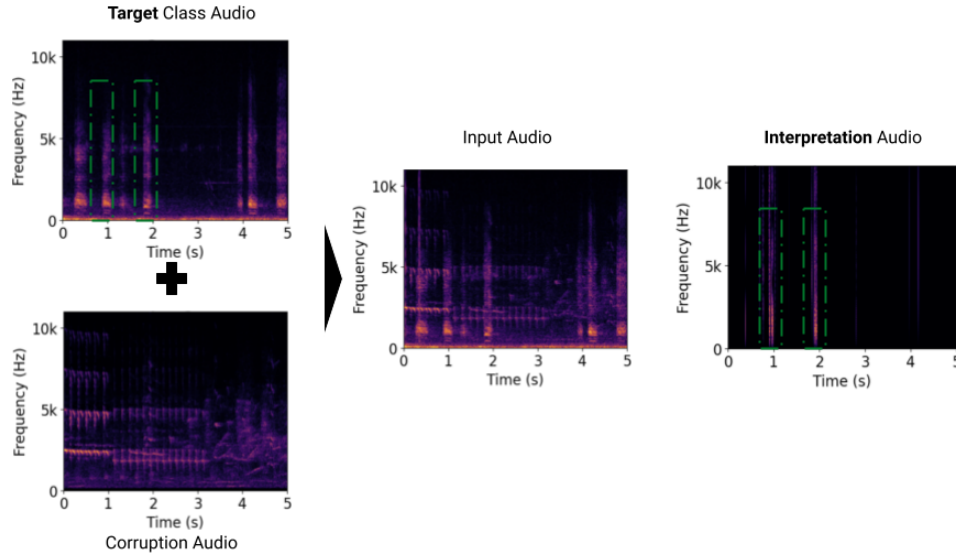


Figure 14: Qualitative illustration of WAM for audio via an Overlap experiment. The audio of the target class ('Crow') is mixed with a corrupting audio ('Chirping birds') to form the input to the classifier. Interpretation audio reconstructed with important wavelet coefficients virtually eliminate signal from the corrupting audio, and also clearly emphasize parts of the target class audio (indicated with green boxes).

C.4 ADDITIONAL PLOTS

Additional illustrations on images. Figure 15 presents additional illustrations of WAM for images. We illustrate the important coefficients (using the IntegratedGradients variant for smoothing the gradients) in the wavelet domain (second column) as a heatmap on the original image (third column), and we show the important components on the image, i.e., what the model needs to see based on the WAM on the fourth column.

Additional minimal images. Figure 16 presents additional examples of minimal images. We can see that the color information does not appear as important for maximizing the model's prediction. On the other hand, the texture and edge information are essential. It would be interesting to replicate this method on a shape-biased model such as those proposed by Chen et al. (2020); Geirhos et al. (2019) to see whether the behavior remains the same or not.

Additional explanations on shapes. Figure 17, Figure 18, Figure 19 and Figure 20 present additional visualization on shapes.

Additional randomization checks. Figure 21, Figure 22, and Figure 23 present additional examples of randomization checks.

Additional audio overlap experiments. Figure 24 and Figure 25 present additional examples of overlap experiments on waveforms.

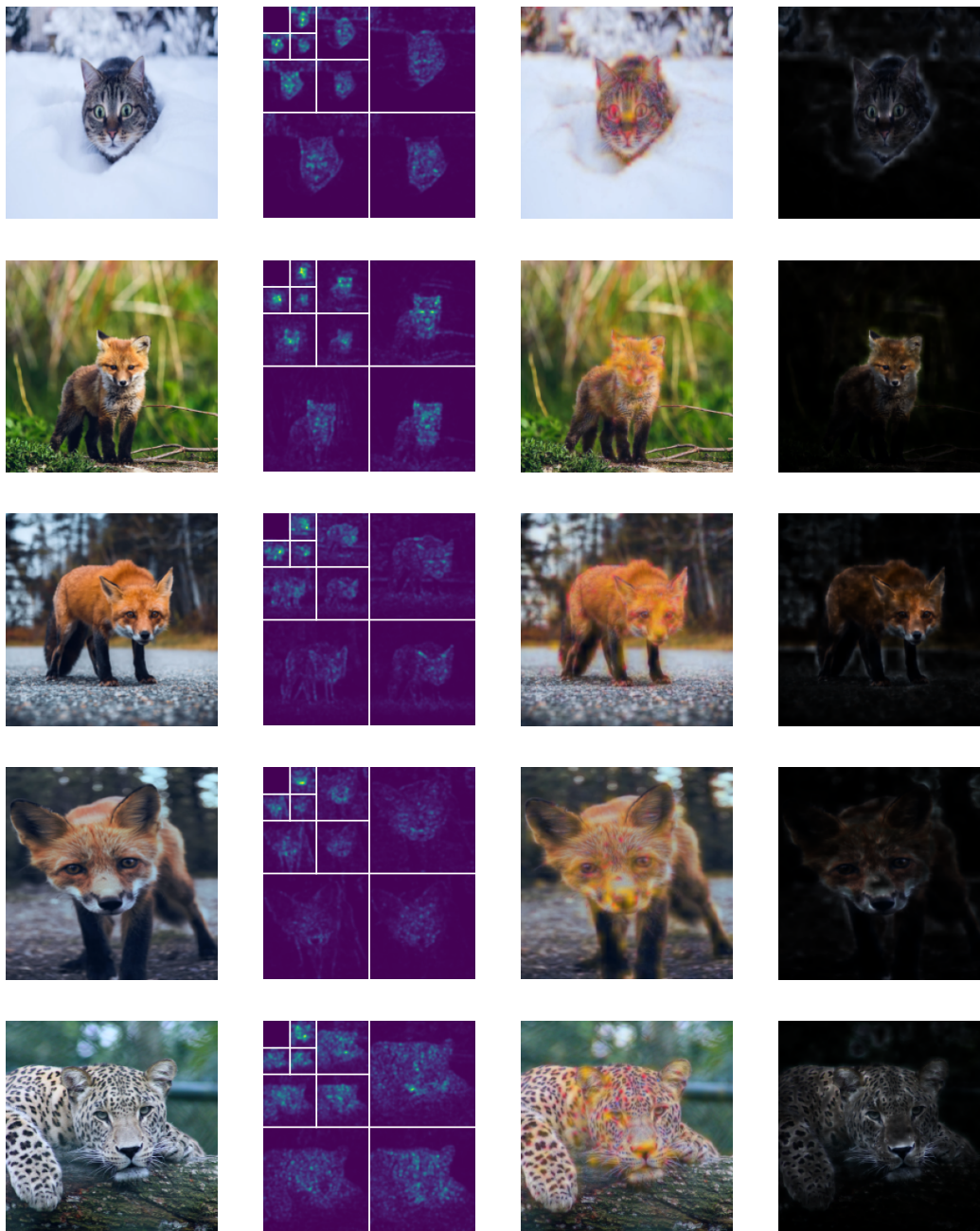


Figure 15: Additional visualizations of WAM for images. First column: original image. Second column: WAM in the wavelet domain. Third column: heatmap in the pixel domain. Fourth column: Filtered image illustrating the image regions that need to be well defined for the model to predict the image label correctly.

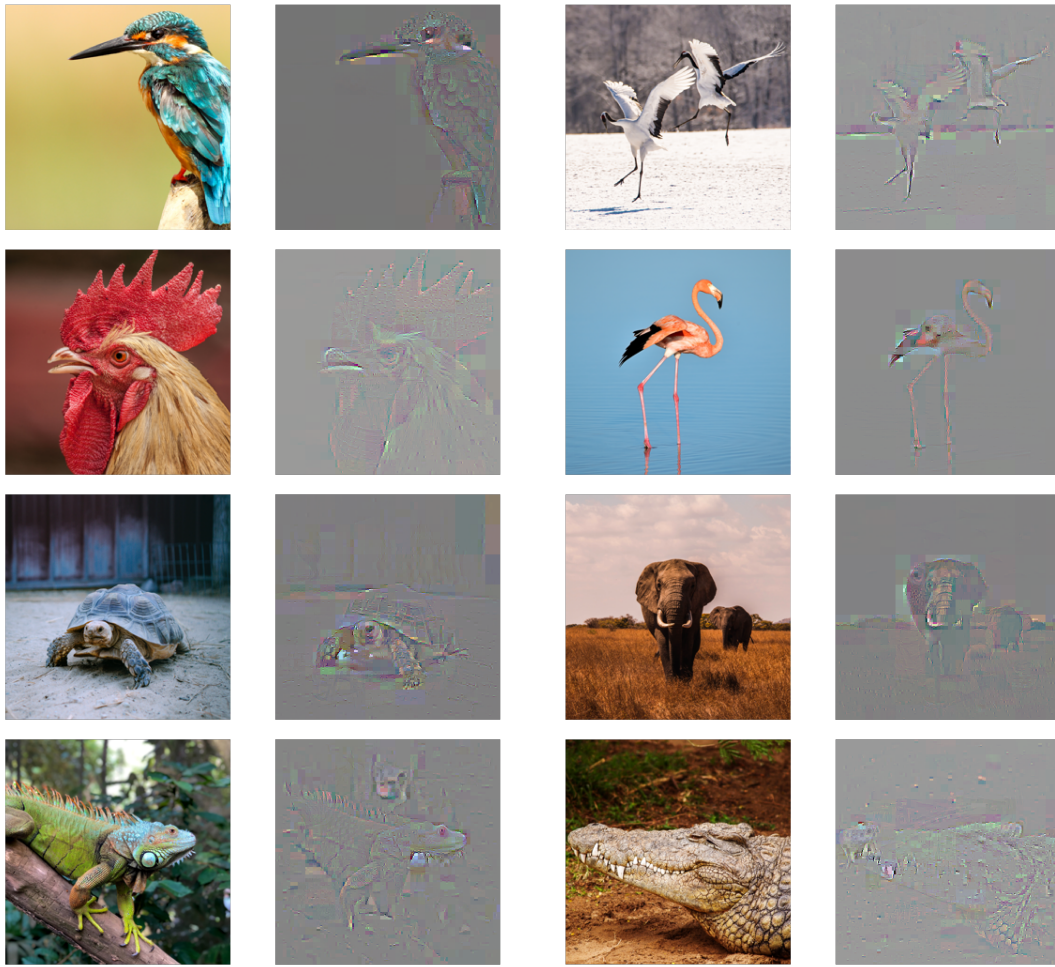


Figure 16: Additional examples of minimal images.

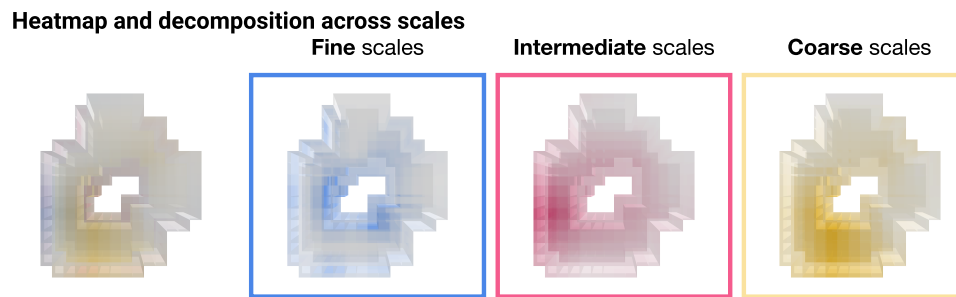


Figure 17: Decomposition of the different important scales on a voxel with WAM.

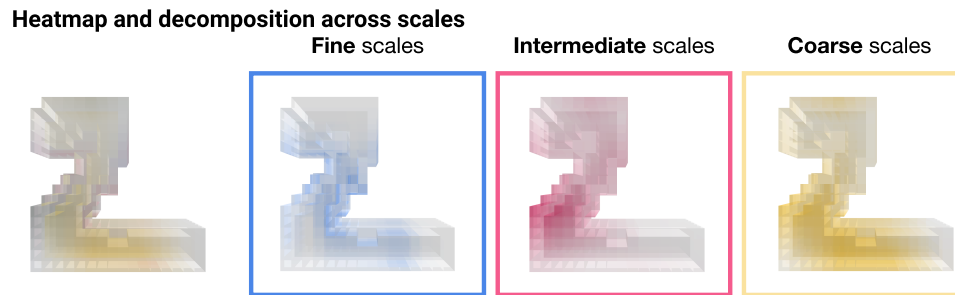


Figure 18: Decomposition of the different important scales on a voxel with WAM.

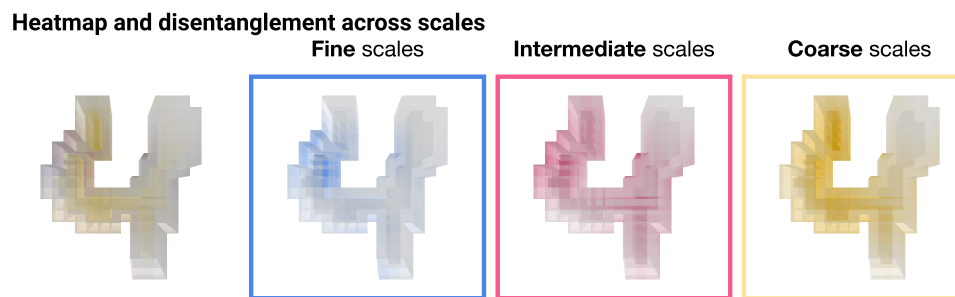


Figure 19: Decomposition of the different important scales on a voxel with WAM.

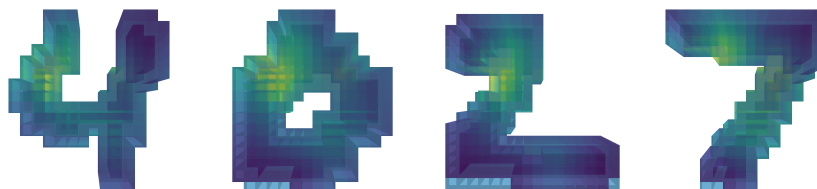


Figure 20: Heatmaps combining the importance at different scales on different voxels from 3D MNIST.

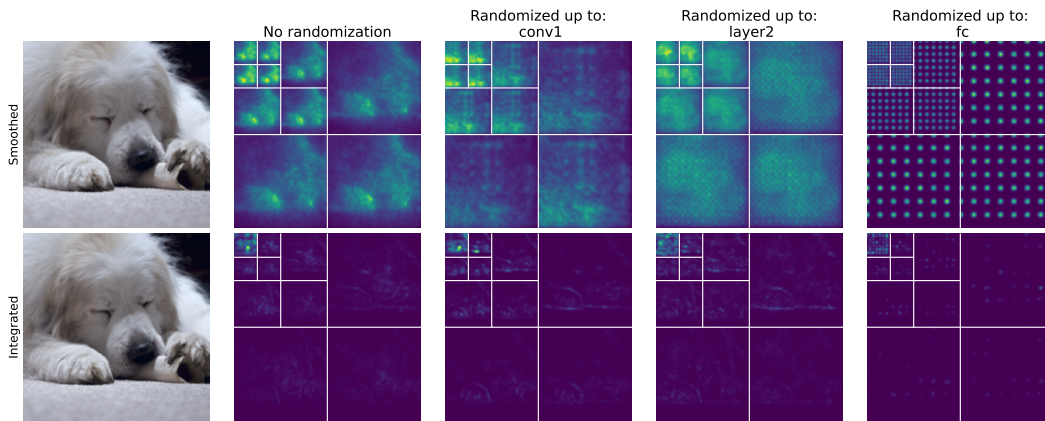


Figure 21: Illustration of the randomization test as a cascade of randomizations of the layers of the classifier. From left to right shows the explanation from the WAM for an increasingly randomized ResNet-18.

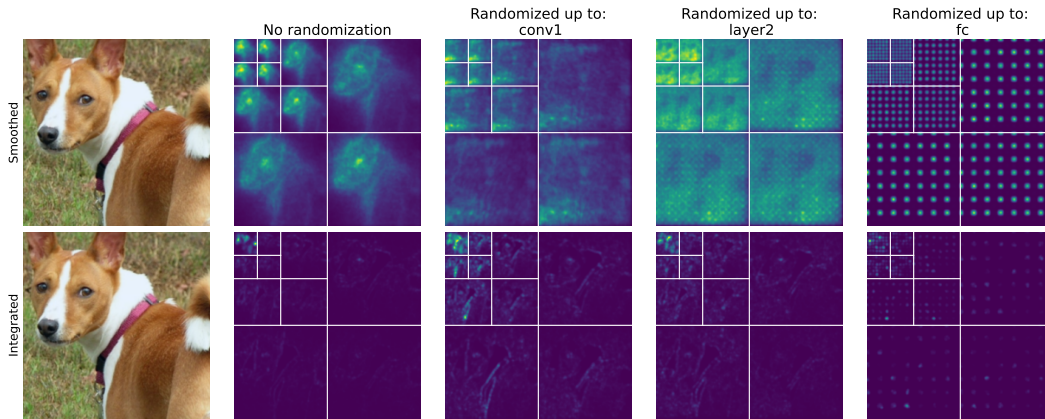


Figure 22: Illustration of the randomization test as a cascade of randomizations of the layers of the classifier. From left to right shows the explanation from the WAM for an increasingly randomized ResNet-18.

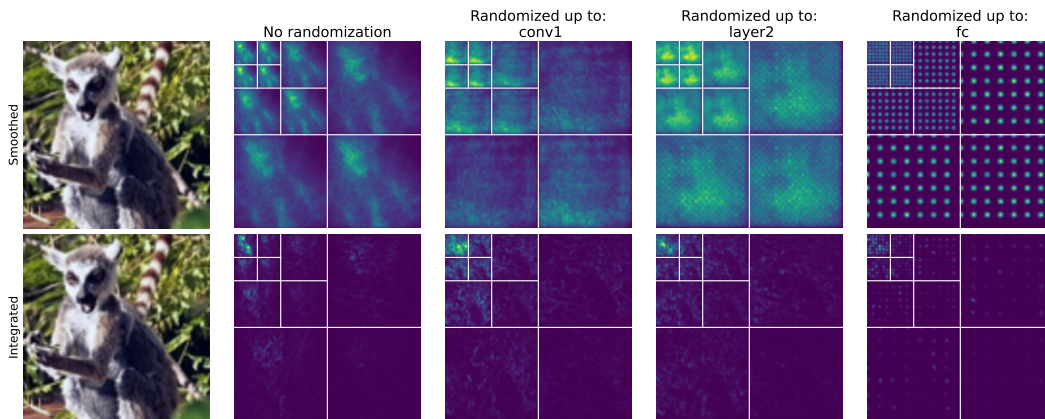


Figure 23: Illustration of the randomization test as a cascade of randomizations of the layers of the classifier. From left to right shows the explanation from the WAM for an increasingly randomized ResNet-18.

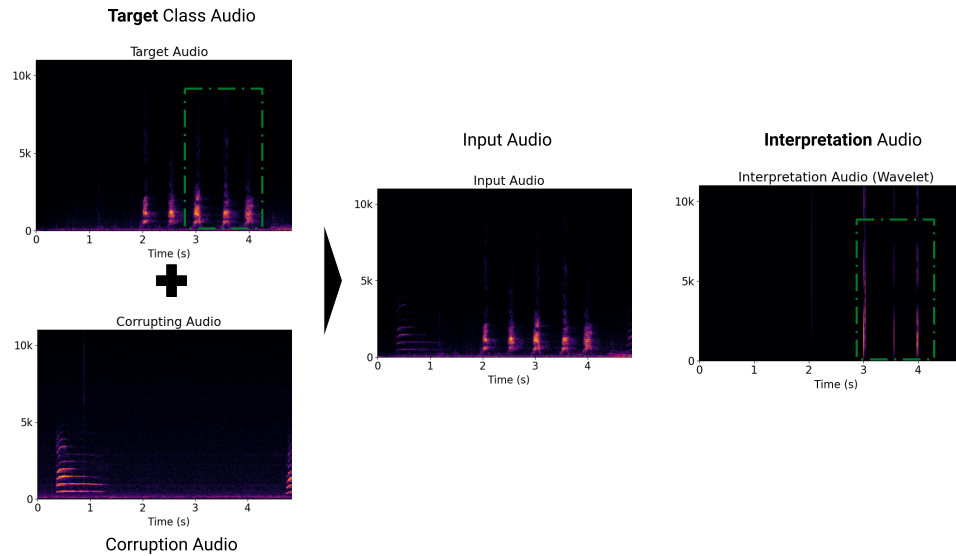


Figure 24: Qualitative illustration of WAM for audio via an Overlap experiment. The audio of the target class ('Dog') is mixed with a corrupting audio ('Cat') to form the input to the classifier. Interpretation audio reconstructed with important wavelet coefficients virtually eliminates signal from the corrupting audio and clearly emphasizes parts of the target class audio (indicated with green boxes).

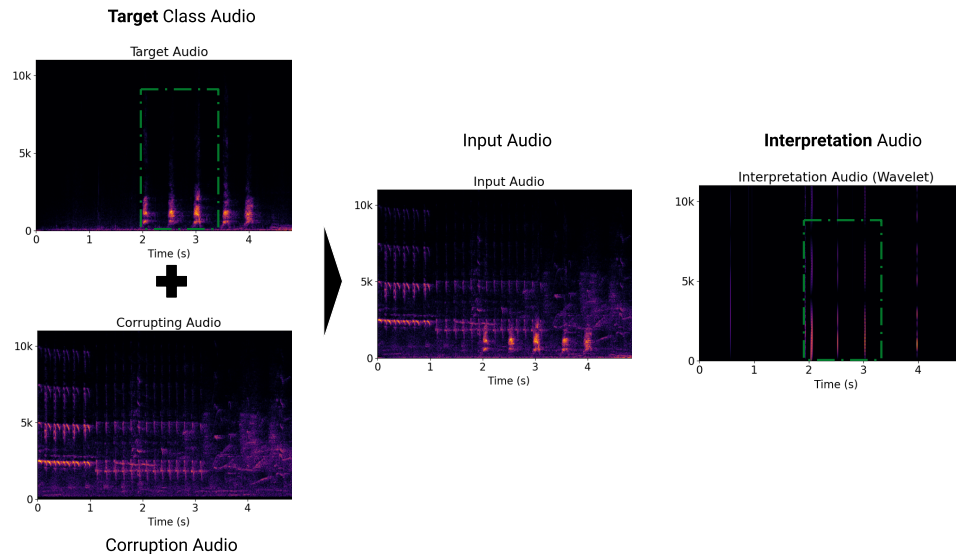


Figure 25: Qualitative illustration of WAM for audio via an Overlap experiment. The audio of the target class ('Dog') is mixed with a corrupting audio ('Rooster') to form the input to the classifier. Interpretation audio reconstructed with important wavelet coefficients virtually eliminates signal from the corrupting audio and clearly emphasizes parts of the target class audio (indicated with green boxes).