Accounting for local morphological fluctuations in the prediction of the transverse elastic behaviour of UD composites

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ACCOUNTING FOR LOCAL MORPHOLOGICAL FLUCTUATIONS IN THE PREDICTION OF THE TRANSVERSE ELASTIC BEHAVIOUR OF UD COMPOSITES

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Abstract: *While* unidirectional (UD) *materials are used for their excellent stiffness and strength in the fibre direction, they suffer from poor transverse properties. In the case of complex loadings, the first damage often appears in the transverse directions. While obtaining longitudinal properties is not a problem, transverse properties are still very difficult to measure reliably experimentally and the use of predictive models is often necessary. The main difficulty lies in taking into account the local morphological fluctuations which greatly influence the transverse behaviour. Using a recently developed model based on a Generalized Self-Consistent Scheme (GSCS) coupled with a Morphologically Representative Pattern (MRP), we propose to use the analytical solutions obtained to predict the effective transverse properties of different microstructural configurations. In a two-phase context, the robustness of the model is verified for phase contrasts up to 10^5, reinforcement volume fractions up to 70% or an incompressible matrix, which defeats many analytical models.*

Keywords: Micromechanical model; Homogenization; Interphase; Porosity; Hybrid

1. Introduction

On all the cross-sections in Figure 1, the phases involved are disconcertingly simple: fibres are perfectly aligned cylinders of revolution that the percolating matrix joins together.

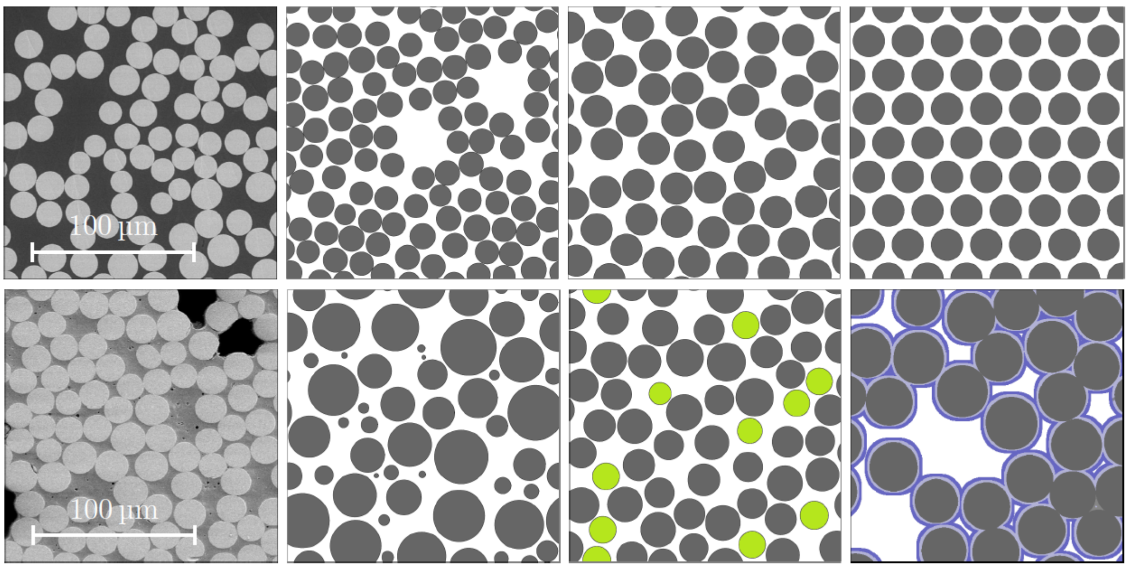

Figure 1. Cross-sections of real (with scale bars) and virtual unidirectional composite materials.

The UD discussed in this paper is the "elementary brick" used to build more complex composite structures. Whether it is a laminated composite, a woven-reinforced composite, or even a discontinuous-reinforced composite, these three configurations all result from the geometric assembly of UD "bricks". The high anisotropy of UD is both a strength and a weakness. It is first of all an asset because the aligned arrangement of the fibres allows an optimal response to any longitudinal load. But it is also a weakness because the arrangement results also in poor mechanical strength in the plane of transverse isotropy. Real mechanical loadings are often complex, far from a uniaxial state of stress. The study of transverse properties is therefore as important as that of longitudinal properties.

For transverse properties, experimental characterisation is always a tedious procedure and the resulting uncertainties are often large. These complications provide a strong practical motivation for the development of models that can be used to predict material performance. However, this does not mean that the models eliminate the underlying difficulties. While the prediction of longitudinal properties of UD is rarely complicated, transverse properties are much more affected by interactions between constituents and the prediction of these properties is always a challenge. Indeed, although simple in nature, UD can exhibit a very wide variety of microstructures as suggested by Figure 1. If the longitudinal properties are only dependent (to the first order) on the intrinsic characteristics and the volume fraction of the phases, it is not the same for the transverse properties. Transverse properties are intimately linked to morphological fluctuations and the spatial distribution of the phases.

This paper builds on the results published in three papers [1-3], and aims to address the issue of model calibration to take into account local morphological fluctuations in the prediction of transverse behaviour of UD composites. After presenting the strategy on a simpler transport-diffusion problem, we propose in a second step to treat the mechanics problem and then to unify the approach.

2. Generalized Self-Consistent Scheme & Morphological Representative Patterns

The general framework of this work is best introduced by modelling the transport phenomenon in a multiphase transverse isotropic material. As an example, a UD brick of composite material is subjected to a molecular diffusion of water. It goes without saying that the present development is applicable to any other transport phenomenon and in particular to thermal diffusion. The strategy adopted in [1-2] allows the effective diffusivity of a composite medium to be predicted. While the components of the diffusivity tensor can be obtained by averaging the properties of the phases, the difficulty lies in how this averaging is carried out. If a simple law of mixtures is sufficient to accurately predict the longitudinal transport properties, a more refined model is required to deal with the transverse properties by taking into account morphological fluctuations. What distinguishes the different existing approaches, (while remaining within a mean-field modelling strategy), is the way they can handle (under undiluted conditions) local morphological fluctuations or a significant contrast between the properties of the phases involved. We have proposed to address these two challenges by exploiting a Generalized Self-Consistent Scheme [4,5] coupled with a morphologically representative model [6].

The GSCS part follows from the work in [7,8] which extends the initial 3 phase approach in [4] to the general case of "n" concentric phases. This work was revisited in [9] with a simplification of the formulation. The n-phases approach allows for example to consider problems of interphases or imperfect interfaces [10]. As mentioned above, in [1] the n-phase model has been coupled with a morphological pattern approach in order to capture transverse morphological fluctuations.

In [2], the model is implemented based on the microstructure and experimental results in [11,12]. In addition to the original 3 phase pattern, it was necessary to take into consideration the matrix areas trapped by an inverse pattern. Figure 2 illustrates this 2-phases/2-patterns model wrapped in the Equivalent Homogeneous Medium (EHM).

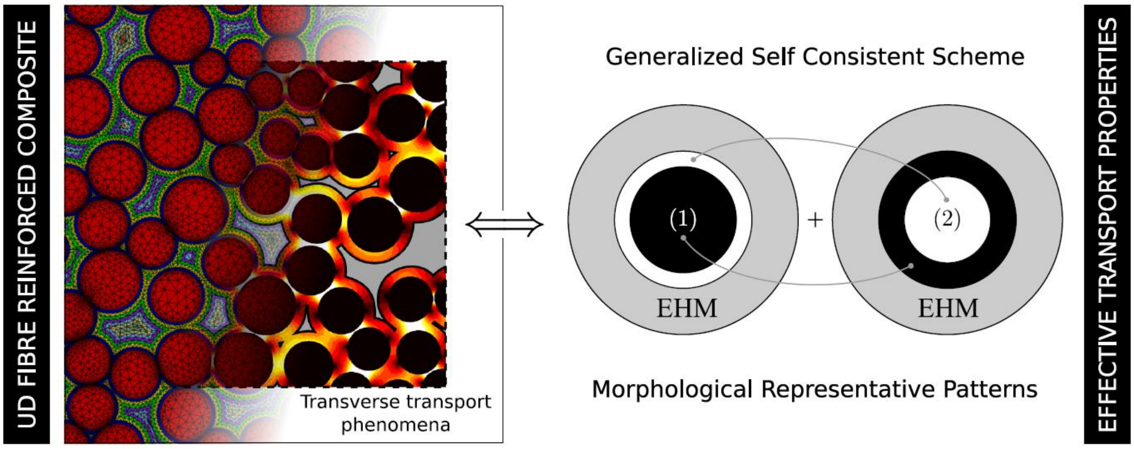

Figure 2. The composite medium (left) is modelled by a 2-phase/2-pattern approach (right). On the left, the flux norm is represented by a colour gradient from pale yellow to black (zero flux). The gray shaded areas correspond to the trapped matrix areas.

The behaviour of this particular case is governed by three morphological parameters: the volume fraction of reinforcement in the composite, f, the proportion of the original direct pattern, m, itself containing a concentration c of reinforcement. By defining f, it is possible to specify the range in which the effective transverse diffusivity can vary. The special case $c = f$ is relatively interesting as we will see in the forthcoming mechanical application, and, in this case:

$$\frac{D_T^{\text{eff}}}{D_T^{\text{matrix}}} = \frac{(1-f)(2m-1)}{(1+f)} \quad \text{if} \quad D_T^{\text{fibre}} = 0$$

By setting f, it is possible to specify the range in which the effective transverse diffusivity can vary (see Figure 3). The peak of the envelope curve corresponds to the classical GSCS model. The identification of the parameter m (and eventually c) remains an essential step. Several strategies based on the analysis of microstructure images have been experimented. Figure 3 shows the full predictive range of the proposed model and confirms the relevance of the proposed coupling between GSCS and MRP strategies.

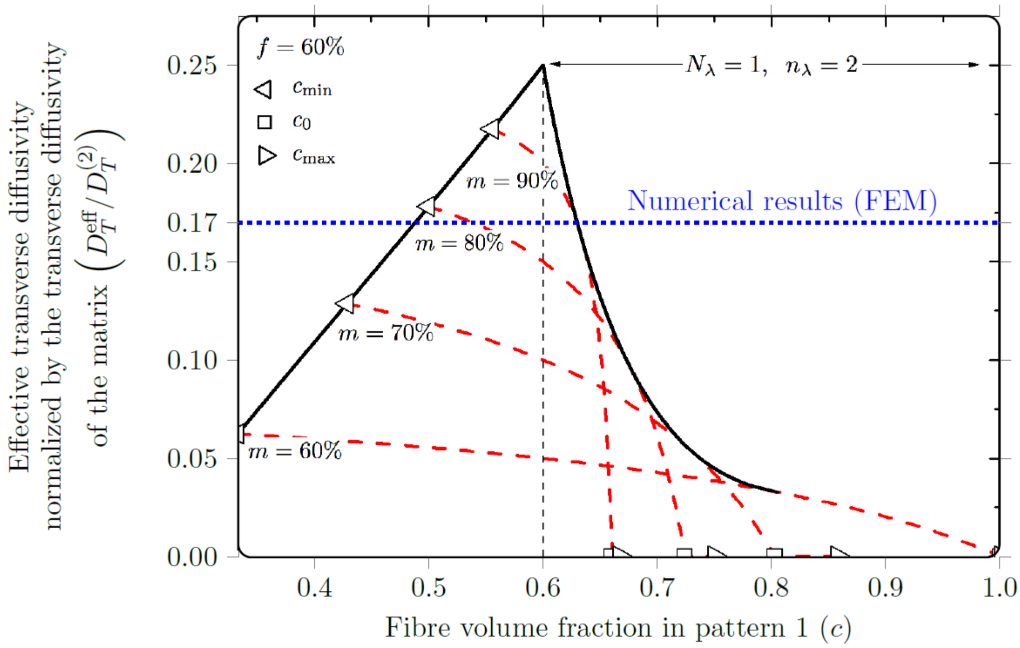

Figure 3. Envelope curve of the effective transverse diffusivity normalized by the matrix diffusivity

3. Extension of the model to mechanics

The previously described GSCS-MRP coupling has been extended to mechanics [3]. The focus was on k_{23}^{eff} and μ_{23}^{eff}, respectively the effective transverse bulk and shear moduli of the material. These two moduli are generally very difficult to obtain experimentally or with a very high uncertainty that makes structural calculations complex. The results [3] show a coupling between these two parameters, making the solution of the problem implicit.

As in the previous example, the model was also simplified to apply to the2-phases/2-patterns case with three parameters: f, m and c. In relation to diffusion, the first question raised was the existence of a unique (m, c) couple for the two elastic parameters studied. For this purpose, rather than very difficult experimental tests, full-field finite element numerical simulations have been carried out on representative microstructures. Uncertainties have been controlled through the use of tools such as the integral range method.

In Figure 4, the effective transverse shear modulus, normalized by the transverse shear modulus of the matrix is plotted as a function of phase contrast. The volume fraction of inclusions is $f = 0.6$; $\nu^{\text{fibre}} = 0.2$, $\nu^{\text{matrix}} = 0.3$. It can be seen from Figure 4 that the direct pattern of the original GSCS fails to capture the behaviour of representative random microstructures. For a given fibre volume fraction, f, but whatever the contrast of the properties between phases, we have shown that it is possible to define an optimised pair (m, c) allowing to predict with satisfaction and simultaneously k_{23}^{eff} and μ_{23}^{eff}. The sensitivity study has shown that in order to remain in a robust performance zone, it is nevertheless very profitable to choose $c = f$. In this case, the resolution remains implicit but the equations are much easier to handle.

Taking into account this configuration ($c = f$) and referring to the contrast conditions of the right part of Figure 4, the largest error for μ_{23}^{eff} is committed for the highest contrast ratio at 10^5 and is of the order of 5%; whereas the GSCS model leads to an error of more than 8% under the same conditions.

For k_{23}^{eff} (not shown on the figure), the error committed for the highest contrast ratio at 10^5 is less than 1%, which is much smaller than the GSCS model which leads to an error of the order of 8% also, under the same conditions. The question on the uniqueness of (m, c) pairs was then raised by extending the context of mechanics to the transport problem described above. By choosing $c = f$ and using the value of m optimised for mechanics, it is possible to predict with accuracy the values of k_{23}^{eff} and μ_{23}^{eff}, but also D_T^{eff}.

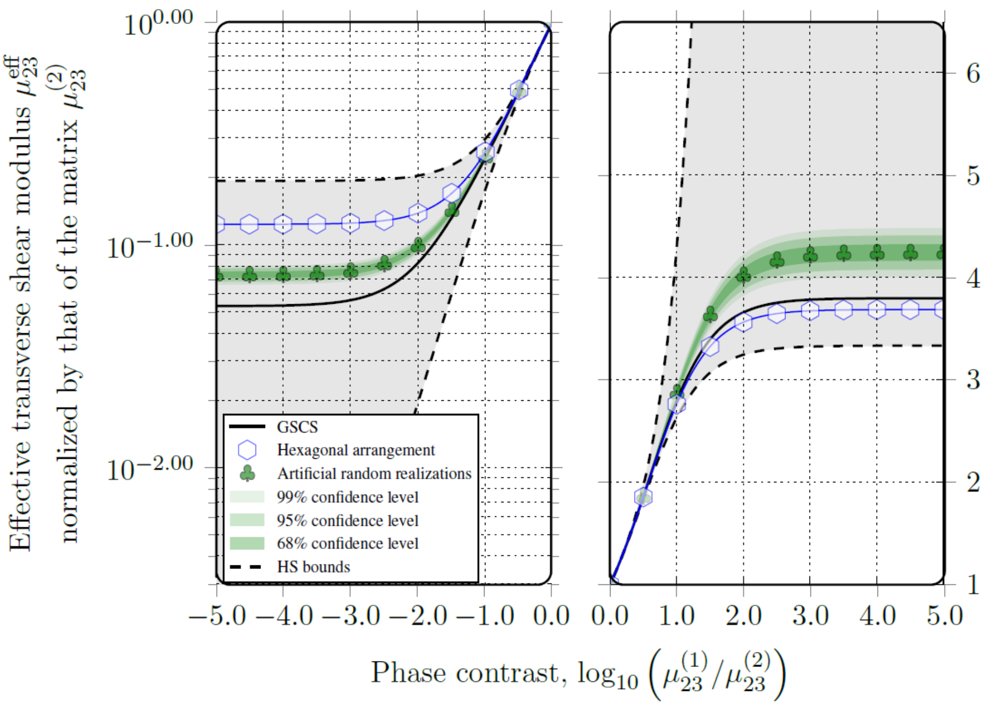

Figure 4. Effective transverse shear modulus, normalized by the transverse shear modulus of the matrix, plotted as a function of phase contrast.

4. Conclusion

The identification of m remains today the main challenge of this work and the use of the geometrical covariogram seems a very promising way. The present approach is appropriate to take into consideration the different microstructures illustrated in Figure 1. For some examples, however, the number of phases or patterns must be increased and the calibration strategy for the new parameters must be defined. For instance, the method has been used with success to model the porosity at different scales of the composite part of a high pressure cylinder. The application to hybrid composites is also a very interesting application and many others configurations are possible.

5. References

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