

Optimizing over Nonlinear Networks with Duality Cuts

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A nonlinear network [1] is a directed graph with arc flows and nodal potentials, involving flow conservation and a nonlinear relation between the potential loss and the flow through each arc. It serves as a basic model for a variety of physical networks like water, gas, or electricity distribution networks. Under realistic assumptions, these relations form the KKT conditions of a convex optimization problem that can be solved for example by the Newton-Raphson method. This approach is used in simulation – to compute a feasible potential-flow – and, by extension, in simulation-based heuristics – typically to optimize the topology of the network w.r.t the existence of a potential-flow. Such optimization problem arises as a strategic level – e.g., to decide the least-cost network design under a static standard or worst-case functional scenario – or at the operational level – to decide the best schedule of the active elements of the network according to a dynamic scenario. Some hybrid or fully integrated mathematical programming approaches have recently been applied in different contexts (see e.g., [2]): in general, the nonconvex potential-flow relations are modelled with the combinatorial topological options. Then, the resulting mixed-integer nonlinear program is solved by any variant of the spatial branching algorithm involving consecutive relaxations of the nonconvex components.

In this presentation, we reformulate these models as bilevel programs by replacing the inner potential-flow equality systems with the corresponding convex optimization subproblems. From the strong duality condition holding on these subproblems, we derive cutting-planes to tighten the relaxation of the integrated model. Finally, we focus on the context of water distribution networks and compare the application of this technique on the design problem, where the strong duality cuts become convex [3], and on the operational scheduling problem, where the reformulation remains nonconvex. Regardless of this, our computations on this latter problem show the interest of these cuts to tighten the relaxation and accelerate the branching algorithm of [4].

References

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