A constitutive model for rubber bearings subjected to combined compression and shear

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R. Bouaziz, C. Ovalle and L. Laiarinandrasana
PSL Research University
Mines ParisTech, Centre des Matriaux, CNRS 7633 BP 87, F-91003 Evry Cedex, France

ABSTRACT: This paper focuses on the finite strain thermo-mechanical modelling of the dynamical behaviour of filled rubbers under combined compression and shear using a multiphysics coupling approach. The mechanical behaviour of such materials depend on the network evolution due to thermo-mechanical loadings and the external environmental conditions (oxidation, ageing ...). An experimental campaign including tensile, compression/shear and compression/torsion tests allows to study the impact of thermal ageing on the material properties, namely the bulk modulus and the shear modulus. This experimental investigation will allow us to model both the relationship between the hydrostatic pressure versus the volume change and the isochoric visco-hyperelastic behaviour as a function of ageing.

1 INTRODUCTION

Laminated rubber bearings are commonly used for the seismic isolation of civil engineering structures. During earthquake, the rubber material is supposed to undergo static compressive loading combined with cyclic shear. In terms of constitutive modelling, rubber is generally assumed to be quasi-incompressible. Nevertheless, in the laminated geometry of bearings, exhibiting a high shape factor (Dorfmann, Fuller, & Ogden 2002), modelling the relationship between the hydrostatic pressure versus the volume change is of prime importance. In this contribution, an attempt is made to investigate a relevant constitutive model, using a multiphysics coupling approach, such as the one developed in (Lejeunes, Eyheramendy, Boukamel, Delattre, Méo, & Ahose 2018), that allows the combined shear and dilational responses to be handled. To this end, an original experimental set-up is used to obtain a reliable database, useful for the finite element modelling.

In the other hand, during long term service, the shear and bulk moduli of the rubber can evolve due to environmental influences such as temperature, humidity, oxygen…The evolution of these mechanical properties can be related to the modification of the rubber network due to the chemical-mechanical interaction (Steinke et al. 2011). In a second part of the study, ageing on rubber samples will be replicated by means of an experimental set-up based on degradation of the material by oxidative ageing. The results will be used to enhance the proposed model by offering a closer look at the ageing effects on the rubber mechanical properties. This may help to improve the residual life-time prediction for related applications.

2 CONSTITUTIVE MODELLING

2.1 *Kinematic*

The proposed constitutive model is based on the thermodynamics of irreversible processes and on the local state assumption. Viscoelastic overstress is introduced through the help of an intermediate configuration defined from the splitting of the deformation gradient $\mathbf{F}(\mathbf{X},t)$, where $\mathbf{X}$ is the position of a material point in the reference configuration. To consider the nearly incompressible behaviour, the deformation gradient $\mathbf{F}$ is split into a volumetric and an isochoric part. The isochoric part is decomposed multiplicatively into purely isochoric elastic parts $\bar{\mathbf{F}}^i_{\mathbf{e}}(\mathbf{X},t)$ and isochoric viscous ones $\bar{\mathbf{F}}^i_{\mathbf{v}}(\mathbf{X},t)$, where i stands for the i^{th} decomposition:

$$\begin{cases} \mathbf{F} = \mathbf{F}^{vol}\bar{\mathbf{F}} \\ \mathbf{F}^{vol} = J^{1/3}\mathbf{1} \\ \bar{\mathbf{F}} = J^{-1/3}\mathbf{F} = \bar{\mathbf{F}}^i_{\mathbf{e}}\bar{\mathbf{F}}^i_{\mathbf{v}} \end{cases} \tag{1}$$

where:

- $\mathbf{F}^{vol}$ and $\bar{\mathbf{F}}$ are, respectively, the volumetric and the isochoric parts of the deformation gradient $\mathbf{F}$.
- $\mathbf{1}$ is the second order unit tensor.

- $J = \det(\mathbf{F})$ is the jacobian of $\mathbf{F}$ that corresponds to the total volume variation.

From physical measurements (mass and dimension) of samples before and after ageing, no ageing-dependent variation of mass or volume was observed. Therefore, we assume that volume variation J can only be attributed to thermal dilatation or mechanical compressibility. We adopt the following splitting of the volume variation $J = J_\Theta J_m$, with

$$J_\Theta = 1 + \alpha_\Theta(\Theta - \Theta_0) \tag{2}$$

$$J_m = \frac{J}{J_\Theta} \tag{3}$$

where α_Θ is the thermal expansion coefficient, Θ_0 is the reference temperature and Θ is the current temperature. The previous linear expansion relation is motivated by experimental evidences that we have obtained from dilatometry experiments.

2.2 *Thermodynamics*

In the following, we adopt the hybrid free energy concept (Lion et al. 2014, Lejeunes et al. 2018, Lejeunes et al. 2018). The main idea is to formulate the free energy from standard state and internal variables. The volume dependency is replaced by a pressure-like variable designated by q. This formalism can be viewed as a generalization of partial Legendre transformation of the free energy.

The hybrid free specific energy is assumed as follows:

$$\begin{aligned}\varphi(\bar{\mathbf{B}}, \bar{\mathbf{B}}^i_{\mathbf{e}}, q, \xi, \Theta) =& \varphi_{iso}(\bar{\mathbf{B}}, \bar{\mathbf{B}}^i_{\mathbf{e}}, \xi, \Theta) \\ & + \varphi_{vol}(q, \Theta) + \varphi_{th}(\Theta) \\ & + \varphi_{chem}(\xi, \Theta)\end{aligned} \tag{4}$$

where $\bar{\mathbf{B}} = \bar{\mathbf{F}} \cdot \bar{\mathbf{F}}^{\mathrm{T}}$ and $\bar{\mathbf{B}}^i_{\mathbf{e}} = \bar{\mathbf{F}}^i_{\mathbf{e}} \cdot \bar{\mathbf{F}}^{i\mathrm{T}}_{\mathbf{e}}$ are respectively the isochoric left Cauchy Green tensor and the i^{th} elastic isochoric left Cauchy Green tensor.

The additive splitting of eq. (4) is motivated by the fact that it can exist purely thermal variations that are stress-free for which no chemical reaction occurs (contribution of φ_{th}). Thermochemical evolutions that are stress free can also be observed (contribution φ_{chem}) and finally thermo-mechanical states can also exist without any chemical reaction. Furthermore, due to the huge difference of stiffness for both hydrostatic or isochoric states we follow (Flory 1961) and we adopt an isochoric/volumetric splitting for the remaining part of the hybrid energy (contributions of φ_{iso} and φ_{vol}). We do not observe impact of the ageing on the thermal properties of the material (heat capacity and thermal dilatation), therefore φ_{th} is assumed to depend only on Θ. This is the same for the volumetric contribution, therefore φ_{vol} depends only on the pressure-like variable q and temperature.

The hybrid energy approach requires to define a supplementary potential β, which allows to relate q, J and Θ with a constitutive law, or a supplementary flow rule if we consider dissipative mechanisms in volume, cf. (Lejeunes et al. 2018) for more details. The hybrid free energy φ can be related to the Helmholtz free energy ψ through the relation:

$$\begin{aligned}\psi(\bar{\mathbf{B}}, \bar{\mathbf{B}}^i_{\mathbf{e}}, J, \xi, \Theta) =& \varphi(\bar{\mathbf{B}}, \bar{\mathbf{B}}^i_{\mathbf{e}}, q, \xi, \Theta) \\ & - \beta(J, \Theta, q)\end{aligned} \tag{5}$$

The combination of the first and the second law of thermodynamics, in the Clausius-Duhem form, leads to the following inequality:

$$\begin{aligned}\phi =\ & \rho\left(\frac{\boldsymbol{\sigma}}{\rho} - 2\left(\bar{\mathbf{B}}\frac{\partial\varphi}{\partial\bar{\mathbf{B}}} + \textstyle\sum_{i=1}^{N} \bar{\mathbf{B}}^i_{\mathbf{e}}\frac{\partial\varphi}{\partial\bar{\mathbf{B}}^i_{\mathbf{e}}}\right)^{\mathrm{D}} + J\frac{\partial\beta}{\partial J}\mathbf{1}\right) : \mathbf{D} \\ - \ & \rho\left(s + \frac{\partial\varphi}{\partial\Theta} - \frac{\partial\beta}{\partial\Theta}\right)\dot{\Theta} + 2\rho\textstyle\sum_{i=1}^{N}\left(\bar{\mathbf{B}}^i_{\mathbf{e}}\frac{\partial\varphi}{\partial\bar{\mathbf{B}}^i_{\mathbf{e}}}\right) : \bar{\mathbf{D}}^{\mathbf{o}i}_{\mathbf{v}} \\ - \ & \rho\left(\frac{\partial\varphi}{\partial q} - \frac{\partial\beta}{\partial q}\right)\dot{q} - \rho\frac{\partial\varphi}{\partial\xi}\dot{\xi} - \frac{\mathrm{grad}_{\boldsymbol{x}}\Theta}{\Theta}\mathbf{q}_\Theta \geq 0 \\ & \forall \mathbf{D}, \dot{\Theta}, \bar{\mathbf{D}}^{\mathbf{o}i}_{\mathbf{v}}, \dot{q}, \mathbf{q}_\Theta \dot{\xi}\end{aligned} \tag{6}$$

where ϕ is the total dissipation, $\boldsymbol{\sigma}$ is the Cauchy stress, $\mathbf{D} = 1/2(\mathbf{L} + \mathbf{L}^{\mathrm{T}})$ is the eulerian strain rate with $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$, ρ is the current density, s is the specific entropy, $\mathbf{q}_\Theta$ is the Eulerian heat flux, $\bar{\mathbf{D}}^{\mathbf{o}i}_{\mathbf{v}}$ is the i^{th} objective eulerian strain rate, defined by:

$$\bar{\mathbf{D}}^{\mathbf{o}^i}_{\mathbf{v}} = \bar{\mathbf{R}}^i_{\mathbf{e}} \bar{\mathbf{D}}^i_{\mathbf{v}} \bar{\mathbf{R}}^{i\mathrm{T}}_{\mathbf{e}} \tag{7}$$

where $\bar{\mathbf{R}}^i_{\mathbf{e}}$ comes from the polar decomposition of $\bar{\mathbf{F}}^i_{\mathbf{e}} = \bar{\mathbf{V}}^i_{\mathbf{e}} \bar{\mathbf{R}}^i_{\mathbf{e}}$.

Assuming that the dissipation is only due to thermal diffusion, mechanical viscosity including Payne effect and chemical evolution, the following constitutive equations can be obtained from (6):

$$\boldsymbol{\sigma} = \overbrace{2\rho\left(\bar{\mathbf{B}}\frac{\partial\varphi_{iso}}{\partial\bar{\mathbf{B}}}\right)^{\mathrm{D}} + \frac{q}{J_\Theta}\mathbf{1}}^{\boldsymbol{\sigma}_{eq}} + \sum_{i=1}^{N} \overbrace{2\rho\left(\bar{\mathbf{B}}^i_{\mathbf{e}}\frac{\partial\varphi_{iso}}{\partial\bar{\mathbf{B}}^i_{\mathbf{e}}}\right)^{\mathrm{D}}}^{\boldsymbol{\sigma}^i_v} \tag{8}$$

$$s = -\frac{\partial\varphi}{\partial\Theta} + \frac{\partial\beta}{\partial\Theta} \tag{9}$$

$$\frac{\partial\varphi_{vol}}{\partial q} = \frac{\partial\beta}{\partial q} \tag{10}$$

where $\boldsymbol{\sigma}_{eq}$ is the time-independent equilibrium stress at which the total stress $\boldsymbol{\sigma}$ is totally relaxed, i.e. $\boldsymbol{\sigma}_{eq} = \boldsymbol{\sigma}(t \to \infty)$, $\boldsymbol{\sigma}^i_v$ is the i^{th} viscous (non-equilibrium) stress, i.e. $\boldsymbol{\sigma}^i_v(t \to \infty) = 0$. The pressure-like variable

q is defined by (10) and we can see from (8) that the hydrostatic pressure is defined from $p = q/J_\Theta$.

We consider that internal variables evolve independently from each other, e.g. (Germain, Nguyen, & Suquet 1983); then, the positivity of each dissipation term is independently required. From (6), we obtain the following inequalities:

$$\begin{aligned}\phi^i_{vis} =& 2\rho\left(\bar{\mathbf{B}}^i_{\mathbf{e}}\frac{\partial\varphi_{iso}}{\partial\bar{\mathbf{B}}^i_{\mathbf{e}}}\right) : \bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}} \\ =& \boldsymbol{\sigma}^i_v : \bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}} \geq 0 \qquad \forall\bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}} \qquad i = 1..N\end{aligned} \tag{11}$$

$$\phi_\xi = -\rho\frac{\partial\varphi}{\partial\xi}\dot{\xi} = A_\xi\dot{\xi} \geq 0 \qquad \forall\dot{\xi} \tag{12}$$

$$\phi_\Theta = \left(-\frac{\mathrm{grad}_{\boldsymbol{x}}\Theta}{\Theta}\right)\mathbf{q}_\Theta = A_\Theta\mathbf{q}_\Theta \geq 0 \qquad \forall\mathbf{q}_\Theta \tag{13}$$

where $\boldsymbol{\sigma}^i_v, A_\xi, A_\Theta$ are thermodynamical forces associated with the thermodynamical fluxes $\bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}}, \dot{\xi}, \mathbf{q}_\Theta$. For the thermal dissipation term, i.e. 13, we assume an isotropic Fourier conduction law such as:

$$\mathbf{q}_\Theta = -k_\Theta\mathrm{grad}_{\boldsymbol{x}}\Theta \tag{14}$$

The conductivity coefficient k_Θ is assumed to be constant and not affected by ageing. There are not experimental evidences supporting this assumption; however, results can be used as an initial approximation.

2.3 *Mechanical behaviour*

An isotropic behaviour is considered so we propose to adopt the following potentials:

$$\begin{aligned}\rho_0\varphi_{iso} =& C_{10}\left(I_1(\bar{\mathbf{B}}) - 3\right) + C_{01}\left(I_2(\bar{\mathbf{B}}) - 3\right) \\ &+ \sum_{i=1}^{N}\frac{\mu_i}{2}(I_1(\bar{\mathbf{B}}^i_{\mathbf{e}}) - 3)\end{aligned} \tag{15}$$

$$\rho_0\varphi_{vol} = -\frac{1}{2}k\left(1 + \frac{q}{k}\right)^2 + q + k \tag{16}$$

$$\rho_0\beta = q(1 - J_m) \tag{17}$$

where C_{10}, C_{20} are the Mooney-Rivlin coefficients that are assumed to be dependent on ξ and Θ, μ_i is the i^{th} shear modulus that depends on ξ and Θ. The bulk modulus k is assumed to be independent on these variables. I_1 and I_2 are respectively the first and the second invariant of the strain tensor and $\rho_0 = J\rho$ is the initial mass density.

2.3.1 *Equilibrium contribution*

From (8) the equilibrium stress is written as:

$$\begin{aligned}\boldsymbol{\sigma}_{eq} =& 2J^{-1}C_{10}\bar{\mathbf{B}}^{\mathrm{D}} \\ &+ 2J^{-1}C_{01}(I_1(\bar{\mathbf{B}})\bar{\mathbf{B}} - \bar{\mathbf{B}}^2)^{\mathrm{D}} + \frac{q}{J_\Theta}\mathbf{1}\end{aligned} \tag{18}$$

2.3.2 *Visco-elastic contribution*

From (8) the i^{th} viscous stress is written as:

$$\boldsymbol{\sigma}^i_v = J^{-1}\mu_i\bar{\mathbf{B}}^{i\ \mathrm{D}}_{\mathbf{e}} \qquad i = 1..N \tag{19}$$

We consider a Maxwell-like flow rule for viscosity as follows:

$$\bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}} = \frac{1}{\eta_i}\boldsymbol{\sigma}^i_v \quad i = 1..N \tag{20}$$

where η_i is the i^{th} viscosity parameter.

The positivity of the viscous dissipation terms ϕ^i_{vis} can be easily shown by inserting (20) in (11). From the time differentiation of $\bar{\mathbf{B}}^i_{\mathbf{e}}$ and the multiplicative splitting of the deformation gradient, the Maxwell flow rule can be also reformulated by the following relation:

$$\dot{\bar{\mathbf{B}}}{}^i_{\mathbf{e}} = \mathbf{L}\bar{\mathbf{B}}^i_{\mathbf{e}} + \bar{\mathbf{B}}^i_{\mathbf{e}}\mathbf{L}^{\mathrm{T}} - 2\bar{\mathbf{V}}^i_{\mathbf{e}}\bar{\mathbf{D}}^{\mathrm{o}^i}_{\mathbf{v}}\bar{\mathbf{V}}^i_{\mathbf{e}} \tag{21}$$

Combining (20) and (21), we can obtain:

$$\begin{cases}\bullet\ \dot{\bar{\mathbf{B}}}{}^i_{\mathbf{e}} = \mathbf{L}\bar{\mathbf{B}}^i_{\mathbf{e}} + \bar{\mathbf{B}}^i_{\mathbf{e}}\mathbf{L}^{\mathrm{T}} \\ \quad -\frac{2}{3}(\mathbf{1} : \mathbf{L})\bar{\mathbf{B}}^i_{\mathbf{e}} - \frac{1}{J\tau_i}\bar{\mathbf{B}}^i_{\mathbf{e}}\bar{\mathbf{B}}^{i\ \mathrm{D}}_{\mathbf{e}} \qquad i = 1..N \\ \bullet\ \bar{\mathbf{B}}^i_{\mathbf{e}}(t = 0) = \mathbf{1}\end{cases} \tag{22}$$

where $\tau_i = \eta_i/2\mu_i$ is a characteristic time of viscosity linked to the i^{th} viscous stress.

3 APPLICATION TO A COMPRESSION-TORSION TEST

3.1 *Analytical modelling*

We consider an homogeneous cylinder. We denote by L_0 its initial length and A its initial radius. In the initial configuration C_0, a material point M of the undeformed cylinder is identified by (R, θ_0, Z) in the initial cylindrical coordinate system (e_R, e_{θ_0}, e_Z). The transformation consists of an axial displacement in the e_Z direction and a torsion angle around the same vector (e_Z) applied on the upper surface S_u. The lower suface S_l is maintained fixed and the lateral surface S_{lat} is free. The length and radius of the deformed cylinder are noted respectively by l and a. By this transformation, the point M becomes M', identified by (r, θ, z) in the deformed cylindrical coordinate system (e_r, e_θ, e_z).

The displacement of M to M' is described by:

$$r = \frac{R}{\sqrt{\lambda}}, \qquad \theta = \theta_0 + \tau Z \tag{23}$$

where λ is the relative elongation and τ is the angle per unit of initial length:

$$\lambda = \frac{l}{L_0}, \qquad \tau = \frac{\phi}{L_0} \tag{24}$$

Considering the case of oedometric compression, we assume that the deformation gradient and the stress solution are homogeneous. As a consequence, the deformation gradient is written as:

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \tau r \\ 0 & 0 & \lambda \end{bmatrix} \tag{25}$$

and the Cauchy Green tensor is given by:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+\tau^2 r^2 & \lambda \tau r \\ 0 & \lambda \tau r & \lambda^2 \end{bmatrix} \tag{26}$$

We consider that the viscoelastic Cauchy Green tensor take the following form:

$$\bar{\mathbf{B}}^i_{\mathbf{e}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{B}^i_{e22} & \bar{B}^i_{e23} \\ 0 & \bar{B}^i_{e23} & \bar{B}^i_{e33} \end{bmatrix} \tag{27}$$

Inserting (27) in (22), a system of differential equations with three unknowns is obtained.

We consider a disc with a radius $A = 12.5$ mm and an initial length $L_0 = 2$ mm. The initial shape factor is given by $S_f = A/L_0 = 6.25$. This disc is submitted to compression by imposing $\lambda = 0.9$ and torsion by imposing a rotation angle $\phi = 10\pi/180$ on the upper surface.

3.2 *Discussion*

In order to study the contribution of shear stress and the hydrostatic pressure as function of ageing, we assume that both the bulk modulus K and the shear modulus G increase with ageing.

Figure 1 shows the evolution of the shear stress and the hydrostatic pressure (absolute value) as a function of the normalized radius for different values of G and K, obtained from the analytical model.

However, in a case of uniaxial compression loading on a disc having a shape factor higher than 5, the hydrostatic pressure curve should not be constant near the edge of the disc due to the well-known barrelling effect (Dorfmann, Fuller, & Ogden 2002). We expect that the absolute value of the hydrostatic pressure decreases between $0.8a$ and a. Figure 2 illustrates the expected curves of hydrostatic pressure as function of the normalized radius.

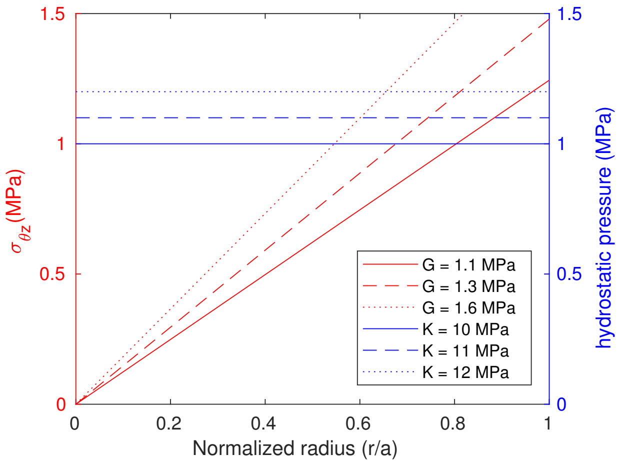

Figure 1: Evolution of the shear stress and the hydrostatic pressure as a function of the normalized radius

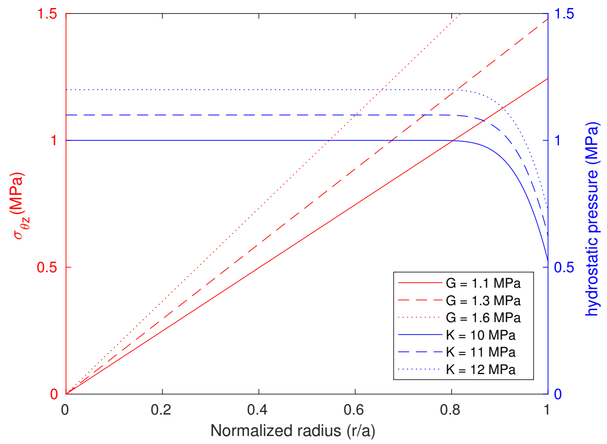

Figure 2: Evolution of the shear stress and the hydrostatic pressure as a function of the normalized radius:(*expected appearance of hydrostatic curves)

Since the barrelling effect cannot be highlighted by the analytical model, the proposed model will be implemented into a finite element code.

On the other hand, the shear-bulk ratio between the shear stress and the hydrostatic pressure is defined as follows:

$$\chi = \frac{\sigma_{\theta z}}{p}$$

Figure 3 shows the evolution of the radius at a fixed bulk modulus $K = 10$ MPa and a shear-bulk ratio $\chi = 1$ as a function of the shear modulus G.

Figure 4 shows the evolution of the ratio χ at a fixed bulk modulus $K = 10$ MPa and on a radius $r = 0.8a$ $\chi = 1$ as a function the shear modulus G.

Figure 5 shows the evolution of the radius r at a fixed shear modulus $G = 1.1$ MPa and a shear-bulk ratio $\chi = 1$ as a function the bulk modulus K.

Figure 6 shows the evolution of the ratio χ at a fixed shear modulus $G = 1.1$ MPa and a radius $r = 0.8a$ as a function the bulk modulus K.

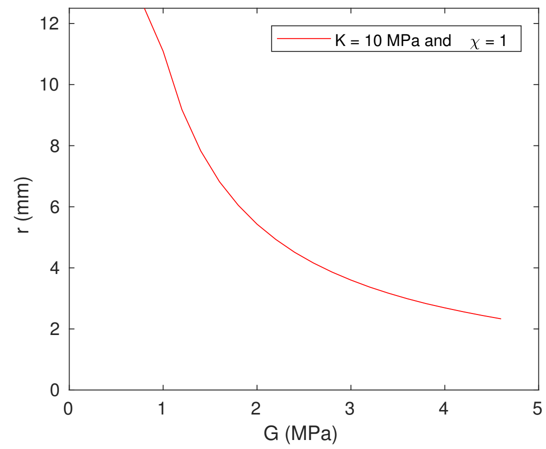

Figure 3: Evolution of the radius at $K = cte$ and $\chi = 1$ as a function of G

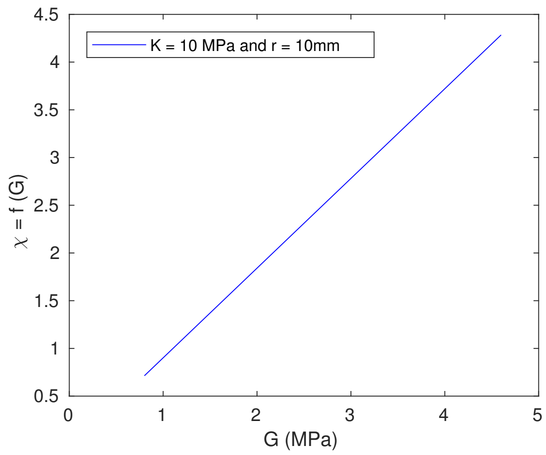

Figure 4: Evolution of χ at $K = cte$ and $r = 0.8a$ as a function of G

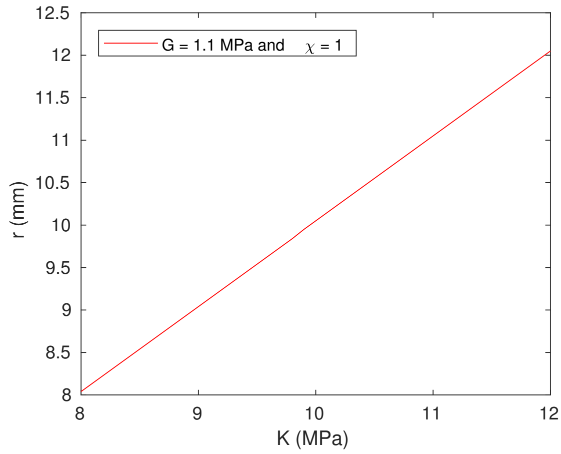

Figure 5: Evolution of the radius at $G = cte$ and $\chi = 1$ as a function of K

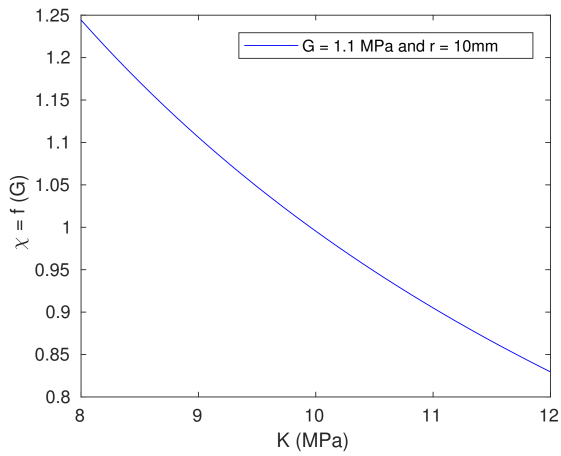

Figure 6: Evolution of χ at $G = cte$ and $r = 0.8a$ as a function of K

4 CONCLUSIONS

In this paper, we presented a constitutive model based on a multiphysics coupling approach for compressible filled rubbers. This model takes into account the network mechanisms that are responsible of the evolution of the mechanical properties. Moreover, it takes into account the evolution of volumetric behaviour as a function of ageing. Finally, in a future work, the model will be implemented into a finite element code and verified with experimental results.

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