



Velké Karlovice, 23 May 2019

Coupled Numerical Modelling of Redox Reactions and Glass Melt Fining Processes

F. Pigeonneau (Mines-ParisTech PSL - CEMEF, Surface du Verre & Interface, Joint lab. CNRS/Saint-Gobain)



1. Glass fining principle



- > 200 kg de CO_2/T of glass:
 - > 25 Nm³ of CO₂/m³ of glass, or 140 m³ de CO₂/m³ of glass at $T = 1400^{\circ}$ C.

► The small solubility of CO₂ ⇒ bubble formation: 10⁸ bulles/m³.



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Redox & fining modelling

1. Glass fining principle

According to Hadamard and Rybczynski¹, the rising velocity is

$$V_T = \frac{\rho g a^2(t)}{3\mu(T)}.$$
 (2)

- How to remove the bubble from the glass bath?
 - Increase T to decrease $\mu(T)$;
 - Enhance a.

► To increase the bubble size, "fining" agents are added:

- Sulfate: Na₂SO₄;
- ► Transition metals: As⁵⁺/As³⁺, Sb⁵⁺/Sb³⁺, etc.
- ► \checkmark *T* promote the reduced state \Rightarrow "fining gases" release O_2 et SO₂.

¹J. Hadamard: Mouvement permanent lent d'une sphère liquide et visqueuse dans un liquide visqueux, in: C. R. Acad. Sci. Paris 152 (1911), pp. 1735–1738; W. Rybczynski: Uber die fortschreitende bewegun einer flussingen kugel in einem zaben medium, in: Bull. de l'Acad. des Sci. de Cracovie, série A 1 (1911), pp. 40–46.

1. Glass fining principle

Oxidation-reduction of glass former liquids Thermodynamic model

3. Mass transfer between the two phases

- 3.1 Unique Bubble
- 3.2 Bubble population
- 4. Synthesis

2. Oxidation-reduction of glass former liquids

2.1 Thermodynamic model



$$d_{i}S = \frac{A}{T}d\zeta, \ A = \mu_{M^{(n+k)+}} + \frac{\kappa}{2}\mu_{O^{2-}} - \mu_{M^{n+}} - \frac{\kappa}{4}\mu_{O_{2}}.$$
 (4)

• The thermodynamic equilibrium $\Rightarrow A = 0$:

$$K_{\rm M} = rac{a_{{
m M}^{n+}} a_{{
m O}_2}^{k/4}}{a_{{
m M}^{(n+k)+}} a_{{
m O}^{2-}}^{k/2}}, \ K_{\rm M}' = rac{C_{{
m M}^{n+}} P_{{
m O}_2}^{k/4}}{C_{{
m M}^{(n+k)+}}}.$$
 (5)

²D. Kondepudi/I. Prigogine: Modern Thermodynamics: From Heat Engines to Dissipative Structures, 2nd, New York 2015.

2. Oxidation-reduction of glass former liquids

2.1 Thermodynamic model

► For several reactions: $\underbrace{Fe^{3+}}_{\text{oxidized state}} + \frac{1}{2}O^{2-} \rightleftharpoons \underbrace{Fe^{2+}}_{\text{reduced state}} + \frac{1}{4}O_2, \quad (6)$ $\underbrace{SO_4^{2-}}_{\text{oxidized state}} \rightleftharpoons \underbrace{SO_2}_{\text{reduced state}} + \frac{1}{2}O_2 + O^{2-}, \quad (7)$ $\underbrace{oxidized state}_{\text{reduced state}} = \frac{1}{T} \left(\mathcal{A}_{\text{Fe}} d\zeta_{\text{Fe}} + \mathcal{A}_{\text{S}} d\zeta_{\text{S}}\right). \quad (8)$

The thermodynamic equilibrium:

$$\mathcal{A}_{Fe} = 0, \quad \mathcal{K}_{Fe} = \frac{C_{Fe^{2+}} P_{O_2}^{1/4}}{C_{Fe^{3+}}}$$

$$\mathcal{A}_{S} = 0, \quad \mathcal{K}_{S} = \frac{P_{SO_2} P_{O_2}^{1/2}}{C_{SO_4^{2-}}}.$$
(9)

2. Oxidation-reduction of glass former liquids

2.1 Thermodynamic model

- The phase rule of Gibbs: $f = C R \varphi + 2$.
 - R chemical reactions to satisfy;
 - C chemical equilibrium to satisfy between each phases:

$$\mu_{O_2}(glass) = \mu_{O_2}(gas) \Rightarrow C_{O_2} = \underbrace{\mathcal{L}_{O_2}}_{\text{Henry coefficient}} P_{O_2}, (10)$$

$$\mathcal{L}_{O_2} = A_{O_2}^{(s)} e^{B_{O_2}^{(s)}/T}$$
. (11)

2. Oxidation-reduction of glass former liquids

2.1 Thermodynamic model

R reactions of oxidation-reduction:

$$\sum_{i=1}^{N_I} \nu'_{ii} A_i + \sum_{j=1}^{N_g} \beta'_{ij} G_j \Longrightarrow \sum_{i=1}^{N_I} \nu''_{ii} A_i + \sum_{j=1}^{N_g} \beta''_{ij} G_j, \ r \in [1; R].$$
(12)

A_i: ionic species $i \in [1; N_i]$; G_j: gas species $j \in [1; N_g]$

$$\frac{\mathcal{A}_r}{\mathcal{R}\mathcal{T}} = -\frac{\Delta G_r}{\mathcal{R}\mathcal{T}} - \ln \prod_{i=1}^{N_l} C_{\mathcal{A}_i}^{\nu_{ri}} \prod_{j=1}^{N_g} C_{\mathcal{G}_j}^{\beta_{rj}}, \qquad (13)$$

$$\nu_{ri} = \nu_{ri}'' - \nu_{ri}', \forall i \in [1; N_i], \ \beta_{rj} = \beta_{rj}'' - \beta_{rj}', \forall j \in [1; N_g].$$
(14)

Phase equilibrium gives:

$$\alpha_j G_j(g) \rightleftharpoons G_j(f), \forall j \in [1; N_g]$$
(15)

$$C_{G_j} = \mathcal{L}_{G_j} P_{G_j}^{\alpha_j}, \ \mathcal{L}_{G_j} = A_{G_j}^{(s)} e^{B_{G_j}^{(s)}/T}.$$
 (16)

2. Oxidation-reduction of glass former liquids 2.1 Thermodynamic model

Thermodynamic equilibrium between the liquid and the atmospheric phase at P_{atm}.

- \blacktriangleright K_r: obtained by chemical equilibrium³, or by electrochemistry⁴.
- *L*_{G_j}: obtained by computation (physically dissolved), by
 equilibrium and chemical analysis (chemically dissolved)⁵.

³M.-H. Chopinet/D. Lizarazu/C. Rocanière: L'importance des phénomènes d'oxydo-réduction dans le verre, in: C. R. Chim. 5.12 (2002), pp. 939–949.

⁴C. Rüssel: Iron oxide-doped alkali-lime-silica glasses, in: Glastech. Ber 66 (1993), pp. 68–74.

⁵F. W. Krämer: Solubility of gases in glass melts, in: Properties of glass-forming melts, 2005, chap. 13, pp. 405–482.

2. Oxidation-reduction of glass former liquids 2.1 Thermodynamic model

Glass with sulfate and low iron concentration. At 1280°C:

$$\mathcal{R}_{\rm Fe} = rac{C_{\rm Fe^{2+}}}{C_{\rm Fe}} = 0.186,$$
 (17)

$$C_{\mathrm{SO}_3^{2-}} = 0.49 \text{ wt \%}.$$
 (18)

- K_r taken from the two references⁶.
- \mathcal{L}_{G_i} taken from the paper of Beerkens.

⁶Chopinet/Lizarazu/Rocanière: L'importance des phénomènes d'oxydo-réduction dans le verre (see n. 3); R. G. C. Beerkens: Analysis of advanced and fast fining processes for glass melts, in: Advances in Fusion and Processing of Glass III, New York 2003, pp. 3–24.

2. Oxidation-reduction of glass former liquids

2.1 Thermodynamic model



Figure 1: $C_{SO_3^{2-}}$ (% masse) vs. T.

Figure 2: \mathcal{R}_{Fe} vs. T.





Mass transfer with multicomponent bubbles.

The bubble volume changes due to the mass gradient between the bubble and the liquid.



Figure 3: C_i vs. r.



Figure 4: C_{O_2} around a rising bubble.

For a rising bubble, C_{G_i} changes over a thin boundary layer, δ :

$$J_{G_i} = \frac{\mathcal{D}_{G_i}\left(C_{G_i}^{\infty} - C_{G_i}^{S}\right)}{\delta} = \frac{2a}{\delta} \frac{\mathcal{D}_{G_i}}{2a} \left(C_{G_i}^{\infty} - C_{G_i}^{S}\right).$$
(19)

The Sherwood number is roughly defined:

$$\mathbf{Sh}_{\mathbf{G}_{\mathbf{i}}} = \frac{\mathbf{2a}}{\delta}, \ Sh_{G_{i}} = f(Pe_{G_{i}}), \ \text{with } \mathbf{Pe}_{\mathbf{G}_{\mathbf{i}}} = \frac{\mathbf{2aV}_{\mathsf{T}}}{\mathcal{D}_{\mathbf{G}_{\mathbf{i}}}}.$$
 (20)



Figure 5: Sh_{O₂} vs. Pe_{O₂} for several C_{Fe} at $T = 1400^{\circ}$ C⁷.

⁷F. Pigeonneau: Mass transfer of rising bubble in molten glass with instantaneous oxidation-reduction reaction, in: Chem. Eng. Sci. 64 (2009), pp. 3120–3129.

 \blacktriangleright H₂O, N₂; CO₂; O₂ and SO₂.

$$\frac{dn_{G_i}}{dt} = 2\pi a \operatorname{Sh}_{G_i} \mathcal{D}_{G_i} (C_{G_i}^{\infty} - \mathcal{L}_{G_i} P_{G_i}^{\beta_{G_i}}), \quad (21)$$

$$\frac{3\mathcal{R}T\sum_{i=1}^{N_g} n_{G_i}}{4\pi a^3} = P_0 + \rho g(H - z) + \frac{2\sigma}{a}, \quad (22)$$

$$\frac{dz}{dt} = \frac{ga^2}{3\nu}. \quad (23)$$



Figure 6: a/a_0 vs. *t* for 2 glasses with different iron contents at $T = 1400^{\circ}C^9$.

⁸F. Pigeonneau/D. Martin/O. Mario: Shrinkage of oxygen bubble rising in a molten glass, in: Chem. Eng. Sci. 65 (2010), pp. 3158–3168. ⁹Ibid.



Figure 7: $a (\mu m)$ vs. $\sqrt{t} (\sqrt{s})$ for a float glass¹⁰.

¹⁰D. Boloré/F. Pigeonneau: Spatial distribution of nucleated bubbles in molten glasses undergoing coalescence and growth, in: J. Am. Ceram. Soc. 101.5 (2018), pp. 1892–1905.

- A unique bubble exchanges $\sim 10^{-8}$ mol/s.
- During the glass melting, $N_b \sim 10^8$ bulles/m³.



Figure 8: N_b vs. 2*a* in a molten glass ($T = 1300^{\circ}$ C).

3.2 Bubble population

The consumption (release) of a bubble population is a sink (source) of gas:

$$SO_4^{2-}$$
 $\stackrel{sink}{\underset{\leftarrow}{\leftarrow}}$ $SO_2 + \frac{1}{2}O_2 + O^{2-}.$ (24)

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The mass transfer between the bubble population and the liquid is crucial to the chemical equilibrium after the melting. Study the coupling between the oxidation-reduction and the fining process

3.2 Bubble population

- **Spatially homogeneous medium** \Rightarrow only *t* is considered.
- Medium = liquid + N_{cl} bubble classes, $N_{b,k}$ ($k = 1, N_{cl}$).
- R chemical reactions are involved:

$$\sum_{i=1}^{N_l} \nu'_{ri} A_i + \sum_{j=1}^{N_{ga}} \beta'_{rj} G_j \Longrightarrow \sum_{i=1}^{N_l} \nu''_{ri} A_i + \sum_{j=1}^{N_{ga}} \beta''_{rj} G_j, \quad (25)$$
$$\frac{dC_{A_i}}{dt} = \sum_{r=1}^{R} \nu_{ri} \dot{\zeta}_r, \text{ pour } i = 1 \text{ à } N_l, \quad (26)$$
$$\frac{dC_{G_j}}{dt} = \sum_{r=1}^{R} \beta_{rj} \dot{\zeta}_r + S_{b,G_j}, \text{ pour } j = 1 \text{ à } N_g. \quad (27)$$

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$$\frac{dn_{G_j}}{dt} = 2\pi a \operatorname{Sh}_{G_j} \mathcal{D}_{G_j} \left(\mathcal{C}_{G_j} - \mathcal{L}_{G_j} \mathcal{P}_{G_j}^{\alpha_{G_j}} \right).$$
(28)

For the bubble class k of density N_{b,k}, the gas consumption is

$$N_{b,k}\frac{dn_{G_j}}{dt} = 2\pi a_k \operatorname{Sh}_{G_j,k} \mathcal{D}_{G_j} \left(C_{G_j} - \mathcal{L}_{G_j} P_{G_j}^{\alpha_{G_j}} \right) N_{b,k}.$$
 (29)

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 (29)

► So, S_{b,Gi} is equal to:

$$S_{b,G_{j}} = -2\pi \mathcal{D}_{G_{j}} \sum_{k=1}^{N_{cl}} a_{k} \operatorname{Sh}_{G_{j},k} \left(C_{G_{j}} - \mathcal{L}_{G_{j}} P_{G_{j},k}^{\alpha_{G_{j}}} \right) N_{b,k}.$$
 (30)

- $\dot{\zeta}_r$ is determined assuming instantaneous reactions.
- ► ∀t, the chemical equilibrium is verified and the consumption or the production of gas defined the chemical state.

$$\sum_{i=1}^{N_{l}} \frac{\nu_{ri}}{C_{A_{i}}} \frac{dC_{A_{i}}}{dt} + \sum_{j=1}^{N_{ga}} \frac{\beta_{rj}}{C_{G_{j}}} \frac{dC_{G_{j}}}{dt} = \frac{1}{K_{r}} \frac{dK_{r}}{dt}, \quad (31)$$

$$\sum_{k=1}^{R} M_{rk} \dot{\zeta_{k}} = \frac{d \ln K_{r}}{dT} \frac{dT}{dt} - \sum_{j=1}^{N_{g}} \frac{\beta_{rj}}{C_{G_{j}}} S_{b,G_{j}}, r \in [1; R] (32)$$

$$M_{rk} = \sum_{i=1}^{N_{l}} \frac{\nu_{ri}\nu_{ki}}{C_{A_{i}}} + \sum_{j=1}^{N_{g}} \frac{\beta_{rj}\beta_{kj}}{C_{G_{j}}}. \quad (33)$$

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• Glass: $T = 1200^{\circ}$ C ($\mathcal{R}_{Fe} = 0.2$, $C_{Fe} = 0.1$ % m et $C_{SO_3} = 0.35$ % m) with 5 bubble classes¹¹:

Class, k	1	2	3	4	5
$N_{b,k} (m^{-3})$	$2.2 \cdot 10^{7}$	1.2 · 10 ⁸	1.2 · 10 ⁸	$5.7 \cdot 10^{7}$	1.4 · 10 ⁷
<i>a_k</i> (m)	$4.9 \cdot 10^{-5}$	$9.4 \cdot 10^{-5}$	$1.5 \cdot 10^{-4}$	$2.3 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$

Table 1: Volume densities and initial radii of the 5 bubble classes.

- Only SO₂ and O₂ are considered.
- Initially, the bubbles are only composed by O₂.
 - 1. dT/dt = 10 K/min ;
 - 2. dT/dt = 1 K/min.

¹¹O. Stahlavsky et al.: The bubble effect on the redox state of glass, in: VIII International Seminar on Mathematical Modelling and Advanced Numerical Methods in Furnaces Design and Operation, Velké Karlovice 2005.



Figure 9: *a* (mm) vs. *T* (°C) for the 5 classes at $\frac{dT}{dt} = 10$ K/min.

Figure 10: *a* (mm) vs. *T* (°C) for the 5 classes at $\frac{dT}{dt} = 1$ K/min.

0.35 K/min namic model C_{SO_3} tot. (% mass) 2603 tot. (% mass) 0.3 0,2 0,15 0,1 1200 1300 1400 T (°C) 1500

Figure 11: C_{SO_3} total (% masse) vs. T (°C).

1600

3.2 Bubble population



Figure 11: C_{SO_3} total (% masse) vs. T (°C).

The liquid is oversaturated in sulfate at high heating rate.



Figure 12: \mathcal{R}_{Fe} vs. T (°C).



Figure 12: \mathcal{R}_{Fe} vs. T (°C).

The liquid is more oxidized at high heating rate.



$$N(t) = N_0 e^{-\alpha t}, \tag{34}$$

At
$$T = 1300^{\circ}$$
C, $\alpha = 4.67 \cdot 10^{-4}$ s⁻¹.

Bastick found at $T = 1320^{\circ}$ C, $\alpha = 5 \cdot 10^{-4}$ s⁻¹.

¹²R. E. Bastick: Laboratory experiments on the refining of glass, in: Symposium sur l'affinage du verre, Paris 1956, pp. 127–138.

4. Synthesis

- The thermodynamic model allows us to easily describe the liquid/gas interactions.
- The kinetic model is based on the instantaneous oxidation-reduction reactions.
- The bubble growth rates change strongly with the gas consumption.
- At high heating rate, the glass chemistry is not in the thermodynamic equilibrium.

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