

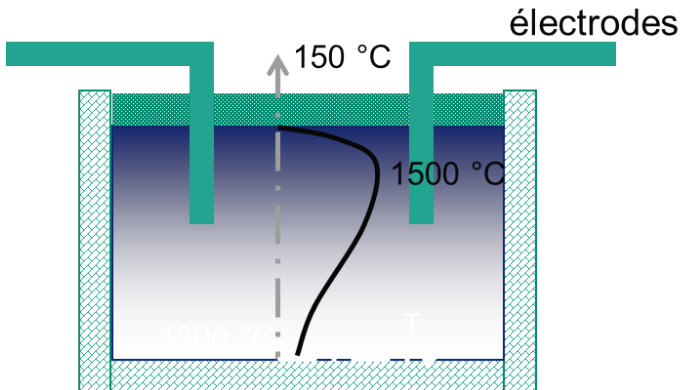
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July, 13th 2018, Saint-Malo ■ ■

■ Thermoconvective instabilities of a non uniform Joule-heated liquid enclosed in a rectangular cavity

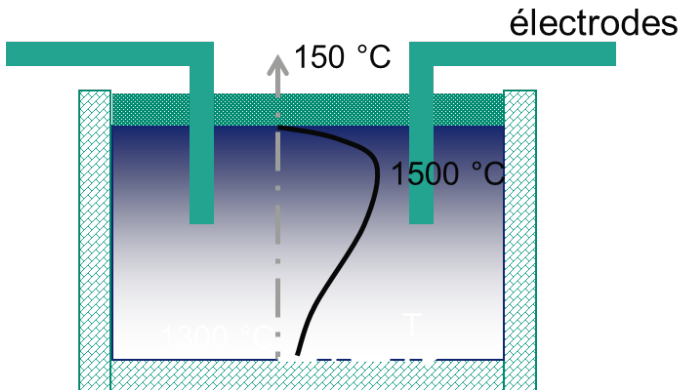
A. Cornet, F. Lop  p   (Saint-Gobain Isover) & F. Pigeonneau

Motivations of the present work



**Joule dissipation provides the heating;
The maximum of T is observed in the bulk;**

Motivations of the present work



Joule dissipation provides the heating;
The maximum of T is observed in the bulk;
Thermally instable.

1. Problem statement
2. Steady-state solutions for $L_e/H = 1$
3. Onset of convection when $L_e/H < 1$
4. Instabilities in a cavity with $L_e/H = 2/3$
5. Conclusion

1. Problem statement I

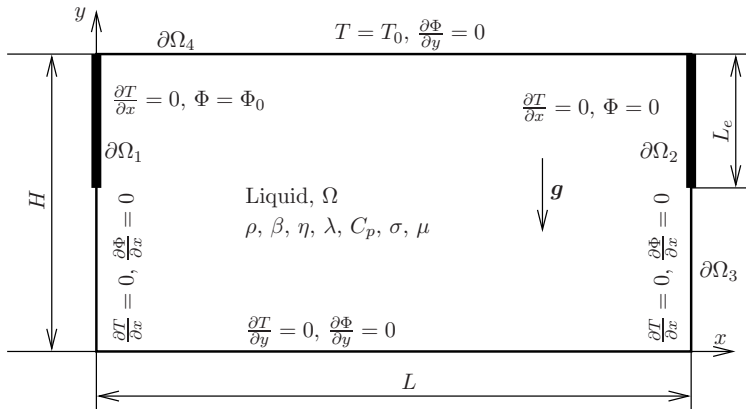


Figure 1: Cavity Ω filled with an electric conductor liquid according to Sugilal, G., et al. (2005). [*Int. J. Therm. Sci.*, **44**:915-925].

▶ Assumptions

- ▶ Boussinesq approximation is used to take into account the buoyancy force.
- ▶ Effects of induced magnetic field \mathbf{B} are neglected.
- ▶ The Lorentz forces are neglected.

▶ Normalization of the problem

- ▶ \mathbf{x} normalized by H ;
- ▶ t by H^2/κ with $\kappa = \lambda/(\rho C_p)$;
- ▶ Φ by Φ_0 ;
- ▶ \mathbf{u} by $\sqrt{\beta \Delta T g H}$;

$$\theta = \frac{T - T_0}{\Delta T}, \text{ with } \Delta T = \frac{\sigma \Phi_0^2 H^2}{2\lambda L^2}. \quad (1)$$

1. Problem statement III

► Balance equations

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

$$\frac{1}{\text{Pr}} \frac{\partial \mathbf{u}}{\partial t} + \sqrt{\frac{\text{Ra}}{\text{Pr}}} \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u} + \sqrt{\frac{\text{Ra}}{\text{Pr}}} \theta \mathbf{e}_y, \quad (3)$$

$$\frac{\partial \theta}{\partial t} + \sqrt{\text{Pr Ra}} \nabla \theta \cdot \mathbf{u} = \nabla^2 \theta + 2L^2 (\nabla \Phi)^2, \quad (4)$$

$$\nabla^2 \Phi = 0. \quad (5)$$

► Dimensionless numbers

$$\text{Ra} = \frac{\beta g \Delta T H^3}{\nu \kappa}, \quad (6)$$

$$\text{Pr} = \frac{\nu}{\kappa}. \quad (7)$$

2. Steady-state solutions for $L_e/H = 1$

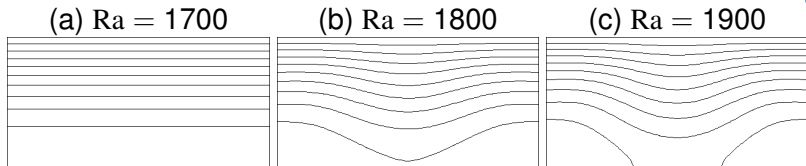


Figure 2: Isolines of θ for (a) $Ra = 1700$, (b) $Ra = 1800$ and (c) $Ra = 1900$.

- ▶ $Ra_{cr} = 1702$.
- ▶ By linear stability, Kulacki and Goldstein¹ found $Ra_{cr} = 1386$.
- ▶ Numerically, Sugilal et al. (2005) found $Ra_{cr} = 1650$.

¹Kulacki, F. A. & Goldstein, R. J. (1975). *Appl. Sci. Res.*, **31**:81-109.

2. Steady-state solutions for $L_e/H = 1$

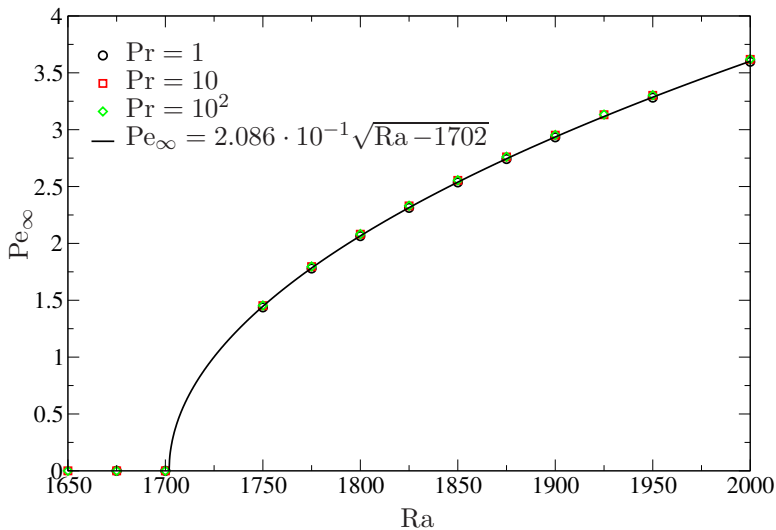


Figure 3: $Pe_\infty = U_{\max} H / \kappa$ vs. Ra for $L/H = 2$, $L_e/H = 1$ and $Pr = 1$, 10 and 100.

3. Onset of convection when $L_e/H < 1$

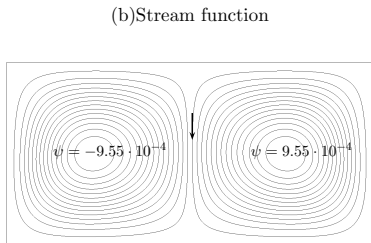
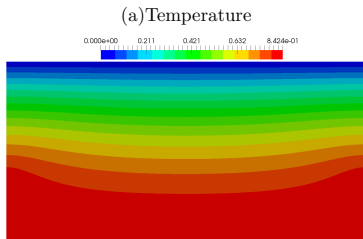


Figure 4: (a) $\theta \in [0, 0.84]$ and (b) stream function ψ for $Pr = 1$, $Ra = 10^2$ and $L_e = 2/3$.

3. Onset of convection when $L_e/H < 1$

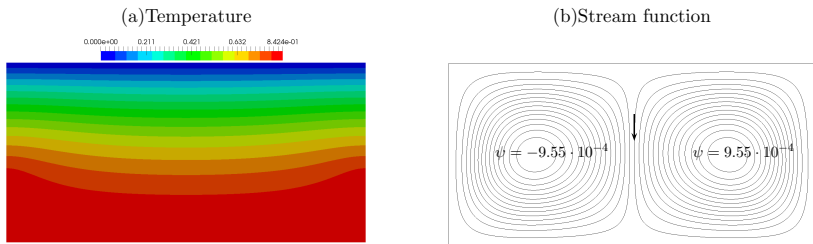


Figure 4: (a) $\theta \in [0, 0.84]$ and (b) stream function ψ for $Pr = 1$, $Ra = 10^2$ and $L_e = 2/3$.

- The threshold disappears when $L_e/H < 1$.

3. Onset of convection when $L_e/H < 1$

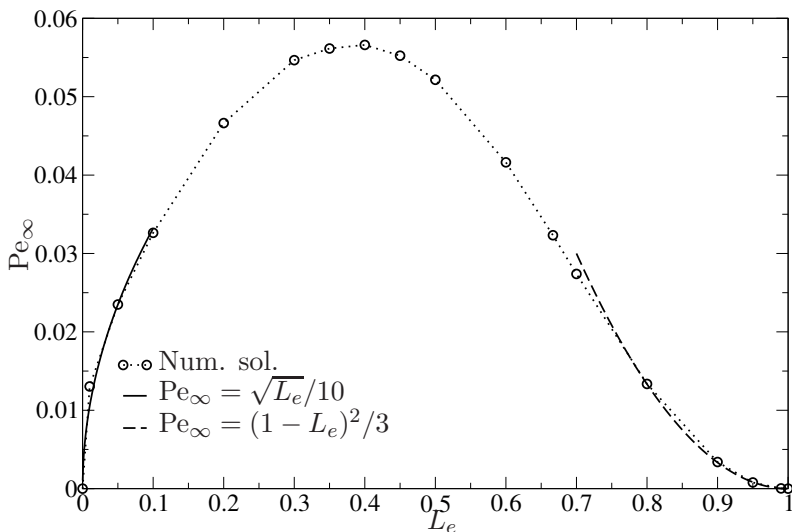


Figure 5: Pe_∞ vs. L_e for $Pr = 1$ and $Ra = 10^2$.

3. Onset of convection when $L_e/H < 1$

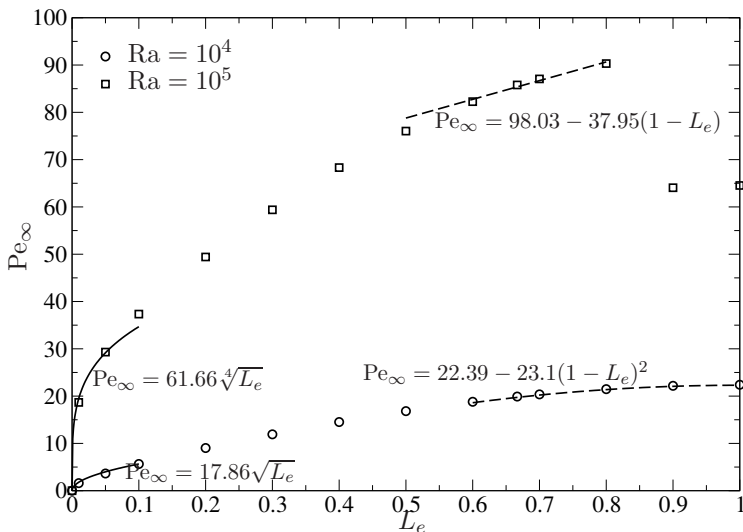


Figure 6: Pe_∞ vs. L_e in the situation where $Ra = 10^4$ or 10^5 and for $Pr = 1$.

3. Onset of convection when $L_e/H < 1$

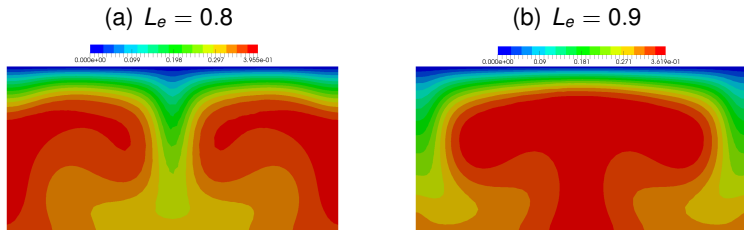


Figure 7: Temperature field obtained for $Ra = 10^5$, $Pr = 1$ and for (a) $L_e = 0.8$, $\theta \in [0; 0.39]$ and (b) $L_e = 0.9$, $\theta \in [0; 0.36]$.

4. Instabilities in a cavity with $L_e/H = 2/3$

Numerical solution for $Ra = 4 \cdot 10^4$ and $Pr = 10^2$

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1. Steady symmetric structure;
2. Steady asymmetric structure;
3. Unsteady asymmetric structure.

4. Instabilities in a cavity with $L_e/H = 2/3$

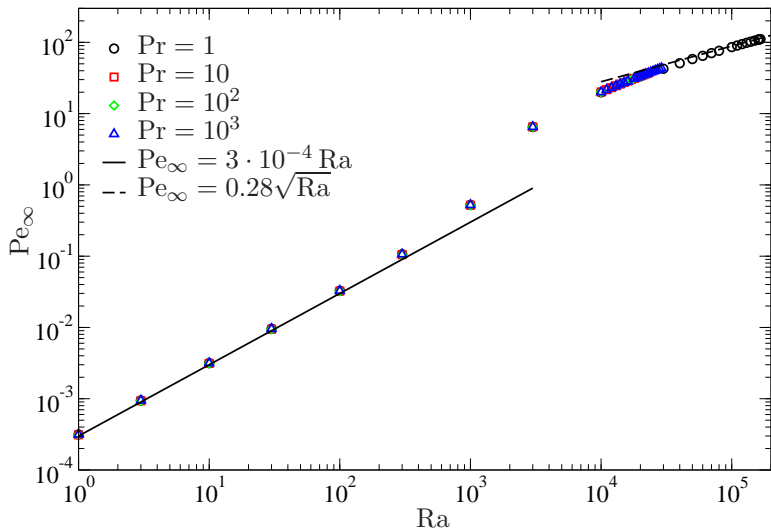


Figure 8: Pe_∞ vs. Ra and $Pr = 1, 10, 10^2$ and 10^3 .

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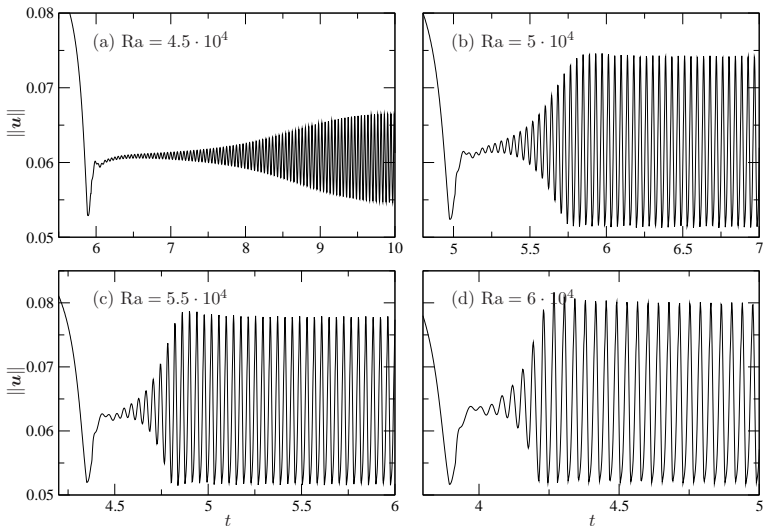


Figure 9: $\|\mathbf{u}\|$ vs. t for $Pr = 10$ and (a) $Ra = 4.5 \cdot 10^4$, (b) $Ra = 5 \cdot 10^4$, (c) $Ra = 5.5 \cdot 10^4$ and (d) $Ra = 6 \cdot 10^4$.

4. Instabilities in a cavity with $L_e/H = 2/3$

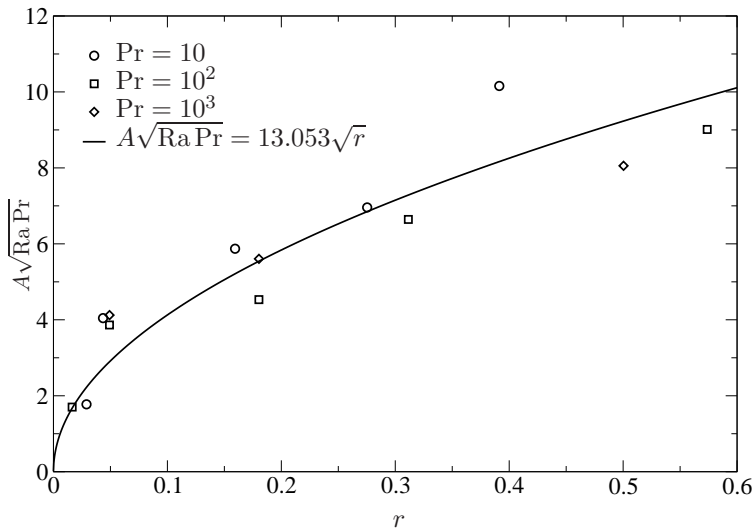


Figure 10: Fundamental amplitude of Fourier spectra $A\sqrt{RaPr}$ vs. $r = (Ra - Ra_{cr2}) / Ra_{cr2}$ for $Pr = 10, 10^2$ and 10^3 .

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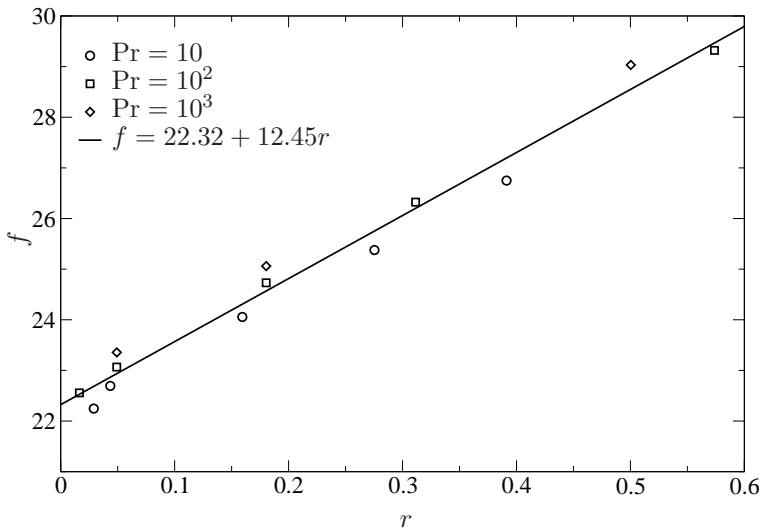


Figure 11: Fundamental frequency of Fourier spectra f vs. $r = (Ra - Ra_{cr2}) / Ra_{cr2}$ for $Pr = 10, 10^2$ and 10^3 .

4. Instabilities in a cavity with $L_e/H = 2/3$

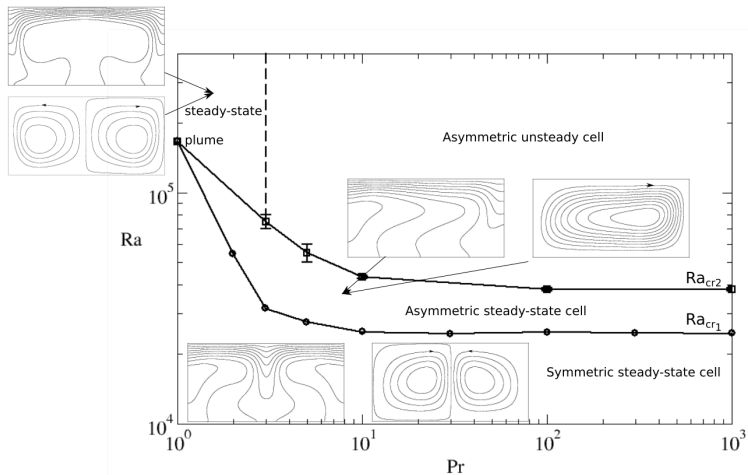


Figure 12: Stability diagram ($Pr, Ra_{cr1,2}$) describing the three main structures obtained numerically when $L_e = 2/3$.

5. Conclusion

- ▶ If $L_e/H = 1$, the critical Rayleigh number is: $Ra_{cr} = 1702$.
- ▶ If $L_e/H < 1$, **convection without threshold**.
- ▶ Three instabilities are established:
 - ▶ The first one is similar to the transcritical bifurcation.
 - ▶ The second is subcritical bifurcation.
 - ▶ The third is similar to the Hopf bifurcation.
- ▶ From the results of oscillation frequencies, we can find:
 - ▶ with physical properties of glass wool, $t_{osc} \approx 1$ h.
- ▶ Heat transfer has been also studied².
- ▶ To be verified in industrial plants.
- ▶ Extend this work to 3D configuration.

²Pigeonneau, F., Cornet, A. & Lop  p  , F. (2018). *J. Fluid Mech.*, **843**:601–636.