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## A novel monolithic Lagrangian approach for modelling crack propagation using anisotropic mesh adaptation

Research Article

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**Abstract:** A monolithic Lagrangian finite element method is explored to model crack propagation. This approach is based on the level-set method coupled with anisotropic remeshing to define the crack faces and tip. Moreover, the  $G\theta$  method is used for computing the strain energy release rate and the propagation direction. Furthermore, a new technique for computing the convection velocity, which is a mixture between the propagation speed and the mechanical velocity, is introduced.

**MSC:** 74R99 • 74S05

**Keywords:** Crack propagation • Mesh adaptation •  $G\theta$  method • Monolithic approach • Lagrangian framework

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### 1. Introduction

Since the last century, a large number of theories and applications have been developed to deal with fracture mechanics and crack propagation [1–8]. The latter gives rise to many difficulties since it leads to an important displacement discontinuity. The Finite Element Method (FEM) [4, 5, 9] has proved its efficiency in dealing with crack propagation by modifying the mesh's topology and performing automatic remeshing [4, 5, 10–14].

Nevertheless, several remeshing-free methods have been proposed: Belytschko has suggested the Element free Galerkin method [3] where the discretisation is achieved by nodal and surface description of the model. The Strong Discontinuity Approach (SDA) [15–17] in which displacement jumps due to the presence of the crack are embedded locally in each cracked finite element without affecting neighbouring elements. The Extended Finite Element Method (XFEM) [18–20] in which the displacement-based approximation is enriched near a crack by incorporating both discontinuous fields and the near tip asymptotic fields through a Partition of Unity method [21]. The arbitrary local mesh replacement method [22] based on two distinct meshes: one that surrounds the propagating crack front and moves with it, and the other one that fills the rest of the domain. However, these methods need improvement in order to

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properly deal with long crack propagation and crack coalescence such as the XFEM. Or also complex configurations such as the Element free method, and multiple cracks such as the arbitrary local mesh replacement method. And some of them, like the XFEM, would eventually need some type of remeshing.

Hence, remeshing techniques are important when dealing with crack propagation. Chiaruttini *et al.* [10] used remeshing along with the Cohesive Zone Model (CZM) to simulate crack propagation in structures subjected to mixed mode loadings. Furthermore, 3D remeshing techniques were used to model properly crack propagation using the computation of a damage indicator [11] or also based on mesh intersection algorithms [12]. Two and three-dimensional remeshing, via the fracture numerical code FRANC [13, 14], were also used to simulate crack growth. Remeshing, combined with nodal relaxation, was developed and used by Bouchard *et al.* [4, 5] and it has proven its efficiency when dealing with complex and multiple crack propagations.

Furthermore, monolithic approaches, which consist in using a unique mesh for the different components, are widely used in various domains. Notably in fluid-structure interaction [23], heat transfer [24], incompressible fluid flows [25], microstructures' behaviour [26, 27]. In all the above-cited applications, the monolithic approach proved its efficiency. Hence, studying its feasibility in crack propagation is proposed in this paper. In this context, two frameworks, the Eulerian and Lagrangian can be compared. The Eulerian framework consists in describing the crack implicitly using a level-set function and thus the air inside the crack is taken into account in the governing equations. Also this level-set function must be convected at each time step in order to simulate its propagation. On the other hand, in the Lagrangian framework the crack is explicitly described by nodes of the mesh and its propagation is performed by moving these nodes, and remeshing.

In this paper, a method for modelling quasi-static crack propagation using level-set functions, anisotropic remeshing and a novel method for choosing the convection velocity is introduced. In the second section, the level-set method and the anisotropic remeshing technique are explained. Also the  $G\theta$  method used for computing the energy release rate along with the kinking angle is detailed. Section 3 introduces our method for computing the convection velocity of the crack. A comparison between results in an Eulerian and a Lagrangian framework is performed. Then the better one is compared with Bouchard *et al.*'s [5] technique. The fourth section concludes this paper.

All numerical calculations mentioned in this paper were performed with CimLib, a finite element C++ library [28].

## 2. Numerical modelling

### 2.1. Level-set and mesh adaptation

In the perspective of modelling crack propagation, a monolithic approach is used. The monolithic method consists of using a unique mesh in which the different domains are considered using a level-set function. Moreover a level-set function is a signed distance function  $\alpha$ , defined over the domain  $\Omega$ , that gives at any node  $X$  of coordinates  $\mathbf{x}$  of the FE mesh the signed distance to the boundary  $\Gamma$ . In turn, the interface  $\Gamma$  is given by the level 0 of the function  $\alpha$ :

$$\begin{cases} \alpha(\mathbf{x}) = \pm d(\mathbf{x}, \Gamma), \mathbf{x} \in \Omega \\ \Gamma = \{\mathbf{x} \in \Omega, \alpha(\mathbf{x}) = 0\} \end{cases} \quad (1)$$

Fig. 1 illustrates the level-set function of a crack.

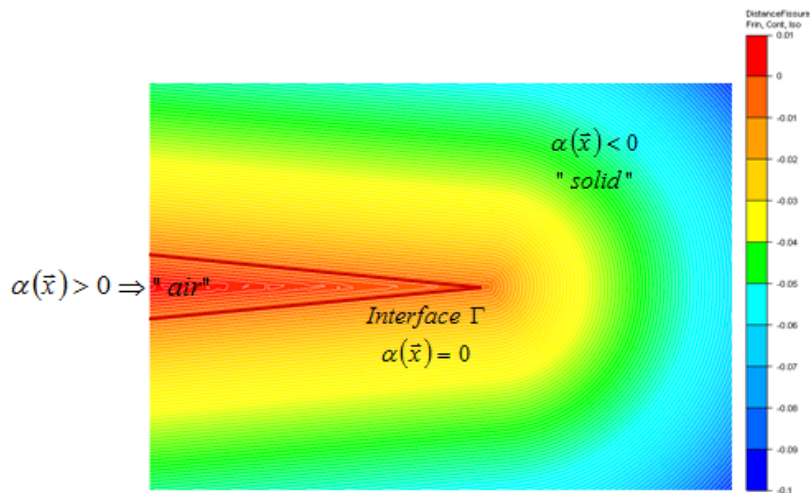


Fig. 1. The level-set function describing a crack

Consequently, different structures are immersed in a larger domain of different material properties so that boundary conditions at the interface can be replaced naturally. Furthermore, the level-set function, defining the interface, is used to mix the different material properties using appropriate mixing laws [26, 29]. Moreover, this function is used for meshing and remeshing operations in order to describe properly the considered interface and also to account for the discontinuities of the physical properties. Mesh adaptation is also needed at the crack tip to insure the computational accuracy of the singular fields. It has to be noted that, contrary to XFEM, these singular fields are not imposed in the shape functions of the elements containing the crack tip.

To this end, it is crucial to pre-adapt anisotropically the mesh at the materials' interface, the crack faces and tip. By doing so, the mesh becomes locally refined, elements are stretched, which enables to sharply define the interface and to save a great number of elements compared to classical local isotropic refinement. This anisotropic adaptation is performed by constructing a metric map. This map is controlled by a directional error estimator based on local interpolation error of the wanted fields which are in this case the level-set functions defining the interface, the crack faces and the pressure field. Furthermore, in our case, different metrics are obtained, one for each field. The intersection of these metrics [30] leads to the one used for mesh adaptation.

In the aim of constructing these metrics, an *a posteriori* error estimator [31] is used. This error estimator uses the eigenvalues and eigenvectors of the recovered Hessian matrix of a given function and a wanted number of elements in order to construct the metric field. In practice, the mesh is generated in several steps using the MTC mesher and remesher developed in [32]. The proposed mesh generation algorithm works well for 2D or 3D complex shapes. It allows the creation of meshes with extremely anisotropic elements stretched along the interfaces. The mesh size is then only refined in the direction of the high physical and mechanical properties gradients. This allows conserving a high accuracy for describing properly mechanical fields and geometry description. During propagation, the mesh is only modified in the vicinity of the interface and near the crack tip which keeps the computational work devoted to the mesh generation low.

Since we will begin by comparing our method in Eulerian and Lagrangian frameworks and the better approach with the one in [5], Fig. 2 shows the result of the mesh adaptation technique on the construction of the initial mesh of the comparison case, which is a pre-cracked rectangular region with an off-center inclusion. Here anisotropic remeshing was applied using the level-set functions of the crack and the disc along with the pressure which is maximal at the crack tip.

## 2.2. Computing the strain energy release rate

The strain energy release rate  $G$  represents the energy required for a unitary crack increase. The criterion states that among all virtual and kinematically admissible crack propagation directions, the real increase is the one which maximizes the strain energy release rate. The kinking angle,  $\theta_0$ , is then determined by:

$$\begin{cases} \left( \frac{dG}{d\theta} \right)_{\theta=\theta_0} = 0 \\ \left( \frac{d^2G}{d\theta^2} \right)_{\theta=\theta_0} \leq 0 \end{cases} \quad (2)$$

Numerous numerical techniques can be used to compute  $G$  [5]. In this study, the  $G\theta$  method [33] is used. It consists in defining two contours  $C_1$  and  $C_2$  around the crack tip. These contours divide the domain near the crack tip into three sub-domains:  $C_{ext}$ ,  $C_{ring}$  and  $C_{int}$  (see Fig. 3).

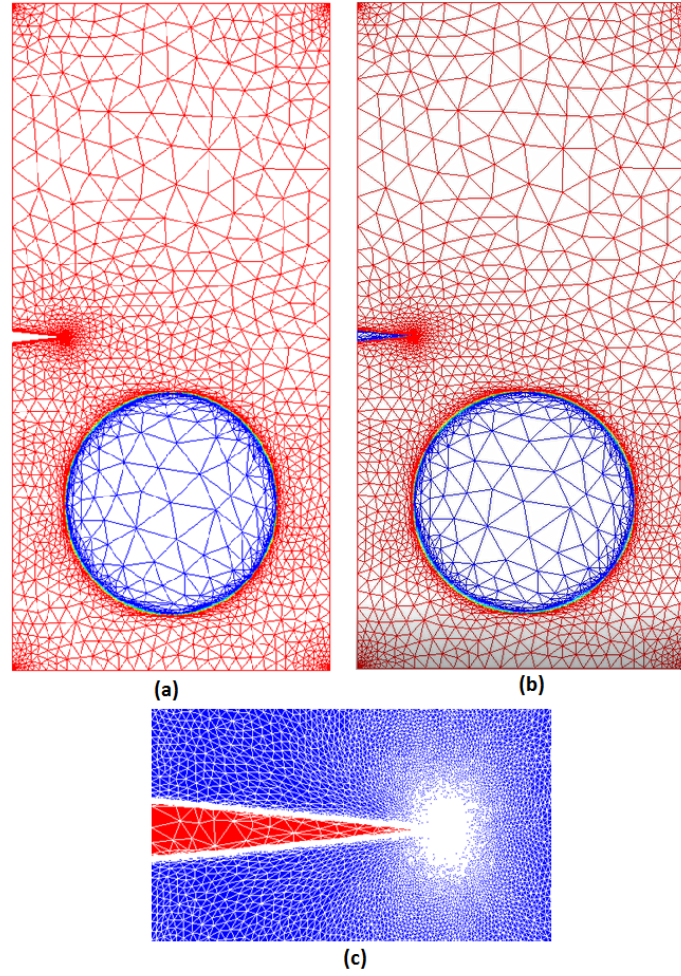
Afterwards, the strain energy release rate computation is performed by solving:

$$G = \int_{C_{ring}} \left[ Tr(\sigma \nabla \mathbf{u} \nabla \Theta) - \frac{1}{2} Tr(\sigma \nabla \mathbf{u}) \nabla \cdot \Theta \right] dC_{ring}. \quad (3)$$

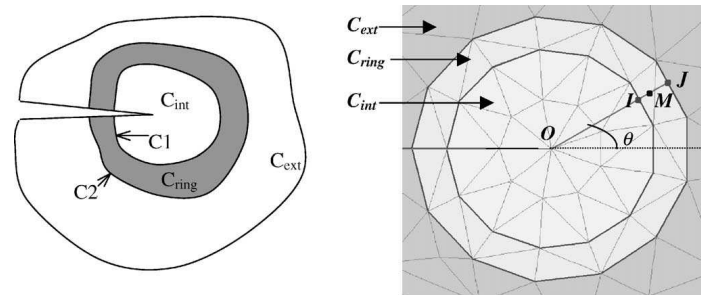
where  $Tr(\cdot)$  is the trace operator,  $\sigma$  is the stress tensor,  $\mathbf{u}$  is the displacement field, and  $\Theta(\theta_1, \theta_2)$  is a virtual unit displacement field that may be expressed as:

$$\begin{cases} \theta_1 = (1 - \frac{IM}{I}) \cos(\theta) \\ \theta_2 = (1 - \frac{IM}{I}) \sin(\theta) \end{cases} \quad (4)$$

Therefore, this virtual field  $\Theta$  modifies only the shape of elements belonging to  $C_{ring}$ . The computation of  $G$  is performed in the  $C_{ring}$  area and a strain energy release rate  $G$  can be computed for each possible direction of crack propagation,  $\theta$ . The real direction of propagation  $\theta_0$  is the one maximizing  $G$  during propagation.



**Fig. 2.** Anisotropic meshing adaptation based on an *a posteriori* error estimation: a) in the Lagrangian framework, b) in the Eulerian framework and c) zoom at the crack tip



**Fig. 3.** Contours and domains used to compute  $G$  with the  $G\theta$  method [5]

### 2.2.1. Comparison with analytical values

The  $G\theta$  method has been implemented in CimLib [28]. At each crack increment,  $G$  is computed for  $\theta$  varying from  $-70^\circ$  to  $70^\circ$  with  $1^\circ$  steps.  $G$ 's curve is increasing and then decreasing (see Fig. 4) so that the determination of the angle  $\theta_0$  corresponding to the maximum strain energy release rate is straightforward.

In order to validate our implementation and use the  $G\theta$  method for computing strain energy release rates, two comparisons with known analytical results were performed. The first with a single edge crack (see Fig. 5 for details and dimensions) and the second for a slant crack (see Fig. 6 for details and dimensions)

In the case of the single edge crack ( Fig. 5), the strain energy release rate is  $G = K_I^2/E$  where  $E$  is the Young modulus and  $K_I$ , the stress intensity factor in mode I, is given by [34]:

$$K_I = \sigma_n F \sqrt{\pi a} \quad (5)$$

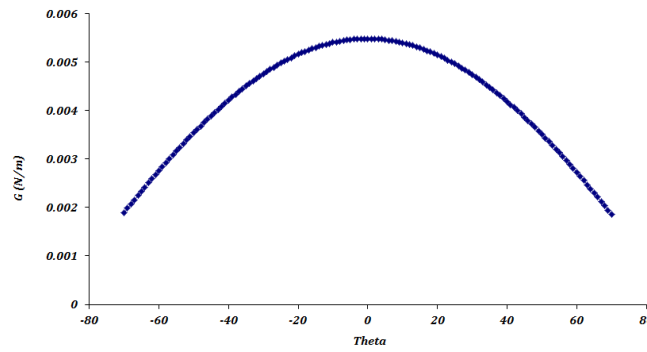


Fig. 4. Curve of G as a function of  $\theta$  in case of a single edge crack

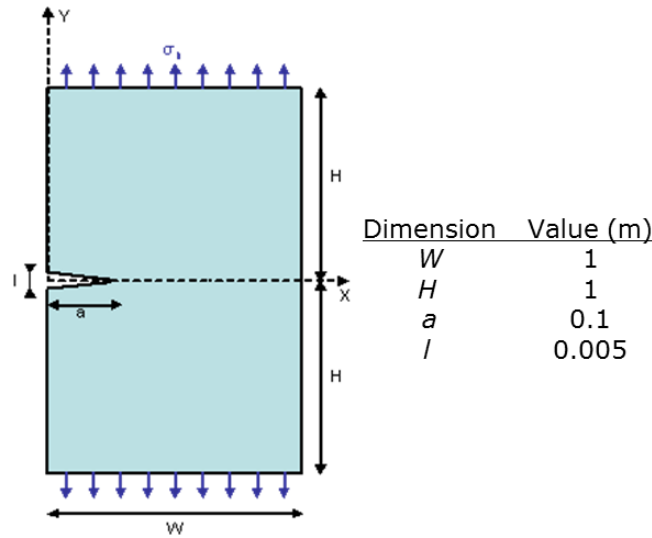


Fig. 5. Domain and dimensions used in the single edge crack case

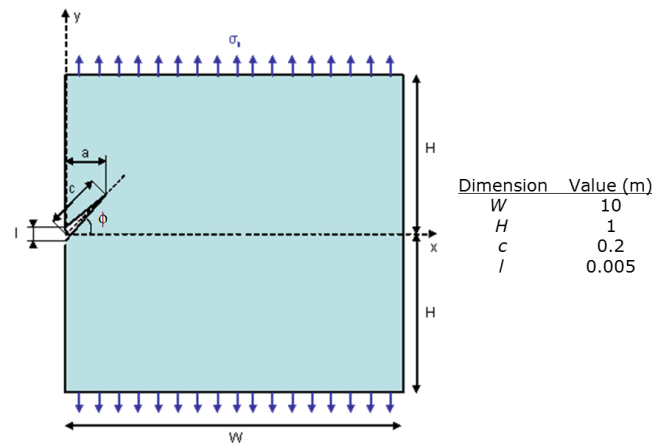


Fig. 6. Domain and dimensions used in the slant crack case

with  $F = 1.0174/\lambda^{3/2}$  where  $\lambda = 1 - aW$  and  $\sigma_n$ , the normal stress, taken to be  $50MPa$ . Analytically,  $G = 5.31 \times 10^{-3} N/m$  and the obtained numerical result was  $G = 5.4 \times 10^{-3} N/m$

In the case of the slant crack,  $G = \frac{(K_I^2 + K_{II}^2)}{E}$  where  $K_I$  and  $K_{II}$  (the stress intensity factor in mode II) are given by [34]:

$$K_{I,II} = \sigma_n F_{I,II} \sqrt{\pi c} \tag{6}$$

with

$$F_I \approx 0.5474 + 0.5738 \cos(1.3225\phi) \tag{7}$$

$$F_{II} \approx 0.4741 \sin(1.4675\phi) \quad (8)$$

where  $\phi = 45^\circ$  and  $\sigma_n$  also taken to be 50MPa.

Analytically,  $G = 6.6 \times 10^{-3} \text{ N/m}$  and the obtained numerical result was  $6.45 \times 10^{-3} \text{ N/m}$

These results validate our implementation and use of the  $G\theta$  method for computing strain energy release rate and hence the kinking angle.

### 3. Convection velocity

#### 3.1. Mechanical equations

After mixing the different material properties, the incompressible Stokes equations modelling the mechanical problem are solved using a stabilized mixed FE method and a P1+/P1 interpolation. These equations can be written:

$$\begin{cases} \mu \Delta \mathbf{v} - \nabla p = -\mathbf{g} & \text{on } \Omega \\ \Delta \cdot \mathbf{v} = 0 \\ \mathbf{v} = \mathbf{v}_{\text{imp}} & \text{on } \Gamma_u \\ \mathbf{v} = -\mathbf{v}_{\text{imp}} & \text{on } \Gamma_l \end{cases} \quad (9)$$

where  $\mathbf{v}$  is the velocity,  $p$  the pressure,  $\mathbf{g}$  the gravity,  $\Omega$  is the computational domain, and  $\mu = \frac{E}{1+\nu}$  is the first Lamé coefficient where  $E$  is Young's Modulus and  $\nu$  is Poisson's ratio. Also  $\mathbf{v}_{\text{imp}}$  is the imposed velocity on  $\Gamma_u$  and  $\Gamma_l$  which are the upper and lower boundaries of the domain respectively.

#### 3.2. Computation of the convection velocity

After obtaining the velocity and the pressure fields from the mechanical problem, the computation of  $G$  infers the calculations of the propagation direction,  $\theta_0$  and hence the speed at the crack tip,  $\dot{a}$  which can be written [35]:

$$\dot{a} = \begin{cases} 0 & \text{if } G < G_c \\ c_R \sqrt{1 - \frac{K_{Ic}}{K_I}} & \text{otherwise} \end{cases} \quad (10)$$

This speed is greater than zero only if  $G \geq G_c$  which is the fracture energy in function of  $K_{Ic}$ , the fracture toughness;  $c_R$  is the Rayleigh wave speed. Hence, a velocity around the crack tip,  $\mathbf{v}_{ct}$ , can be given by:

$$\mathbf{v}_{ct} = \dot{a}(\cos\theta_0, \sin\theta_0). \quad (11)$$

This velocity insures the proper propagation of the crack but not the crack opening. The latter can be insured by the mechanical velocity,  $\mathbf{v}$ . These facts lead us to use a Lagrangian convection velocity which is the combination of  $\mathbf{v}_{ct}$  and  $\mathbf{v}$ . This convection velocity, denoted  $\mathbf{v}_{convec}$ , is given by:

$$\mathbf{v}_{convec} = \begin{cases} \mathbf{v} & \text{if } \dot{a} = 0 \iff G < G_c \\ \mathbf{v}_{ct}(H(\beta)) + \mathbf{v}(1 - H(\beta)) & \text{if } \dot{a} \neq 0 \iff G \geq G_c \end{cases} \quad (12)$$

where  $\beta$  is the level-set function of a small circle centered at the crack tip and  $H(\beta)$  a smoothed Heaviside function used for mixing the different velocities on a certain predefined thickness.

#### 3.3. Results

In all the below-studied cases, anisotropic mesh adaptation is performed, at each time step, to maintain proper propagation and computation of the singular fields at the crack tip. Hence each time step can be illustrated by the scheme in Fig. 7.

Moreover, a rectangular domain with an off-center circular inclusion described in Fig. 8 will be used in all the comparative cases.

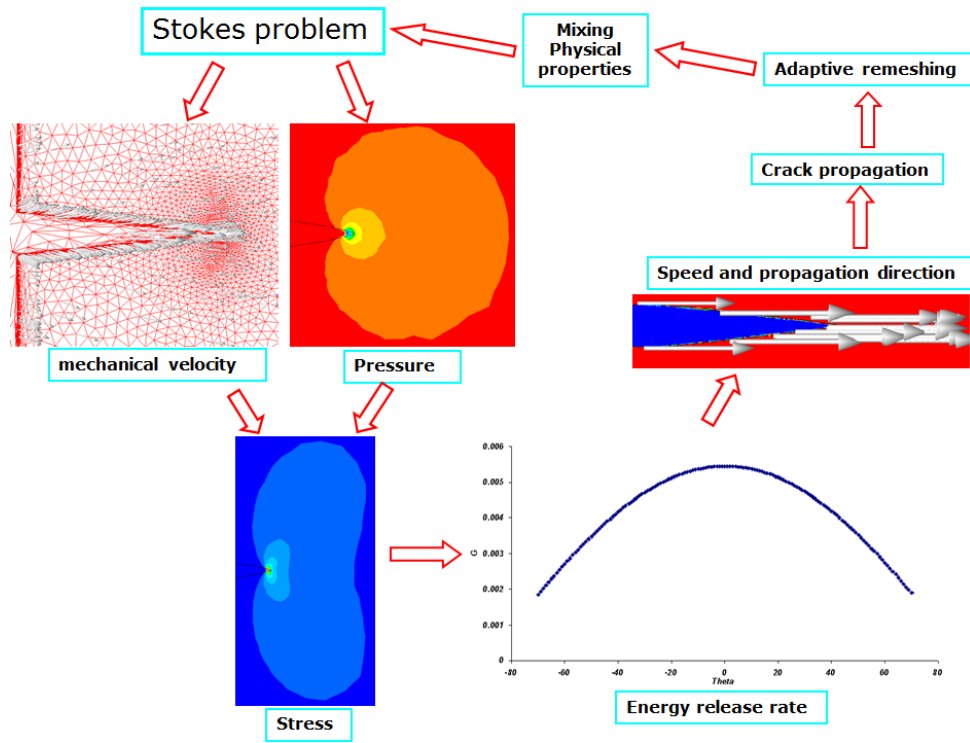


Fig. 7. The scheme of a time step

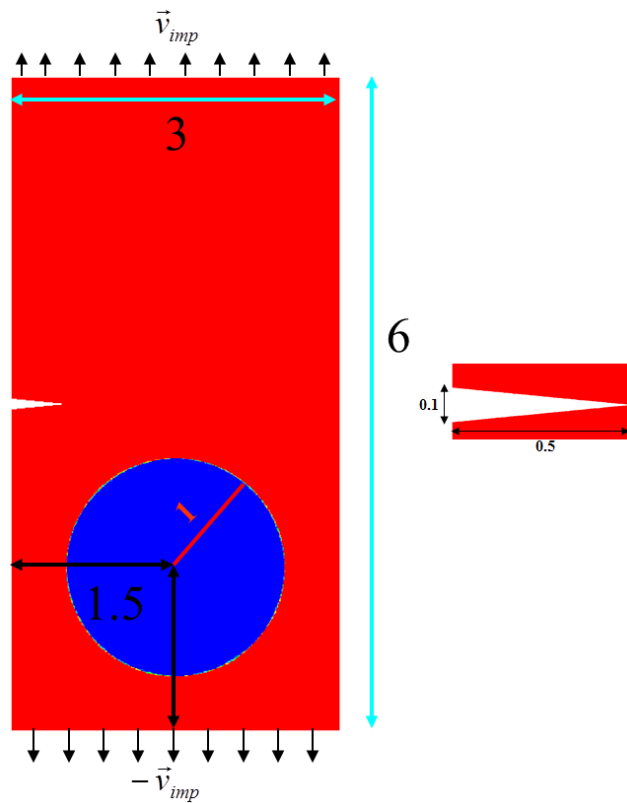
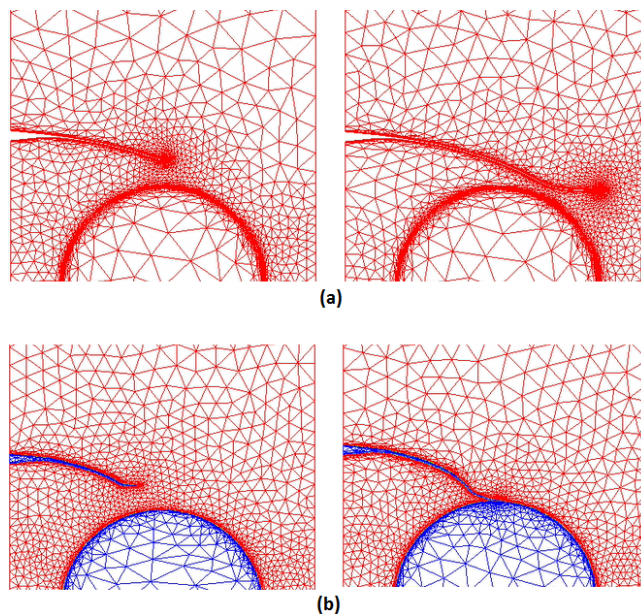


Fig. 8. The domain of the comparative case

### 3.3.1. Comparison between Eulerian and Lagrangian

Before comparing our method with another one, a choice between the Eulerian and Lagrangian frameworks has to be made. Hence, two crack propagation cases (one in each framework) in which the circular inclusion in the pre-described domain is less rigid than the matrix, were performed. Fig. 9 shows the comparison between the two frame-





**Fig. 9.** Comparison between the Lagrangian and Eulerian framework during crack propagation: a) the Lagrangian one and b) the Eulerian one

works at different time steps.

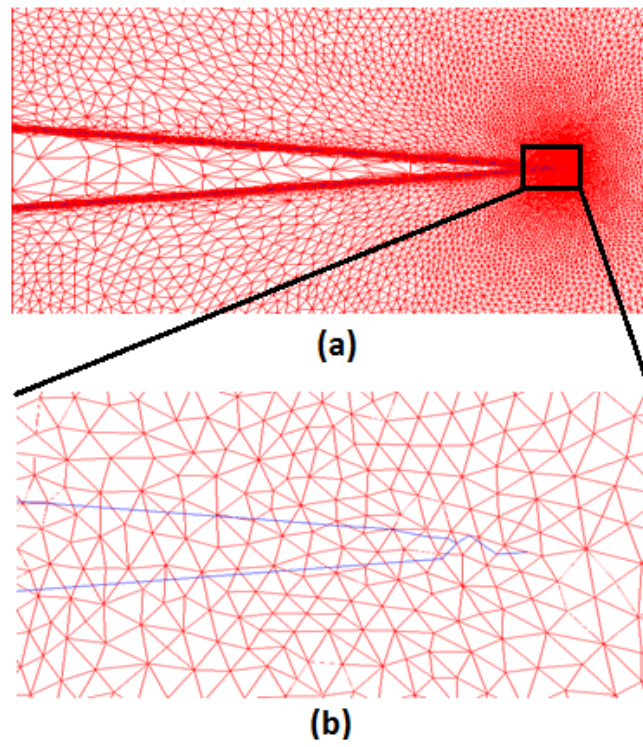
In this case, according to Bouchard *et al.* [5], the crack has to be drawn by the inclusion but not to the point of intersection. It can be seen that using a Lagrangian framework, the crack moves away from the inclusion before hitting it which is not the case in the Eulerian framework. This is due to the fact that in an Eulerian framework a level-set function defines the crack and this function needs to be convected or transported at each time step in order to simulate the propagation.

This type of convection is very delicate because it is largely affected by the mesh size and the time step and since the level-set function would, in general, lose its characteristic as a distance function and has to be reinitialized. This is problematic when the remeshing technique depending on the distance property is used at the interface. Further details can be found in [36]. A solution to this problem might lie in the use of the direct reinitialization technique [37]. It consists in discretizing the interface (isozero of the level-set function) into a collection of simple elements and, for every node, computing the distance to all elements of the collection and storing the smallest one which becomes the updated value of the distance function. This method was considered very time consuming [38] but efficient parallel implementation and computation reduced drastically its computational times [37].

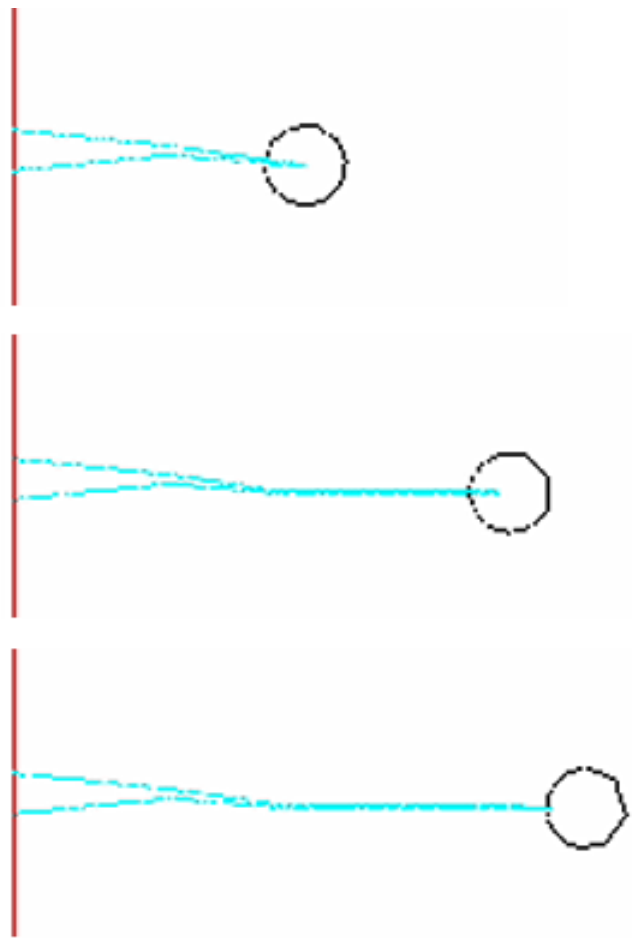
Also capturing accurately the crack tip via a level-set function is very hard since it may lie inside an element of the mesh (see Fig. 10) unless extreme refinement near the tip which, on the other hand, would lead to expensive computational times.

Body-fitted meshing [39] used for analysis of void growth and coalescence could be a solution if used near the crack tip. It consists on defining the interface (the isozero of the level-set) by nodes of the mesh and redefining this interface after convection using body-fitted remeshing. In our case, this might solve the problem of capturing the crack tip since it would be a node of the finite element mesh.

Furthermore, in our case, in addition to the level-set defining the crack, a level-set function defining the circle used for mixing the velocities near the crack tip is used. This circle needs also to be displaced along with the crack tip which makes things more complicated because any small difference in the velocity at neighboring nodes might lead to misplacement of this circle. Fig. 11 shows the case where the circle is convected faster than the crack leading to an unusual propagation of the crack.



**Fig. 10.** (a) The fine mesh near the tip (b) the inaccurate definition of the level-set function defining the crack at the tip



**Fig. 11.** Convection of the crack and the circle near its tip.

Since working in a Lagrangian framework was less complicated and lead to better results than an Eulerian one, the following case will be based on the Lagrangian one. It is important to mention that working in the Eulerian framework could eventually give good results if explored furthermore but surely needs extreme remeshing especially near the crack tip or needs the use of one of the above-mentioned possible solutions. On the other hand, working in the Eulerian framework might be necessary for some applications notably the hydraulic fracking [40, 41] where a fluid is present inside the crack and its pressure on the crack's faces needs to be computed.

### 3.3.2. Comparison with Bouchard *et al.* [5]

Our method, in the Lagrangian framework, is compared with a case presented in [5] (see Fig. 8). The influence of this inclusion on the crack path is studied. The rectangular domain was submitted to a tensile test. The circular inclusion may be more or less rigid than the matrix.  $R$  is defined as the ratio of the matrix and inclusion's Young modulus:  $R = E_{matrix}/E_{incl}$ .

Figs. 12 and 13 illustrate the comparison between our method and Bouchard *et al.*'s [5]. Fig. 12 shows that for an inclusion less rigid than the matrix ( $R = 10$ ), the crack is attracted to the inclusion. Conversely, if the inclusion is more rigid than the matrix ( $R = 0.1$ , Fig. 13), the crack is moving away from the inclusion.

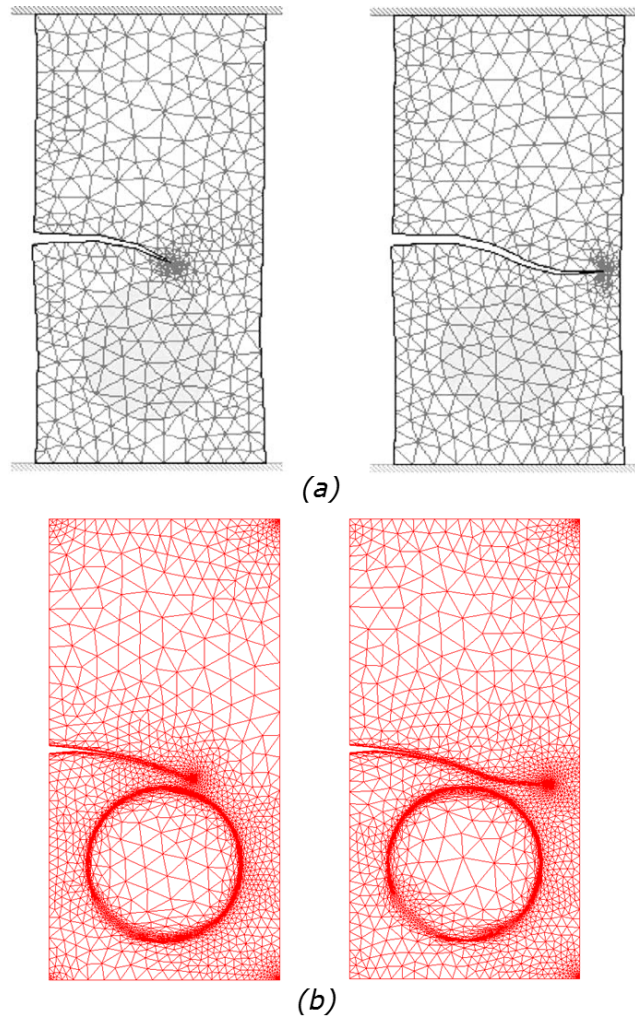
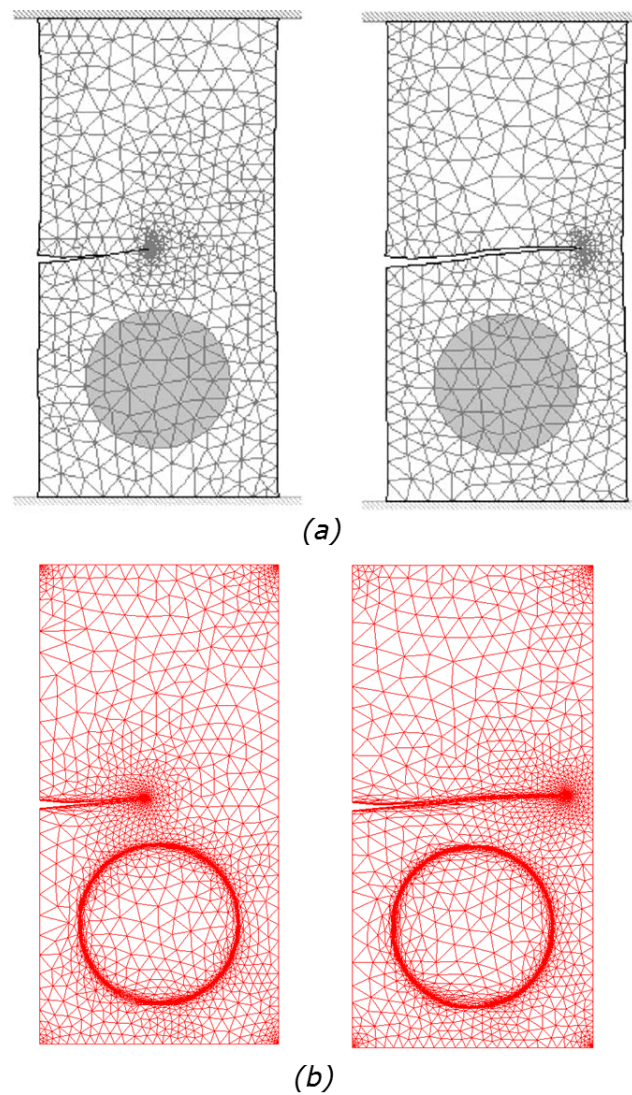


Fig. 12. Comparison between our method and the one in [5] for  $R = 10$ : a) Results of [5] and b) our results



**Fig. 13.** Comparison between our method and the one in [5] for  $R = 0.1$ : a) Results of [5] and b) our results

Both figures show a very good agreement between the two methods. This validates that our technique computes the strain energy release rate and the kinking angle in a proper manner. Furthermore, our choice of Lagrangian convection velocity models correctly crack propagation.

#### 4. Conclusion

A monolithic method was introduced to model crack propagation. This process is based on the use of level-set functions for interface description, on anisotropic mesh adaptation and on the choice of an appropriate convection velocity. The latter is a combination between the mechanical velocity and the speed at the crack tip. This speed is linked to the strain energy release rate which was computed by the  $G\theta$  method along with the kinking angle.

Furthermore, in the monolithic method, two frameworks, the Eulerian and Lagrangian, were compared. In our case, working in the Lagrangian framework was less complicated since it needed less remeshing and since the crack was convected naturally with the mesh. Also since in the Eulerian framework, extremely fine meshes are needed to properly describe the crack faces and tip. And the crack, described implicitly by a level-set function, is hard to convect.

Moreover, our monolithic Lagrangian method proved its efficiency in correctly modelling the propagation of the crack after comparing it with the method in [5]. Nevertheless, the Eulerian framework might be essential in some applications like the hydraulic fracturing in order to take the fluid inside the crack into consideration. A solution to the weak points of the Eulerian framework could be in using a body-fitted immersed volume method at the crack tip as described in [39] or also in use of direct reinitialization [37] to solve the problem of level-set convection.

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