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# The Motion of AC and DC Plasma Arcs under Transverse Cross-Fields: An Analytical Approach

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**Abstract:** Magnetic and dynamic cross-fields have been widely used in plasma torches for multiple purposes: reduction of electrode erosion, increase of the operating power and possibility of control and stabilization of the plasma arc. This paper presents a new analytical approach to model the behaviour of the arc under cross-fields. It is very useful because it provides basic information on the arc without having to resort to costly simulations. An MHD numerical model is used for validation.

**Keywords:** Plasma Physics, AC and DC Plasma Arcs, Cross-fields, Plasma Torches

## 1. Introduction

Multiple investigations have been carried out over the past decades in order to understand the behaviour of the plasma arc under the effect of transverse cross-fields, but the theoretical grounds remain very weakly established, in particular for the AC arc. The study conducted in this paper is part of a wide research that PERSEE is leading in the field of plasma physics and applications [1]. The analytical approach adopted in this paper has great advantage over MHD simulation because it can provide basic information about the arc motion and its characteristics by using simple analytical expressions and equations, without having to resort to costly MHD computations [2].

It is well known that arcs can be stabilized by curvature when exposed to cross-fields. Many theoretical studies have investigated the DC curved arc under transverse fields using the technique of identification of the energy equation (Elenbaas-Heller equation) in the Frenet-Serret referential [3-5]. In this paper, we will extend this technique to solve for the motion of the curved AC arc. All the characteristics and properties of the AC arc are obtained from expressions found in [6]. The analytical model is validated using an MHD model. This approach is of extreme importance in order to establish stability and possibly control of the voltage of the arc.

## 2. General assumption

The study has been conducted for a tip-to-tip electrode torch as shown in figure 1. It's common fact that the model includes several coupled electromagnetic and dynamic equations with insurmountable complexity due to the non-linearity that can be found therein. Therefore, in order to solve the model analytically, some stringent assumptions must be established first:

- Negligible radiation [7].
- Negligible near-electrode phenomenons; they have little influence on the behaviour of the arc column.

- Constant plasma thermal diffusivity, viscosity and arc radius [2,6]

## 3. Governing equations

We will be thoroughly looking into three main equations that govern the motion of the arc under transverse cross-fields: the equation of conservation of momentum, the energy balance and the Maxwell-Faraday equation. They are respectively expressed here below. Then we will solve them in the Frenet-Serret coordinate system figure 2.

$$\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot (\rho \vec{V} \otimes \vec{V}^T + \vec{P}) = \nabla \cdot \vec{\tau}_{Maxwell} \quad (1)$$

$$\frac{1}{\lambda} \left( \frac{\partial S}{\partial t} + \vec{W} \cdot \nabla S \right) = \Delta S + \sigma E^2 \quad (2)$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (3)$$

$\vec{P}$  and  $\vec{\tau}_{Maxwell}$  are the pressure and Maxwell tensors.  $\rho$  and  $\lambda$  are respectively the density and the thermal diffusivity.  $\vec{E}$  is the electric field. It's important to notice that the second equation, known as the Elenbaas-Heller equation, describes the energy balance.  $S$  is the heat potential and is given by the following relation [2-4,6,8]:

$$S = \int_{T_0}^T k(T') dT' \quad (4)$$

Here  $T_0$  is a reference temperature and is taken to be the temperature above which conduction occurs.  $k$  is the thermal conductivity.

$\vec{W}$  in equation (2) corresponds to the relative velocity of the surrounding gas flow with respect to the arc velocity and it is expressed by:

$$\vec{W} = \vec{V}_g - \vec{V}_a \quad (5)$$

Moreover, we already have an expression of  $S$  for the AC arc that is found in [6]:

$$S(r, t) = \frac{I_0 J_0(q_1 \frac{r}{a})}{2\sqrt{B}\pi a J_1(q_1)} \sqrt{1 - \sin\delta \sin(2\omega t + \delta)} \quad (6)$$

Here,  $\delta = \text{arccot}(\omega\theta)$  and  $\theta = \frac{a^2}{\lambda q_1^2}$  are characteristics of the arc.  $a$  is the arc radius and  $q_1$  is the first zero of the Bessel function  $J_0$ .  $I_0$  is the maximum current. A similar expression is found in [8] for the DC arc:

$$S(r) = \frac{I_0 J_0(q_1 \frac{r}{a})}{2\sqrt{B}\pi a J_1(q_1)} \quad (7)$$

It's noteworthy to realize that in both cases, the electric conductivity  $\sigma$  is assumed to have a linear dependence on the flux potential  $S$  given by  $\sigma = BS$  [2,6,8]. Expressions (6) and (7) result from solving equation (2) for a straight arc column. The same expressions are used to solve (2) again by identification in the Frenet-Serret referential.

#### 4. Derivation of the model

Using a Taylor expansion, the Bessel function of the zeroth order can be reduced to second order terms as follows:

$$J_0\left(q_1 \frac{r}{a}\right) = 1 - \left(\frac{q_1 r}{2a}\right)^2 \quad (8)$$

Sifting carefully through equation (3), we can rewrite it:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \approx \vec{0} \quad (9)$$

Electro and magneto quasi-static conditions are satisfied because the AC arc is subject to short distances and very low frequencies [2].

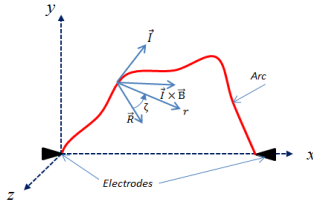


Fig.1. Tip-to-tip torch

As shown in figures 1 and 2, in the general 3D Frenet-Serret coordinate system, torsion can occur, leading to certain twisting. We denote by:

$$\zeta = \eta - \int_0^s \chi(s) ds \quad (10)$$

$\eta$  is the angle that provides zero-twisting and  $\chi(s)$  is the torsion at  $s$ .

In the Frenet-Serret referential, equation (9) yields:

$$E_s = \frac{E_{s0}}{1 - \kappa r \cos \zeta} \approx E_{s0}(1 + \kappa r \cos \zeta) \quad (11)$$

$\kappa(s)$  is the curvature at  $s$ .

Using expressions (6), (8) and (11) and the Appendix A of [2], equation (2) leads by identification to:

$$v_{va}(s, t) = \lambda \kappa \left( 5 - \frac{4 \cos \delta \cos(2\omega t + \delta)}{1 - \sin \delta \sin(2\omega t + \delta)} \right) + v_{vg}(s, t) \quad (12)$$

Equation (12) is of great relevance because it governs the motion of the curved AC and DC arcs. [3-5] obtained the same equation in the particular case of a DC arc. It's important to take note that  $v_v = v_{vg} - v_{va}$  is the normal component of the relative velocity from relation (5) in the direction of curvature. We denote by:

$$C_\delta(t) = 5 - \frac{4 \cos \delta \cos(2\omega t + \delta)}{1 - \sin \delta \sin(2\omega t + \delta)} \quad (13)$$

This expression casts equation (12) into a more simplified form:

$$v_{va}(s, t) = C_\delta(t) \lambda \kappa(s, t) + v_{vg}(s, t) \quad (14)$$

$C_\delta(t)$  is plotted in [2] for various values of  $\delta$ .  $\delta = \pi/2$  corresponds to the DC case.

To proceed further, we have to estimate  $v_{vg}(s, t)$  which will be obtained by solving the integral form of equation (1) in the Frenet-Serret referential. In order to solve for the shape of the arc afterwards, equation (14) must be written in an absolute coordinate system (Cartesian or polar) in the observer's referential.

#### 5. Computation of the velocity of the plasma gas flow

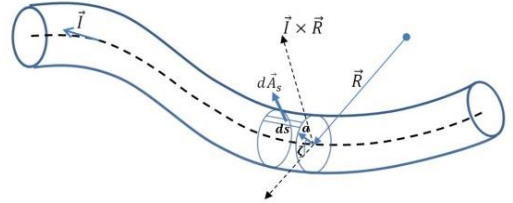


Fig.2. Arc parameters in the Frenet referential

As mentioned in the previous section, equation (1) must be solved in the Frenet-Serret coordinate system to evaluate  $v_{vg}(s, t)$ . However, it's very tedious and painstaking to deal with the differential form of equation (1). Therefore, we will only investigate its integral form over a control volume englobing an elementary slice of the current carrying arc of length  $ds$  as shown in figure 2. It's equally important to mention that the pressure tensor  $\vec{\bar{P}}$  comprises the static pressure tensor and the viscous stress tensor:  $\vec{\bar{P}} = \vec{\bar{p}} - \vec{\bar{\tau}}_{viscous}$ . Thus, the integral form of (1) can be expressed by:

$$\iiint_V \nabla(\rho \vec{V} \otimes \vec{V}^T + \vec{\bar{P}} - \vec{\bar{\tau}}_{Maxwell}) dV_s = \vec{0} \quad (15)$$

The contribution of the time-derivative of  $\vec{V}$  is omitted because the dynamic inertia is significantly greater than the electrical inertia. Body forces such as gravity are neglected as well, given the fact that Archimedes' number is much less than unity [4] (no natural convection). No relative motion is employed in this expression ( $\vec{V}$  is used instead of  $\vec{W}$ ) because the motion of the arc corresponds to the motion of a temperature profile not of a solid body [5]. Using Ostrogradsky's theorem we get:

$$\oint_S (\rho \vec{V} \otimes \vec{V}^T + \vec{p} - \vec{\tau}_{viscous}) \overrightarrow{dA}_s = \oint_S (\vec{\tau}_{Maxwell}) \overrightarrow{dA}_s \quad (16)$$

In the Frenet-Serret coordinate system  $\overrightarrow{dA}_s$  (the elementary lateral surface) is given by:

$$\overrightarrow{dA}_s = \left( (1 - a \kappa \cos \zeta) \vec{u}_r - \frac{da}{ds} \vec{u}_s \right) a d\zeta ds \quad (17)$$

Assuming that the arc radius  $a$  is constant and  $\vec{u}_r = \cos \zeta \frac{\vec{R}}{R} + \sin \zeta \frac{\vec{I} \times \vec{R}}{I.R}$ , and that an external magnetic field  $B_0(s)$  is applied we get from [9]:

$$\frac{d \oint_S (\vec{\tau}_{Maxwell}) \overrightarrow{dA}_s}{ds} \approx \vec{I} \times \vec{B}_0 \quad (18)$$

We limit our approximation to first order terms. All the expressions are calculated at  $s$ , but we omit its mention for simplicity sake ( $\vec{I}$ ,  $\vec{R}$  and  $\vec{I} \times \vec{R}$  represent respectively the tangent, normal and binormal directions).

For the left hand side of equation (16), the expression of the integral is more complex. However it can be approximated using a drag coefficient, especially when the flow injection is weak flow (weak dynamic field) and the arc is mainly driven by the applied magnetic field. In this case we have [9]:

$$\frac{d \oint_S (\rho \vec{V} \otimes \vec{V}^T + \vec{p} - \vec{\tau}_{viscous}) \overrightarrow{dA}_s}{ds} \approx C_D a \rho_g v_{vg}^2 \frac{\overrightarrow{v}_{vg}}{v_{vg}} \quad (19)$$

Here  $C_D$  is a drag coefficient. For low Reynold's flows it is inversely proportional to Reynold's and therefore to  $v_{vg}$  and proportional to the viscosity  $\mu$  of the plasma gas. This is assumed in to be the case for curved arcs under the effect of magnetic cross fields and weak arc blowing (or dynamic injection field). Equating expressions (18) and (19) leads to:

$$v_{vg} = \frac{KIB}{\mu} \quad (20)$$

This relation is found in [2-4].  $K$  is a constant. Here  $C_D$  is considered to be dependent on the inverse of Reynold's.

For arcs curved by dynamic transverse fields with no applied external magnetic fields and assuming that the self-induced magnetic field is negligible, the above expression doesn't apply anymore because the RHS of equation (16) would become negligible. So the LHS must be zero. Accounting for small contribution of the effect viscosity and pressure change, we obtain that:

$$\frac{d \oint_S (\rho \vec{V} \otimes \vec{V}^T + \vec{p} - \vec{\tau}_{viscous}) \overrightarrow{dA}_s}{ds} \approx (\rho_g v_{vg}^2 - \rho_\infty v_{v\infty}^2) \frac{\vec{R}}{R} \approx 0 \quad (21)$$

$v_{v\infty}$  is the upstream velocity of the flow. Hence:

$$v_{vg} = v_{v\infty} \sqrt{\frac{\rho_\infty}{\rho_g}} \quad (22)$$

This expression is found in [2,4]. Note that the viscous forces could always be taken into account. Expressions of  $v_{vg}$  involving them are derived in [2].

## 6. Solutions for the planar arc

Equation (14) must be written in an absolute referential in order to determine the motion of the arc. Subsequently, using a Cartesian or polar coordinate system, equation (14), for the 2D planar case, can be written as:

$$\begin{cases} \frac{y}{(1+y'^2)^{\frac{1}{2}}} = \frac{c_\delta(t) \lambda y''}{(1+y'^2)^{\frac{3}{2}}} + v_{vg}(s, t) & (Cartesian) \\ \frac{r \dot{r}}{(r^2+r'^2)^{\frac{1}{2}}} = \frac{c_\delta(t) \lambda (-r r'' + 2r'^2 + r^2)}{(r^2+r'^2)^{\frac{3}{2}}} + v_{vg}(s, t) & (polar) \end{cases} \quad (23)$$

Mathematical details can be found in [2]. In the 2D case, no torsion is involved so  $\zeta = \eta$  and  $\overrightarrow{v}_{vg}$  and  $\vec{R}$  are collinear. The values of the plasma gas velocity  $v_{vg}$ , exiting the arc in the normal direction, can be estimated from section 5 according to the type of cross-field to which the arc is exposed. The "dot" and the "prime" derivatives in equation (23) designate partial derivatives with respect to time and space respectively. Expressions regarding the effects of the self-induced magnetic field are obtained in [2]. A certain dimensionless number found in the cited reference determines their prevalence over the resorting viscous forces. The analytical model that we have presented so far has been validated by means of MHD computational work that we will present in the following section along with the results.

## 7. MHD computational work

The software used for simulation is "code\_Saturne"; a powerful solver for electric arc simulation based on the finite volume method. The gas employed is air at a pressure of 1 bar. Transient simulations are adopted with an absolute time step of 1  $\mu$ s. The current is imposed at 50 A for both AC and DC cases. For the AC case, the frequency is chosen to be 50 Hz. The geometry employed

is displayed by figure 3. For more details on the boundary conditions, the mesh and the equations involved in the model, it is highly recommended to consult [2].

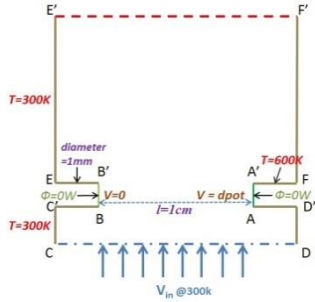


Fig. 3. Geometry used in the MHD model

## 8. Results and comparison

Figure 4 and table 1 show a good agreement for the AC arc between results obtained from the MHD numerical simulation and the analytical model presented in this paper. Other results can be found in [2], but are omitted here for the sake of simplicity. The cross-field used is a dynamic-type (gas injection). Also, other simulations have been run for self-induced and external magnetic cross-field. Again, they have all turned out to be congruent with the analytical model. It is important to mention here that air properties and formulas used to carry out the computations of the arc parameters for the analytical model are found in [2,6,8]. The Cartesian form of equation (23) is solved using MATLAB.

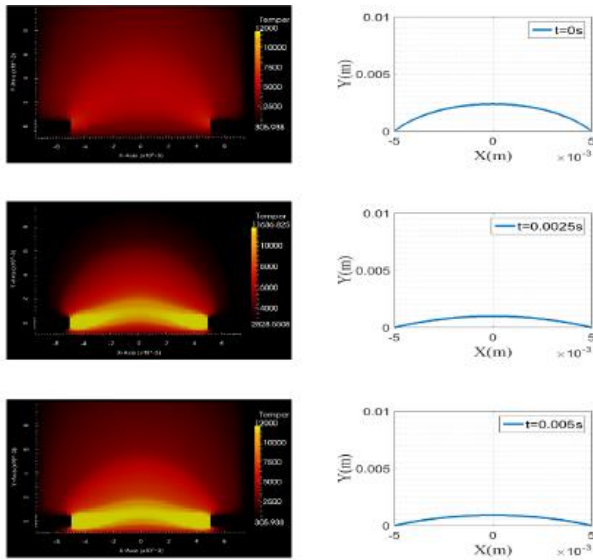


Fig. 4. MHD (left) and analytical (right) results for a blown 50 A 50 Hz- air plasma arc (@ 1bar) with an injection velocity  $V_{in} = 1m.s^{-1}$  at  $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$S$  is the heat potential,  $T$  the temperature,  $Y$  the maximum displacement at electrode mid-length,  $a$  the arc radius and  $E$  the electric field. For the analytical approach, they are computed using equations (4), (6) and (23). For the calculation of  $a$  and  $E$  methods and

expressions can be found in [2,6,8]. The reference temperature for  $S$  in our case is  $T_0 = 4000K$ .

Table.1.Comparison table between MHD and analytical results for a blown 50 A 50 Hz- air plasma arc (@ 1bar) with an injection velocity  $V_{in} = 1m.s^{-1}$  at  $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$\omega t$	0		$\pi/4$		$\pi/2$	
Results	MHD	Theory	MHD	Theory	MHD	Theory
$S_{max}(W.m^{-1})$	-	740	-	10940	-	15980
$T_{max}(K)$	5450	5280	7460	7120	11710	11680
$Y_{max}(mm)$	2.3	2.4	1.5	1.4	0.95	0.95
$a(mm)$	-	-	2	2.2	2.3	2.2
$E(V.m^{-1})$	70	0	2950	2790	2450	2580

## 9. Conclusion

An analytical model has been established using a theoretical approach. It allows us to treat the complex physical problem using a rather simple method and obtain precious information about the arc, without having to resort to costly MHD simulations. Analytical and MHD results showed good agreement for several case studies. This allows us to validate our analytical approach. Slight differences between the MHD and the analytical model could be noticed. This could be attributed to the stringent assumptions used in the analytical case and the boundary conditions used in the MHD simulations. Further refinements to the model must be made by including radiation and the variation of the arc radius. Moreover, this approach in its current form does not account for instabilities caused by turbulence in large scale industrial plasmas. Nevertheless, it constitutes a handy tool for basic process design of industrial plasma torches and can be insightful in various plasma applications such as welding, cutting, circuit breakers, etc.... It can also potentially enable us to control plasma arcs by means of cross-fields.

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