Extraction of heating control rules from the dynamic programming method for load shifting in energy-efficient building

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1. ABSTRACT

In France, 40 % of buildings are heated with electrical devices causing high peak load in winter. In this context, optimal strategies (under constraints related to comfort and maximum heating power) have been developed using the dynamic programming method in order to shift electricity consumption used for heating, taking advantage of the building thermal mass. However, this exact optimisation method is computationally intensive and can hardly be applied to real-time control. Complementary statistical techniques exist that allow for the extraction of logistic decision models from the optimal control simulation results. These rule extraction techniques model the relationship between explanatory variables and a response variable. In this study, a generalised linear model was used because it is able to mimic the general characteristics of the dynamic programming results with good precision and greatly reduced computational effort (150 times faster than the dynamic programming method).

Keywords: Rule extraction, optimal control, load shifting.

2. INTRODUCTION

2.1 Existing control schemes

In modern construction of buildings, the main objectives for the control systems are to save energy (Nygard Ferguson, 1990), to increase comfort (Mathews et al., 2000) and to reduce peak electricity demand (Greensfelder et al, 2011). To meet such objectives, control systems have to be able to anticipate the weather, the occupancy, and the solar and internal gains. Dounis and Caraiscos (2009) reviewed many advanced control systems meeting such objectives. For instance, during a summer period, control systems are used to maintain comfort using passive cooling (Braun et al, 2001), to reduce energy consumption of air conditioning (Chahwane, 2011), or to control solar protections (Nielsen et al, 2011). During a winter period, control systems are used to decrease the energy consumption of the heating system (Le, 2008) or to reduce peak demand (Malisani et al. 2011).

2.2 Load shifting

Recently, numerous efforts have been made to reduce electricity peak-demand. In Europe, these peaks mostly appear during winter periods and are due to heating systems. For example in France, the building sector represents 68 % of the final electricity consumption (ADEME, 2012). To guarantee the grid stability, some studies have been done on electrical load shifting.

Thanks to electricity demand-side response (DSR), the consumer demand for energy can be modified through various methods such as financial incentives or education. Many economical models are used by the demand side response programs. Two categories may be distinguished: time based programs and incentive based programs (Marwan et Kamel, 2011; Federal Energy Regulatory Commission, 2006). Examples of application of time based programs are Time Of Use (with fixed electricity prices for off-peak and peak hours), or Real Time Pricing (with variable electricity tariffs). For incentive based programs, an example is the Direct Load Control, which allows to turn specific appliances on and off during peak demand periods.

At the level of the individual houses, the electricity peak reduction can be achieved thanks to a careful architectural design to efficiently manage solar gains (Nygard Ferguson, 1990). An advanced control system can also be used to reduce heating consumption. Such control can be based on power tariff (Hämäläinen et al., 2000; Pineau et Hämäläinen, 2000) or the use of the thermal mass of the building to shift part of electricity consumption (Wyse, 2011; Hong et al., 2011). For instance, Favre and Peuportier (2014) used the dynamic programming method to shift the building consumption. The proposed method consisted in over-heating the building in the hours before the peak based on weather forecast, and occupancy and internal gains schedules for the next 7 days. However, this exact optimisation method is time-consuming and can hardly be applied to real-time control.

2.3 Rule extraction

In developing an operational strategies framework, exact optimisation results can be used to extract simplified control rules that are implementable in real-time.

This approach was first applied in water resource management. The application was to developed simplified control rules for reservoir management based on the results of offline model predictive control (MPC) (Wei et Hsu, 2009). The approach has recently been used in the building context. For instance, May-Ostendorp et al. (2013) used many data mining techniques (generalised linear models, classification and regression trees and adaptive boosting) to extract rules from offline MPC results for a mixed mode building operated during the cooling season. To our knowledge, this approach was never applied to shift the heating consumption in building.

The present study is based on the results of Favre and Peuportier (2014). and its objective is to develop operational strategies to shift the heating load in building. A new methodology is proposed to extract decision models from dynamic programming results and then compare them.

3. MODELS

3.1 Thermal model of the building

The building is modelled considering spatial zones of homogenous temperature. For each zone, each wall is meshed according to the finite volume technique with a uniform temperature and thermal capacity. Another mesh is added for the zone's air and furniture. Energy conservation equations are written on each mesh within the building and form a system of equations:

$$C_i \, {}^{dT_i}/_{dt} = Gains - Losses \tag{1}$$

with

- C_i the thermal capacity of the node i,
- T_i the temperature of the node i,
- $Gains$ the solar and internal gains (due to heating, occupancy and other appliances),
- $Losses$ the heat losses by conduction, convection and radiation.

Repeating energy conservation equations for each mesh leads to a linear time-invariant system (Peuportier and Blanc-Sommereux, 1990), temporal variation terms being added in the simulation:

$$\begin{aligned} C\dot{T}(t) &= AT(t) + EU(t) \\ Y(t) &= JT(t) + GU(t) \end{aligned} \tag{2}$$

with

- T the node temperature vector,
- C the diagonal thermal capacity matrix,
- U the driving forces (climate parameters, heating, etc.),
- Y the output vector (indoor temperatures accounting for air and wall surfaces),
- A, E, J, G the state, input, output and feedforward matrices, respectively.

In order to perform simulation, it is important to know the occupancy of the building which defines the emission of heat by the inhabitants and appliances, and the thermostat setpoint influencing the heating equipment. Another important aspect is the weather model influencing heat losses and solar gains. The data regarding house occupancy and weather models were included in the driving forces vector U.

The high order linear model (2) needed to be reduced because its state dimension was too large to allow a fast convergence of the optimisation algorithm. A reduction method (modal reduction) was thus applied to lower the state dimension. In this work, the building energy simulation tool COMFIE was used (Peuportier and Blanc-Sommereux, 1990).

3.2 Optimisation algorithm

The dynamic programming method was developed by Bellman (1957). It is a sequential optimisation method which examines all possible ways to solve an optimisation problem and provides, given a discretisation, an optimal set of commands over a period.

To apply dynamic programming, a state variable describing the system is used and discretised temporally:

$$x(t) = x_t \in X_t, X_t \subset \mathbb{R}^{N_S} \tag{3}$$

where X_t is the set of possible states and N_S the dimension of X_t. The control vector u can be chosen in a set $U_t \subset \mathbb{R}^{N_C}$ (the set of possible controls) where N_C is the dimension of the control vector:

$$u(t) = u_t \in U_t, U_t \subset \mathbb{R}^{N_C} \tag{4}$$

One can act on the system state through the control variable u. The state space equation of the dynamical system $f(.)$ is thus:

$$x(t) = x_t, \qquad x(t+1) = f(x(t), u(t)) \tag{5}$$

A value function v_t is defined, which is the cost to go from $x(t)$ to $(t+1)$:

$$v_t(x_t, x_{t+1}) \tag{6}$$

Under these assumptions, a finite-horizon decision problem takes the following form:

$$V_0^t = \max\,[\sum_{j=0}^{t-1} v_j\left(x_j, x_{j+1}\right)] \tag{7}$$

subject to the constraints (3) and (4) and the state space equation (5). V_0^t denotes the optimal value that can be obtained by maximising the objective function subject to the assumed constraints. The dynamic programming method is then applied to break this decision problem into smaller sub-problems. Bellman's principle is thus used: "*An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision*".

Equation (7) becomes:

$$V_0^t = \max\left[v_0(x_0, x_1) + \sum_{j=1}^{t-1} v_j\left(x_j, x_{j+1}\right)\right] \tag{8}$$

*Figure **1*** shows how the dynamic programming operates:

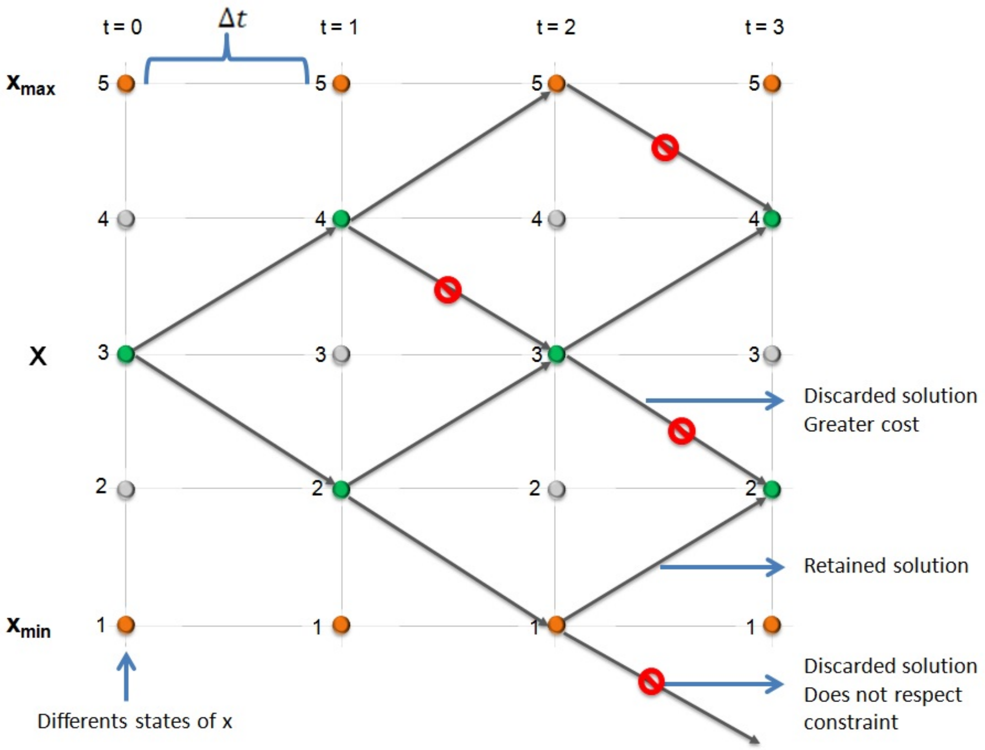

Figure 1: Dynamic programming description

In *Figure* **1**, the state variable x is discretised in five states. Optimisation aims at minimising the state variable. In this example, thanks to Bellman's principle, four solutions can be discarded either because they trespass the constraints or because they reach the same states as solutions with lower costs.

For the application of dynamic programming in building context, the chosen state variable and the cost function are defined in §5.2.1.

3.3 Rule extraction: Generalised linear model

The generalised linear model (GLM) framework was used to derive simplified decision models from the dynamic programming results allowing a small computational expense adapted to real time control. GLM models the relationship between regressors x_j (explanatory variables) and response y. It consists of three elements:

- a random component (the response y is assumed to be generated from a particular probability distribution),
- a deterministic component (a linear combination of explanatory variables x_j),
- a link function (that provides the relationship between the linear combination of explanatory variables and the mean of the distribution function).

We have thus to estimate the following model:

$$f_l(E[y]) = \sum_j a_j x_j + b \tag{9}$$

with

- $f_l(.)$ the link function,
- $E[y]$ the expected value of y.

The unknown parameters a_j and b are typically estimated with maximum likelihood (Gill, 2004).

4. METHODOLOGY

The following section describes techniques employed to extract decision models from dynamic programming results. Dynamic programming was used to generate training data and validation data to identify a GLM's parameters and to evaluate its performance, respectively. It was done in two stages.

As a first step, Test Reference Year-type (TRY) weather data were used to perform optimisation using dynamic programming and to elaborate an optimal strategy. This optimal strategy (training data) was then used to identify the GLM's parameters with the same weather data.

As a second step, the predictive capacity of the model was measured on locally recorded weather data. We compared the GLM's results with the optimal strategy calculated by dynamic programming (validation data).

4.1 Model identification

Model identification was done in a four-step process (*Figure 2*). First, all data used by dynamic programming were collected (Test Reference Year-type weather data, electricity tariff, occupancy). Secondly, the optimal strategy was elaborated using dynamic programming (training data). Thirdly, we identified the GLM's parameters thanks to optimal strategy. Fourthly, the resulting control models were implemented within the simulation platform (COMFIE).

4.2 Model comparison

Model comparison was done in a three-step process (*Figure 3*). Firstly, all data used by the optimisation method and the GLM were collected (local weather data, electricity tariff, occupancy). Secondly, we performed optimisation with dynamic programming and GLM to determine optimal strategy and operational strategies respectively. Finally, we compared performances of operational strategies against the optimal strategy.

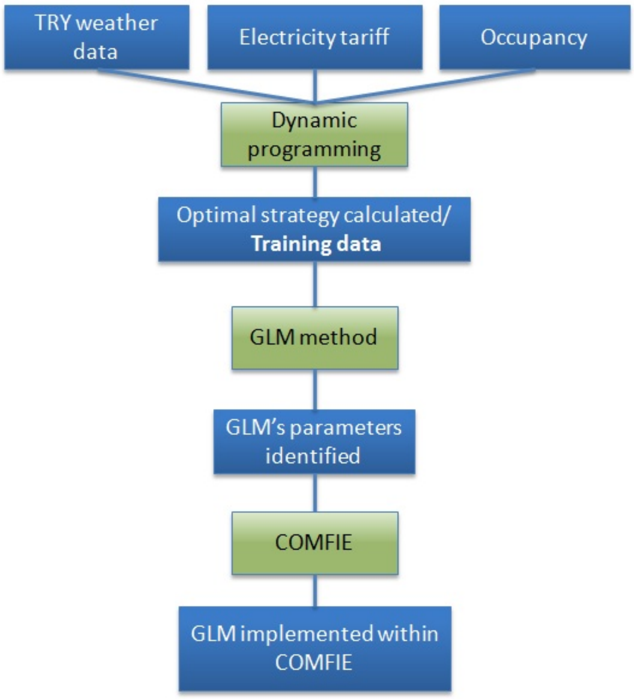

Figure 2: Model identification

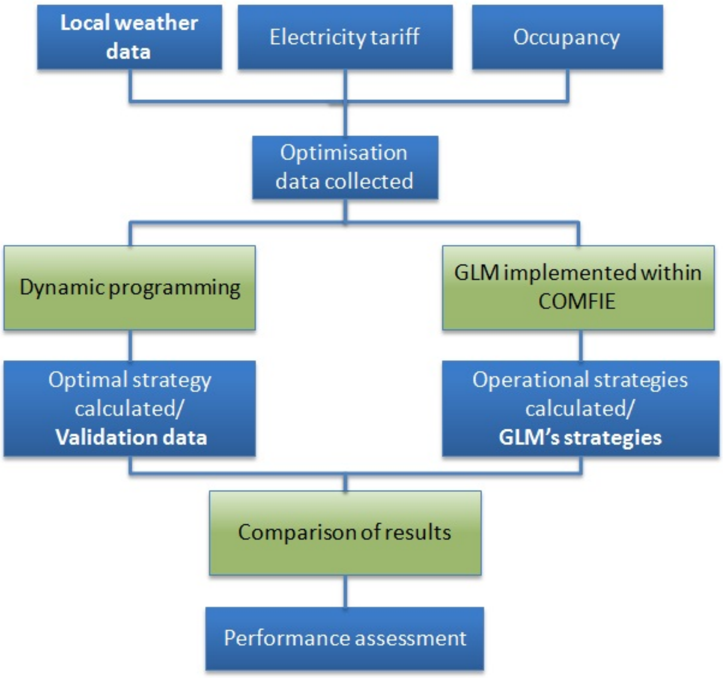

Figure 3: Model comparison

5. CASE STUDY

5.1 Building description

The building under study is a single-family house based on an actual experimental passive house being part of the INCAS platform built in Le Bourget du Lac, France, by the National Solar Energy Institute (INES). The studied house has two floors and a total living floor area of 89 m² (*Figure 4*). The North facade has only two small windows whereas 34 % of the South facade is glazed. The building's façades include double ($U_{gw} = 1.1\ W.m^{-2}.K^{-1}, SF = 0.6$) and triple on the North ($U_{gw} = 0.7\ W.m^{-2}.K^{-1}, SF = 0.45$) glazing windows of various dimension. The south façade also includes solar protection for the summer period. The house is highly insulated with a high thermal mass as shown in *Table **1***.

Table 1: Building description

	External wall	**Ground**	**Intermediate floors**	**Ceiling**	**Interior partition**
Composition	15 cm thick 20 cm of extruded polystyrene	20 cm concrete slab 20 cm external insulation	16 cm concrete screeds and girders 12 cm concrete slab floor	40 cm of glass wool	4 cm of glass wool
U ($W.m^{-2}.K^{-1}$)	0.15	0.15	2.2	0.09	0.96

The main thermal bridges as well as the building air tightness have been carefully designed and implemented. The house is heated by an electrical resistance integrated in an efficient heat recovery ventilation system. According to thermal simulation results using the thermal model described in §3.1, the annual heating load is 14 $kWh.m^{-2}$.

Figure 4: view of the house (west and south façades)

5.2 Optimisation parameters

5.2.1 Dynamic programming parameters

The chosen state variable for dynamic programming is the total energy E_t stored in the building, calculated as follows(Favre et Peuportier, 2014) :

$$E_t = \sum_{i=0}^{N_{nodes}} C_i(T_i - T_{ref}) \tag{10}$$

with

- T_{ref} the reference temperature chosen at 0°C,
- N_{nodes} the number of nodes

An upper and lower bound of this state variable was defined according to its initial value. Then it was discretised in 800 nodes.

To ensure thermal comfort in the building, indoor temperature had to be maintained between 19°C (T_{min}) and 26°C (T_{max}). We considered a typical four people family occupancy: the building was non-occupied only during the working days from 8 a.m. to 5 p.m.. Each occupant emitted 80 W due to their metabolism, and internal gains from appliances were also considered during occupied hours. The heating power was in the range of 0 W (P_{min}) to 5000 W (P_{max}).

The model of the building was mono-zonal and the optimisation was done over 34 days (which was the maximal decision problem's horizon solved by dynamic programming), with one hour time step, to generate training and validation data. The goal of dynamic programming was to minimise the heating cost of the building by determining a set of commands (heating power P) with constraints on thermal comfort and heating power. Thus, the finite-horizon decision problem took the following form:

$$\min_P \sum_{t=0}^{t=tf} C_{elec\_t} P_t \tag{11}$$

with constraints

$$T_{min} \leq T(t) \leq T_{max} \tag{12}$$

$$P_{min} \leq P(t) \leq P_{max} \tag{13}$$

with

- P_t the heating power at time step t,
- C_{elec_t} the electricity cost at time step t.
- tf the duration of the optimisation period

5.2.2 *Electricity tariff*

To shift electricity demand, a time-of-use pricing was considered (*Table **2***):

Table 2: Electricity prices

	Off-peak hours	**Peak hours**	**High peak hours**
Hours	12 a.m. to 9 a.m.	9 a.m. to 5 p.m. 10 p.m. to 12 a.m.	5 p.m. to 10 p.m.
Cost per kWh (€)	0.0864	0.1275	0.255

5.2.3 *Weather data*

Meteonorm data from Chambery (to generate training data) and local weather conditions data (to generate validation data) measured at the Chambéry airport which is 300 meters away from the building, were used to perform simulations. Test Reference Year-type (TRY) were used to develop GLM because these weather data represent the typical long-term weather patterns. Thus, GLM's results were adjusted with the long-term average climatic conditions. Then, we used local weather conditions data to assess GLM's behaviour in real conditions.

Meteorological features are summarised in *Table **3***.

Table 3: Weather data

	Training data	**Validation data**
Minimum temperature (°C)	-9.10	-14
Average temperature (°C)	1.44	-0.22
Maximum temperature (°C)	11.50	11.33
Average global horizontal irradiance ($W.m^{-2}$)	58	60
Maximum global horizontal irradiance ($W.m^{-2}$)	486	569

5.3 Skill evaluation

Objective criteria for evaluating the predictive quality of the model were required. Therefore, the following indicators were used to assess its performance:

- the mean absolute error (MAE), between heating powers P calculated by dynamic programming and GLM,
- the average heat power,
- the cumulative cost,
- the percentage of high peak hours which are load shifted,
- the percentage of peak hours which are load shifted,
- the thermal discomfort rate TI_{min} representing the number of hours when the indoor temperature falls below 19°C (in %),
- the thermal discomfort rate TI_{max} representing the number of hours when the indoor temperature rises above 26°C (in %).

6. RESULT ANALYSIS

6.1 Explanatory variables

Explanatory variables that can be measured in building were used to develop GLM. Thus, to determine the heating power P at time step $t + \Delta t$, we used explanatory variables at time step $t + \Delta t$: outdoor temperature T_{out}, global horizontal irradiance I_{gh} and electricity tariff (C_{elec}). Explanatory variables at time step t were also considered: indoor temperature T_{in}, and heating power.

6.2 Models developed

To apply generalised linear model (GLM), we had to define the link function. That is why we changed the response variable as a proportion of maximum heating power (5000 W). For example, a heating power at time step $t + \Delta t$ of 2500 W, corresponds to a predicted variable by GLM of 50 %. The statistical model used by GLM is thus a multiple logistic regression and the link function is the logit function $f_l(x) = \ln\left(\frac{x}{1-x}\right)$. This model was used to relate the proportion of maximum heating power to predictor variables x_j through the logistic link function:

$$\ln\left(\frac{E(P)}{1 - E(P)}\right) = \sum_j a_j x_j + b \qquad (14)$$

Five models were developed, each one using all or some of the training data (*Table 4*). Training data were divided into three groups: off-peak hours training data (TD_{OPH}), peak hours training data (TD_{PH}) and high peak hours training data (TD_{HPH}).

In the implementation, the heating power at time step $t + \Delta t$ was set at 0 W in the following cases:

- during high peak hours for GLM_2, GLM_3, GLM_4 and GLM_5 models,
- during peak hours for GLM_3 and GLM_5 models.

These choices were done in order to ensure load shifting during peak and high peak hours.

Table 4: Training data

	Off-peak hours	**Peak hours**	**High peak hours**
GLM_1	TD_{OPH}	TD_{PH}	TD_{HPH}
GLM_2	TD_{OPH}	TD_{PH}	-
GLM_3	TD_{OPH}	-	-
GLM_4	TD_{OPH}	TD_{PH}	TD_{HPH}
GLM_5	TD_{OPH}	TD_{PH}	TD_{HPH}

As a more specific example, GLM_3 and GLM_5 models were different because they did not have the same training data. Indeed, GLM_3 was trained only on off-peak hours training data (TD_{OPH}) whereas GLM_5 was trained on complete training data (TD_{OPH}, TD_{PH}, TD_{HPH}). However, in the implementation, the heating power at time step $t + \Delta t$ was set at 0 W during high peak hours et peak hours for both models. The same logic was applied for GLM_2 and GLM_4 models.

6.3 Results

Each GLM model was implemented in the building energy simulation tool COMFIE. *Table 5* summarises GLM models' results obtained on validation data. The dynamic programming reference results are described in the DP column.

The resulting model predictions of GLM_1 and GLM_2 and the original optimised sequence are presented in *Figure 5*. We can clearly observe that GLM_1 and GLM_2 did not follow the dynamic programming's behaviour. For example, we can see that the GLM_2 model performed significantly worse than dynamic programming, with a very high thermal discomfort rate TI_{max} (93 %) and an indoor temperature exceeding 30°C (30.6°C). GLM_1 had a similar behaviour with a significant cumulative cost (137 €) and a high mean absolute error (111 %).

Figure 6 shows that predictions of GLM_4 and GLM_5 are also different from the optimised results. For instance, we can see that GLM_4 and GLM_5 had a significant cumulative cost (105 € and 92 € respectively) and a high average power (1347 W and 1309 W respectively). Moreover, GLM_4 and GLM_5 had a relatively large mean absolute error (88 % and 71 % respectively).

However, *Figure 7* illustrates the interesting behaviour of GLM_3. Firstly, due to its design, no electricity was consumed during high peak hours and peak hours. Secondly, it had a cumulative cost and an average heat power close to dynamic programming (72 € and 1023 W compared to 68 € and 936 W for DP). Thirdly, its mean absolute error (40 %) and mean relative error (9 %) were reasonable. Finally, GLM_3's computational time was 150 times smaller than the dynamic programming method, using a desktop computer.

Table 5:GLM models' results

	GLM_1	GLM_2	GLM_3	GLM_4	GLM_5	DP
Average heat power (W)	1353	1811	1023	1347	1309	936
Cumulative cost (€)	137	158	72	105	92	68
High peak hours load shifted (%)	0	100	100	100	100	99
Peak hours load shifted (%)	0	0	100	0	100	88
TI_{min} (%)	0	0	8	0	0	0
TI_{max} (%)	3	93	0	2	0	0
T_{min} / T_{max}	21.2 / 26.15	21.4 / 30.6	18.4 / 23.8	20.7 / 26.1	19.6 / 26	19 / 23.4
MAE (%)	111	153	40	88	71	-

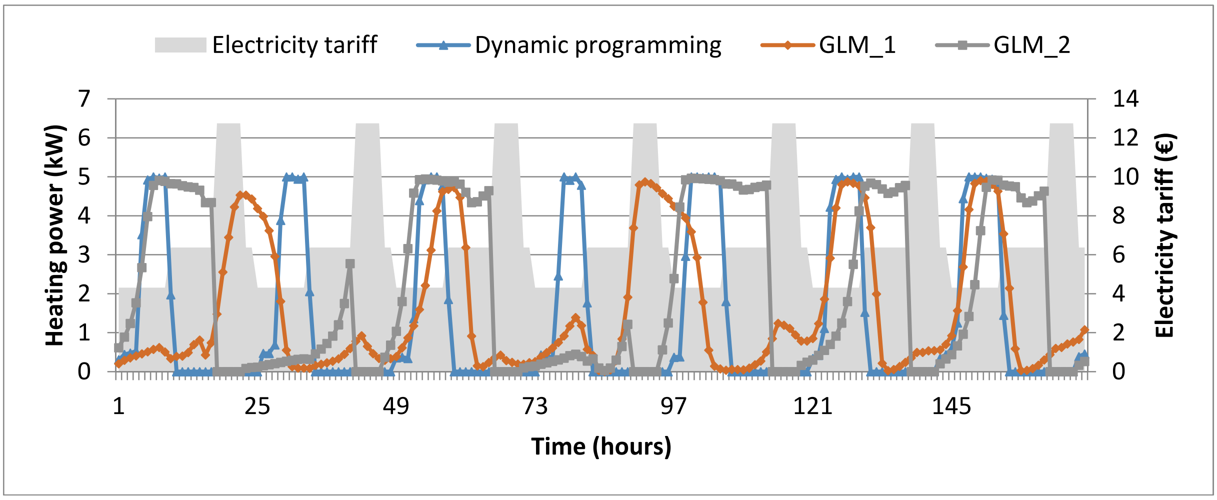

Figure 5: Heating power calculated by dynamic programming, GLM_1 and GLM_2 (Third week)

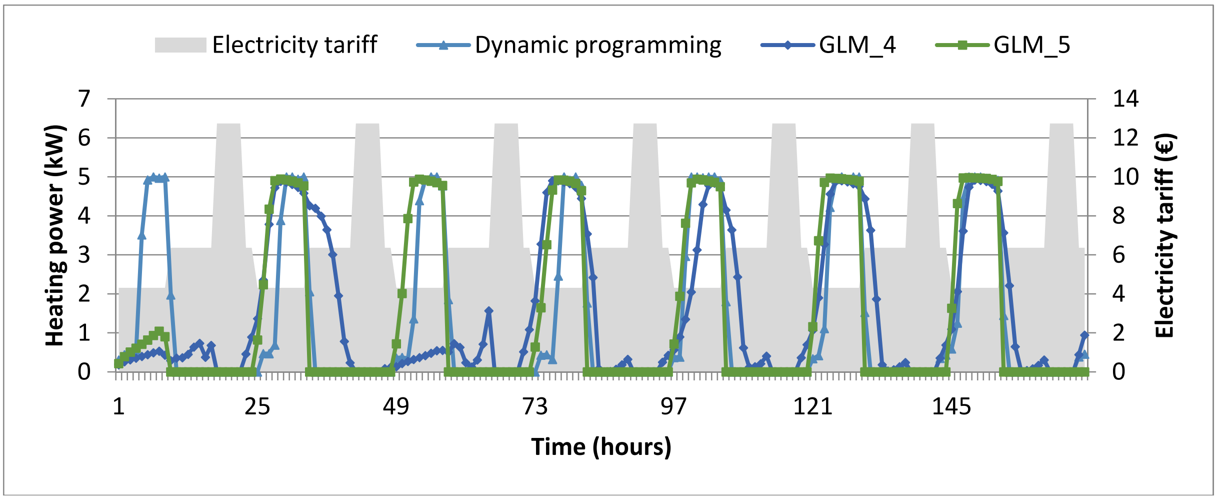

Figure 6: Heating power calculated by dynamic programming, GLM_4 and GLM_5 (Third week)

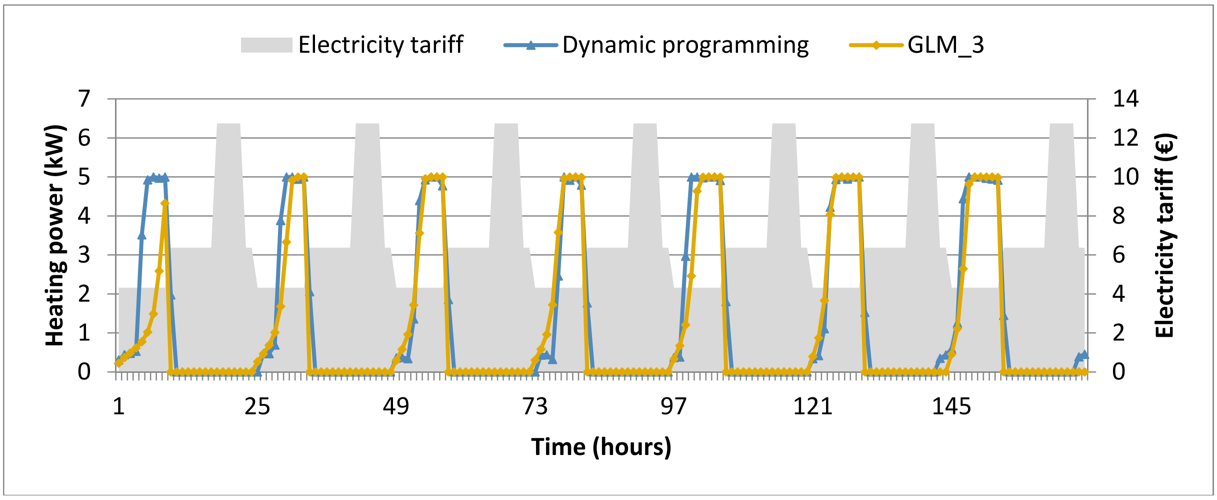

Figure 7: Heating power calculated by dynamic programming and GLM_3 (Third week)

The GLM_3 model presented a satisfactory behaviour and seemed a possible candidate to be used as simplified control system. However, on some occasions, it did not respect the thermal comfort constraints. Therefore, an improved controller was considered that switched heating on as soon as the indoor temperature was below 19°C.

6.4 Application controller

An ideal on-off controller was considered. It was applied during peak and high peak hours as GLM_3 did not work during these periods. Its control law switched between the minimum heating power (0 W) and the maximum heating power (5000 W). The ideal on-off controller was switched on when the indoor temperature fell below 19°C -ε and was switched off when the indoor temperature rose above 19°C + ε (in order to respect the 19°C set point temperature). Assuming that ε tended toward 0, the deadband of the on-off controller (±ε) tended toward 0. The use of this ideal on-off controller aimed at assessing maximum performance of GLM_3 + controller.

The control law was the following:

- During off peak hours

$$P(t+\Delta t) = \begin{cases} GLM\_3 & T_{in}(t) \le T_{max} \\ 0 & T_{in}(t) > T_{max} \end{cases} \tag{15}$$

- During peak and high peak hours

$$P(t+\Delta t) = \begin{cases} \text{Controller on} & T_{in}(t) < 19°C - \varepsilon \\ \text{Controller off} & T_{in}(t) \ge 19°C + \varepsilon \end{cases} \quad \varepsilon \to 0 \tag{16}$$

The obtained results are shown in ***Table 6***. We can notice the interesting behaviour of GLM_3 + controller. Firstly, thanks to the on-off controller, GLM_3 + controller respected temperature constraints (the lowest temperature reached was 19°C). Then, we can see a slight deterioration of peak hours and high peak hours shifted. For example, GLM_3 + controller had 92 % of high peak hours which were load shifted in comparison with the 100% of GLM_3 (and the 99 % of dynamic programming). Similarly, GLM_3 + controller had 95 % of peak hours which were load shifted. It was less efficiency than GLM_3 (100 %) but it was better than dynamic programming (88 %). Finally, GLM_3 + controller had a cumulative cost (72.9 €) and an average heat power (1029 W) close to GLM_3 and dynamic programming.

Table 6: GLM_3 + controller results

	GLM_3	**GLM_3 + Controller**	**DP**
Average heat power (W)	1023	1029	936
Cumulative cost (€)	72.2	72.9	68
High peak hours load shifted (%)	100	92	99
Peak hours load shifted (%)	100	95	88
TI_{min} (%)	8	0	0
TI_{max} (%)	0	0	0
T_{min} / T_{max}	18.4 / 23.8	19 / 23.8	19 / 23.4
MAE (%)	40	41	-

Consequently, adding an on-off controller with GLM_3 enabled to improve the GLM_3's behaviour and to respect temperature constraints. *Figure **8*** shows the GLM_3's behaviour both with and without the on-off controller. To plot GLM_3+controller's graph, the heating power was averaged over one hour time periods.

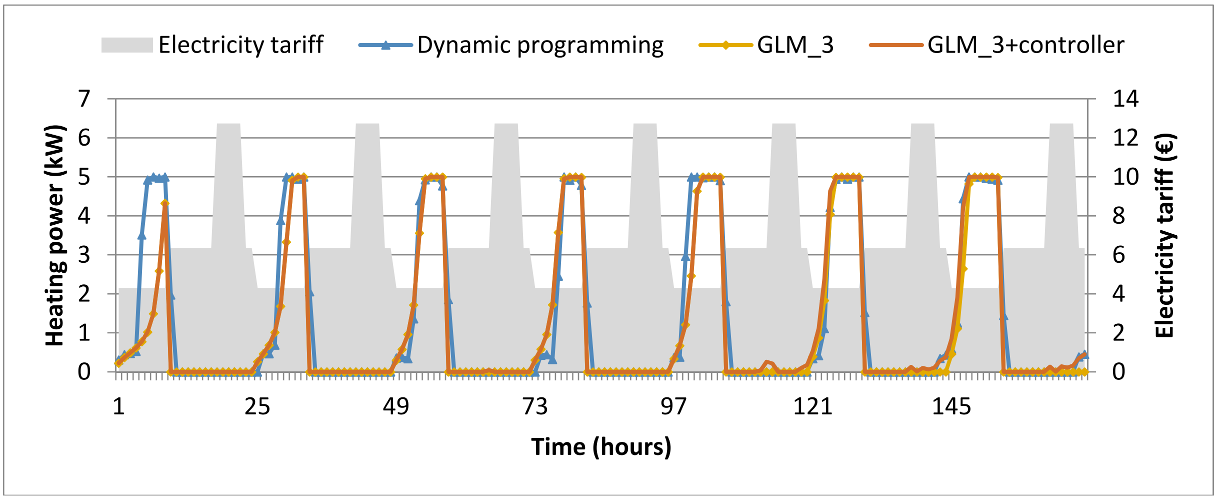

Figure 8: Heating power calculated by dynamic programming, GLM_3 and GLM_3 + controller (Third week)

7. CONCLUSION

Dynamic programming method has been used to study load shifting of heating systems in an energy-efficient building. Due to its computational expense, a statistical technique (generalised linear model) has been introduced that allow for the extraction of logistic decision models from the dynamic programming results. This method models the relationship between explanatory variables and a response variable. The results showed that generalised linear models were able to imitate the general characteristics of the dynamic programming results, with a much smaller computational expense and limited overshooting of the setpoint. To improve the GLM's behaviour, an on-off controller was added that switched heating on as soon as the indoor temperature did not respect temperature constraints. The results showed that the GLM+on-off controller respected temperature constraints and that there were a slight deterioration of peak hours and high peak hours shifted. Therefore, rule extraction (generalised linear model) is a promising technique for developing operational control strategies. Given their simple mathematical formulation, generalised linear models coud be implemented in real time building systems control.

8. NOMENCLATURE

8.1 Latin

A state matrix

a_j regression parameters

b regression parameter

C diagonal thermal capacity matrix [J.K^{-1}]

C_{elec_t} electricity price at time t [€]

C_i thermal capacity of node i [J.K^{-1}]

$E[.]$ expected value (first moment)

E input matrix

E_t total energy stored in the building [J]

$f(.)$ dynamical system

$f_l(.)$ link function

G feedforward matrix

$Gains$ solar and internal gains [W]

I_{gh} global horizontal radiation [W.m^{-2}]

J output matrix

$Losses$ heat losses by conduction, convection, and radiation [W]

N_c dimension of U_t

N_{nodes} number of nodes

N_S dimension of X_t

SF Solar Factor (glazing transmittance) [-]

P heating power [W]

P_{max} maximum heating power [W]

P_{min} minimum heating power [W]

P_t heating power at time t [W]

T node temperature vector [°C]

T_i temperature at node i [°C]

T_{in} indoor temperature [°C]

T_{max} maximal temperature [°C]

T_{min} minimal temperature [°C]

T_{out} outdoor temperature [°C]

T_{ref} reference temperature [°C]

TI_{max} high thermal discomfort rate [%]

TI_{min} low thermal discomfort rate [%]

u control vector

U driving forces [°C] / [W]

U_{gw} window overall heat transfer coefficient [W.m^{-2}.K^{-1}]

U_t set of possible controls at time t

v_t value function at time t

V_0^t finite-horizon decision problem

x state variable describing the system

x_j explanatory variables

X_t set of possible states at time t

y output variable

Y outputs vector [°C]

8.2 Greek

Δt time step

$\pm\varepsilon$ controller's dead band [°C]

8.3 Abbreviations

DP Dynamic Programming

GLM Generalised Linear Model

MAE Mean Absolute Error

TD_{HPH} High peak hours training data

TD_{OPH} Off-peak hours training data

TD_{PH} Peak hours training data

TRY Test Reference Year

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