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# Application of P.O.D. and D.E.P.O.D. model reduction methods to semi-thick sheet metal forming

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**Abstract.** Model reduction allows transforming a high-dimensional model into a very low-dimensional approximation, which makes it possible to significantly decrease the computational time while preserving the physics of the problem. In this paper, it is applied to a metal forming process, the semi-thick sheet bulging test, where computing time can be quite significant when using the associated inverse analysis. The problem is characterized by large deformations, development of instability, implicit velocity / pressure formulation and unstructured meshes of tetrahedrons. A Proper Orthogonal Decomposition (POD) is implemented in the commercial software Forge®. The proposed reduced order model (ROM) involves a reduced basis for both velocity and pressure fields to account for the incompressibility constraint. The ROM by POD allows reducing the number of unknowns to less than 12 while keeping the results accuracy and precisely predicting the sheet instability development. The ROM remains accurate for a wide range of material parameters variations. The DEPOD approach allows further extending the ROM validity range, through the information of additional snapshots in the vicinity of the initial ones.

## 1 Introduction

Simulation techniques have brought huge contributions to engineering problems. Despite major progress in numerical methods and computers capacities, accurate solutions require refined space discretizations, which consequently increase computational time. Model Order Reduction (MOR) seeks to overcome this issue by approximating the high dimensional problem with a lower one, following several techniques as proposed in literature [1-2]. This work is based on the Proper Orthogonal Decomposition (POD); it does not aim to develop a new variant of the method but instead to study its capacity to treat metal forming problems including high non-linearities, large deformations and unstructured meshes.

## 2 Bulging Rheological Test

The considered metal forming problem is the bulged test (see Figure 1 and 2) often used to identify the material behaviour through inverse analysis. An oil pressure is enforced on the metal sheet while the sides of the sheet are maintained by a blank holder. The inverse analysis requires carrying out several simulations of the forming problem with different material parameters, so resulting into large computational time.

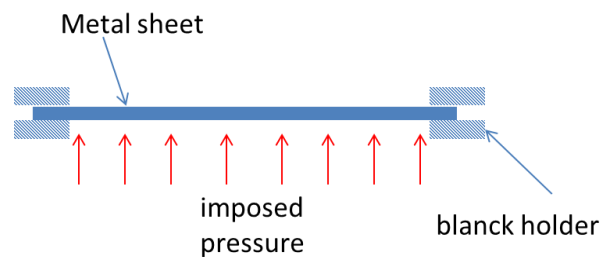


Figure 1. Boundary conditions for the bulging test.

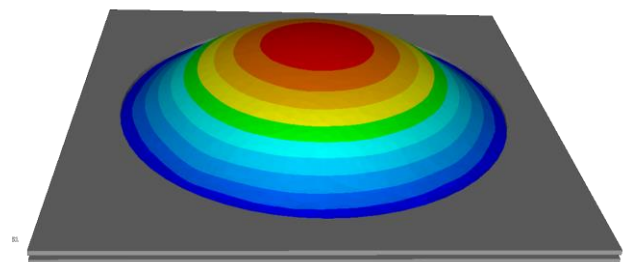


Figure 2. Velocity field at the end of the bulged test (stable case).

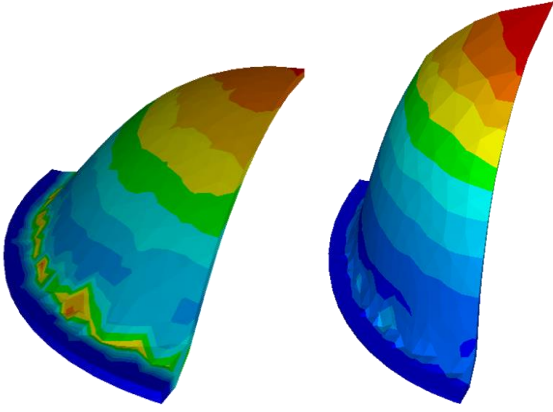
The circular sheet is 1 mm thick with a 50 mm radius. The material, formed under hot conditions, is assumed to follow the Norton-Hoff law (2). Its consistency  $K$  is equal

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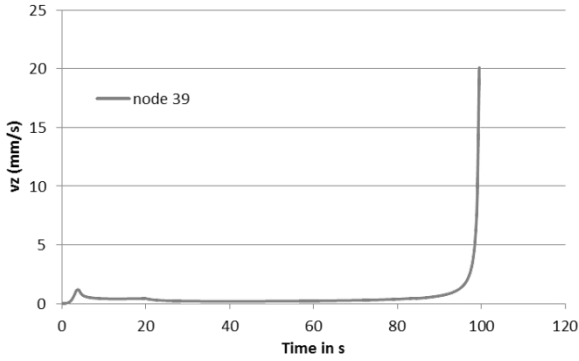
to 202 MPa.s and its strain rate sensitivity  $m$  is equal to 0.15. Two oil pressure cycles are considered from the initial value of 0.3 MPa at  $t = 0$  s.

1. **Stable cycle:** linear increase to 3 MPa at  $t = 20$  s followed by a linear increase to 4.125 MPa at  $t = 50$  s
2. **Unstable cycle:** same linear increase to 3 MPa at  $t = 20$  s followed by a linear increase to 6 MPa at  $t = 100$  s.

Figure 3 shows the deformed bulged sheets at the end of the process in both cases, computed with an unstructured finite element mesh of tetrahedra with 700 nodes. Figure 4 shows the sudden increase of velocity at the central node of the sheet with the “unstable” cycle and that is triggered by the thickening of the sheet before tearing.



**Figure 3.** Isovalues of the equivalent cumulated strain at the end of the process. Left: stable case. Right: unstable case.



**Figure 4.** Time evolution of the vertical velocity at the centre of the sheet in the unstable case.

### 3 Metal Forming Problem Equations

Inertia and gravity forces can be neglected in the problem equations (1.a). Elasticity is also neglected during large material deformations under hot conditions, so that the material is fully incompressible (1.b), where  $\sigma$  is the stress tensor and  $v$  the material velocity.

$$\begin{cases} \nabla \cdot \sigma = 0 & \text{over } \Omega \\ \nabla \cdot v = 0 & \text{over } \Omega \end{cases} \quad (1)$$

Norton-Hoff constitutive equation is given by (2) where  $\varepsilon(v)$  is the strain rate tensor,  $\bar{\varepsilon}$  the equivalent strain and

$p$  the hydrostatic pressure. Boundary conditions are summarized in equation (3), and the resulting weak form of the mixed velocity / pressure problem is presented in equation (4).

$$\sigma = 2K \left( \sqrt{3} \dot{\varepsilon} \right)^{m-1} \varepsilon(v) - pI \quad (2)$$

$$\begin{cases} \sigma \cdot n = 0 & \text{over } \partial\Omega_{free} \\ \sigma \cdot n = p_0 & \text{over } \partial\Omega_{pressure} \\ v = 0 & \text{over } \partial\Omega_{blankholder} \end{cases} \quad (3)$$

$$\begin{cases} \forall w, \int_{\Omega} 2K \left( \sqrt{3} \dot{\varepsilon} \right)^{m-1} \varepsilon(v) : \varepsilon(w) d\omega \\ \quad - \int_{\Omega} p \operatorname{div}(w) d\omega = 0 \\ \forall q, - \int_{\Omega} q \operatorname{div}(v) d\omega = 0 \end{cases} \quad (4)$$

The computational domain  $\Omega$  is discretized using an unstructured mesh made of tetrahedra and equation (4) by the Mini-Element P1+/P1 formulation [3]. The resulting system of  $(v_h, p_h)$  equations is nonlinear:  $R$  denotes the residual of the equations and  $J$  its Jacobian. The matrix formulation of the linear system solved at each iteration of the nonlinear resolution algorithm can be written as:

$$J \delta q = -R, \text{ with } J = \begin{bmatrix} J_{vv} & J_{vp} \\ {}^t J_{vp} & J_{pp} \end{bmatrix}, \text{ and } \delta q = \begin{pmatrix} \delta v \\ \delta p \end{pmatrix} \quad (5)$$

### 4 Reduced Order Model (ROM)

Proper Orthogonal Decomposition (POD) methods generally aim at finding a reduced equivalent system to (5) while preserving the accuracy of the solution. It consists in building a reduced orthonormal basis  $(N_r)_{r=1, \bar{m}}$  with  $\bar{m} < \mathcal{N}$  (number of dof) that minimizes the sum of the squared projection error over the chosen snapshots, following the snapshot POD approach introduced by Sirovich in [4]. At every snapshot time  $(t_i)_{i=1, t_n}$ , the velocity and pressure fields are stored respectively into two matrices  $Q_v$  and  $Q_p$ :

$$Q_v = \begin{bmatrix} v(x_1, t_1) & \cdots & v(x_1, t_n) \\ \vdots & \ddots & \vdots \\ v(x_{\mathcal{N}}, t_1) & \cdots & v(x_{\mathcal{N}}, t_n) \end{bmatrix} \quad (6)$$

$$Q_p = \begin{bmatrix} p(x_1, t_1) & \cdots & p(x_1, t_n) \\ \vdots & \ddots & \vdots \\ p(x_{\mathcal{N}/3}, t_1) & \cdots & p(x_{\mathcal{N}/3}, t_n) \end{bmatrix} \quad (7)$$

The reduced basis is derived from the Singular Value Decomposition (SVD) of  $(Q_i)_{i \in \{v,p\}}$  (8), where the  $U_i$  matrix contains the reduced basis vectors  $(N_r)_r$ .

$$Q_i = U_i \Sigma_i (V_i)^t, \quad i \in \{v,p\} \quad (8)$$

Selecting the  $\bar{m}$  first vectors for velocity and pressure provides the  $A$  reduction matrix (9). The solution in the reduced space writes  $q = Aa$  (10) and the reduced matrix formulation of the problem is expressed by (11):

$$A = \begin{bmatrix} U_v & 0 \\ 0 & U_p \end{bmatrix} \quad (9)$$

$$q = Aa, \text{ with } q = \begin{pmatrix} v \\ p \end{pmatrix}, \text{ and } a = \begin{pmatrix} a_v \\ a_p \end{pmatrix} \quad (10)$$

$${}^tAJA\delta a = -{}^tAR \quad (11)$$

In order to extend the validity range of the ROM in respect to a specific material parameter coefficient such as  $m$ , computing snapshots for different  $m$  values is necessary. In the DEPOD approach [5], the new  $m$  value is obtained from a slight modification of the initial value (12), and the new combined snapshot matrix  $(\tilde{Q}_i)_{i \in \{v,p\}}$  includes results from both simulations (13) where  $\beta$  is a numerical coefficient:

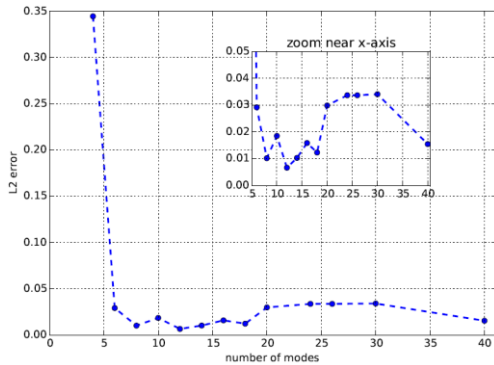
$$m^2 = m^1 + \delta m \quad (12)$$

$$\tilde{Q}_i = \left[ Q_i^1, \beta \frac{Q_i^1}{Q_i^2 - Q_i^1} (Q_i^2 - Q_i^1) \right], \quad i \in \{v,p\} \quad (13)$$

## 4 Applications

### 4.1 Modes Selection

Snapshots are extracted at every time step, Figure 4 shows the relative  $L^2$  error on the velocity field at the end of the process, according to the number of velocity and pressure modes. The error decreases very rapidly and the minimum value is obtained with 12 modes. This number is then selected for following computations.

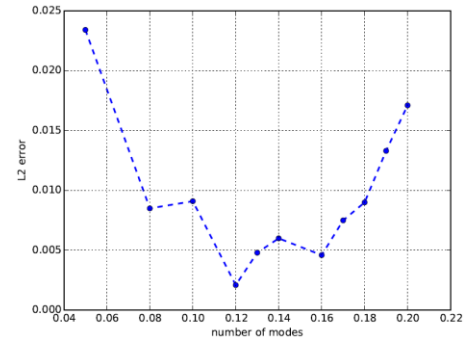


**Figure 4.** Approximation error of the ROM versus the number of velocity/pressure modes.

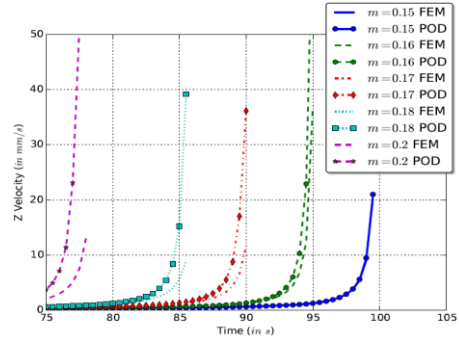
### 4.2 Accuracy of the Reduced Order Model

The ROM constructed in this way is now used with different values of the  $m$  coefficient of the Norton-Hoff constitutive equation (2). Figure 5 shows that for  $m$  ranging between 0.08 and 0.18 the relative  $L^2$  error is less than 1%, which is more than acceptable. This domain can be further increased by the DEPOD approach. A new simulation is run with  $m=0.16$  in order to compute new snapshots and to extract new modes. The errors for  $m=0.05$  and  $m=0.2$  are then reduced to less than 1%.

In the unstable case, Figure 6 shows that the moment for which the instability occurs depends significantly on the  $m$  value and is quite well predicted by the ROM.



**Figure 5.** Approximation error of the ROM according to the strain rate sensitivity coefficient value of the material.



**Figure 6.** Time evolution of the vertical velocity at the centre of the sheet in the unstable case for different values of the material strain rate sensitivity coefficient with the ROM and full model.

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