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Abstract

In a context of market coupling, we study analytically the impact of wind farm concentration and of the uncertainty resulting from the introduction of renewable energy on the procurement total cost, on the market welfare and on the ratio of renewable procurements to conventional supplies. Markets having incomplete information on the quantities of renewable energy produced by the other markets, we show that the providers have incentives to buy information regarding the variability of the other markets' productions. Provided this information could be certified and sold by an external operator, we derive analytically the optimal price for such certified information, depending on the required confidence level.

Keywords: Uncertainty ; Optimization ; Energy Markets ; Intermittent Sources

1 Introduction

Following energy market liberalization, market coupling, developed jointly by power exchanges and transport operators, aims at improving the use of available cross-border capacity and promises a greater harmonization of prices between countries. It creates a unique platform for daily electricity transactions. It implicitly allocates interconnection capacity in the day-ahead and spot timescales until a uniform market clearing price is achieved or the available capacity is fully utilized. In this latter case i.e., when capacity becomes limiting, a congestion rent is paid to the grid operator to give him incentives to invest in capacity upgrading. The increasing introduction of renewable energy in the energy mix, required by the governments, complexifies the coupling mechanisms, due to their strong dependence on exogenous factors such as weather conditions. In the following parts of the Introduction, we describe the major steps of the energy market liberalization, the main challenges associated with the increasing penetration of renewable energy and, finally, give some insights about existing coupling mechanisms.

Energy market liberalization: Energy markets, and especially electricity markets, were traditionally considered as natural monopolies, due to the huge investments required and relatively low marginal costs, and hence they were managed by national

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firms that owned both production plants and distribution network. Some interconnections were created between the States, but only for adjustment purpose.

However, since the end of the 1990s the European Union (EU) decided to progressively liberalize energy markets, i.e., gas and electricity, and to create a global competitive European market. To that purpose, the EU set three versions for the energy packages. The last one, adopted in 2009, concerns:

> First, the effective separation between transport network management, on the one hand, and production and energy supply, on the other hand, in order to avoid that any single firm took the control of the entire production, of the supply chain and also, to promote the competition
>
> Second, the regulation, especially through the creation of the Agency for the Cooperation of Energy Regulators [25], in order to enhance the interconnections between energy markets and to increase the security of the supply in case of blackouts
>
> Third, the cooperation between transmission system operators

In France, one consequence of the electricity market liberalization is that the different activities that were handled by a sole national company, such as production, transport, distribution and electricity selling, are now dispatched between several firms. Precisely, production and distribution are provided by private companies, whereas transport remains a national regulated activity. Many actors are involved on each market: producers, who sell energy on the market ; providers, who buy energy on the market and distribute it to direct consumers ; direct consumers, which demand will be aggregated ; transmission system operators who manage the transport network. Another consequence is that the prices that were set by the State are now determined directly by counterparts on long term contracts, or on liberalized pool markets, by equilibrium between the demand and the supply.

These markets occur at different times, ranging from long terms (years or months) to day-ahead and real time, also known as spot markets. In general, energy prices become larger and capacities smaller when we come closer to real time [19]. Finally, the exchanges between the different States are much larger than before, and the interconnections can be easily congested because they were created for adjustment purpose and not for trading purpose [9], [20].

The increasing penetration of renewable energy: In addition, the European Commission has adopted in 2008 the EU Energy and Climate Package. Its two main objectives are setting up a more sustainable common European energy policy and combating climate change. More precisely, three targets have been set regarding the reduction by 20% of greenhouse gases, the increase of energy efficiency up to 20% and the reaching of 20% of renewable energy in the total energy consumption in the EU, by 2020 [26].

The resulting increasing penetration of intermittent, unpredictable renewable en-

ergy such as wind energy, induces significant challenges for markets[1]. Their erratic generation makes them very difficult to forecast using conventional statistical approaches since they rely on many exogenous factors such as weather conditions [15], [18]. This salient issue seriously threatens the constant equilibrium between supply and demand, that is a guarantee for the efficient operation of the underlying grid. Modeling renewable energy production as random individual sequences, requiring no underlying stochastic assumptions, we studied the impact of distributed learning on the demand and supply equilibrium in [15], [16]. In a multi-period market context, Nair et al. explicitly characterized the impact of a growing renewable penetration on the procurement policy by considering a scaling regime that models the aggregation of unpredictable renewable sources [19]. They introduced a scaling regime for wind penetration, which models the effect of aggregating the output of several wind generators. A key feature of their model is that it allows to take into account the relative concentration (or, inversely, scattering) of the intermittent energy sources being aggregated [27]. Based on this scaling model, they studied how the optimal reserves, the amount of conventional generation produced, as well as the cost of procurement, scale with increasing wind penetration.

Market coupling: Competitiveness, sustainability and energy supply security are essential issues in the pursuit of European energy market integration and the creation of a single energy area. Energy markets were initially liberalized autonomously at a national level, with domestic scope, but there has been a growing need for an optimal management of cross-border transmissions and congestions. However, optimal network governance of a centralized autority depends on the balance between different interests accross countries [8]. Market functioning in terms of competition among producers can be obstructed by limited transmission capacity at the borders of the interconnected markets. Therefore two mechanisms have been put forward to solve the allocation of such scarce-border capacity: the first is implicit auctioning, and the second is coordinated explicit auctioning which has not been implemented yet. The latter system will allow countries to keep their power exchanges running, but it is found to be less efficient than the former system when uncoordinated [8]. The implicit auction mechanism adopted in Europe is designed to include cross-border trades in the day-ahead auction mechanisms on individual power exchanges, to avoid inefficiency. Different implementations of the implicit coupling mechanism exist such as no coupling, volume coupling, one way price coupling, etc. The price coupling implementation, which is the focus of this article, optimizes cross-border flows to reflect energy-only price differences between the coupled markets. It implicitly allocates the interconnection capacity in the day-ahead and real timescales until a uniform market clearing price is achieved or the available capacity is fully utilized [2]. Successful examples of implicit auctioning with price coupling are those within the Nordic area, those between Spain and Portugal, and those involving the Central Western European power markets (CWE) i.e., the Netherlands, Belgium, France, Luxembourg and Germany electricity

[1]Other sources of renewable energy, more easily predictable, such as those coming from marine currents, are still under study. Déporte et al. studied a way to collect it through a wavering membrane [6]. Although this approach seems promising to overcome the uncertainties associated with renewable energy production, much remains to be done.

markets.

From a theoretical point of view, the study of market coupling using bottom-up approaches is still in its infancy. Oggioni and Smeers analyzed and simulated market coupling in a toy network and a two zone splitted market assuming different zonal decompositions, coordination of transmission system operators, and a lack of strategic behaviors. Implementing a Nash equilibrium and social welfare, they observed that market coupling can be weak and proposed a benchmark for market efficiency using the nodal pricing system developed in the US [13], [21]. A well-known issue which makes the development of economic models challenging concerns the occurence of negative price spikes at times when very low demand meets high supply [7], [8]. The negative price spike frequency has increased with the introduction of renewable energy which production is very difficult to predict and hence, barely impossible to adapt to sudden changes in the demand. The occurence of negative price spikes cannot be deemed a residual phenomenon [7], [8]. Such spikes can also be generated by factors contributing to exceptional slumps in demand, limited flexibility of power plant operations, limited transmission capacities, etc.

Taking the end user point of view, market coupling should have a good influence on his bill [22]: by maximizing the use of cross-border interconnection capacity, market coupling increases the level of market integration and facilitates the access to low-cost generation by consumers located in high-cost generation countries, such as Italy. However, the associated congestion management costs might tend to increase significantly in the future given a higher share of intermittent renewable production and potential divergences in the developments of transmission and generation infrastructure [13].

Article organization: The literature dealing with energy market economic models can be roughly divided into two parts: market coupling mechanisms [2], [7], [8], [9], [10], [20], [21], [22], as well as models for multi-period markets with uncertain supply [3], [11], [19]. In this article, we try to combine both approaches. We propose a model taking into account the relative concentration of the intermittent energy sources being aggregated. It is described in Section 2. Considering $N \in \mathbb{N}^*$ coupled energy markets, we first study how this scaling impacts the market optimal reserves, optimal conventional energy productions, optimal prices on day ahead and spot markets in Section 3. In practice, the markets do not communicate complete information on the value of their reserve and renewable energy procurement to their neighboring markets [20]. This can be particularly disadvantageous for the providers seeking to minimize their total costs. Using numerical simulations, we show that, for each provider, the knowledge of the other market renewable procurement forecast errors, minimizes his total cost. Assuming that an external operator has the opportunity to certify such an information according to a certain confidence level, we determine analytically the optimal price of such a certified information in Section 4.

2 The model

We consider $N \in \mathbb{N}^*$ neighboring energy markets. Each market $i = 1, ..., N$ covers a specific country. It is associated with a node and is interconnected to the other markets

through lines managed by a transmission operator. The resulting graph constitutes the underlying network, also known as the super grid. The market rules can be designed in a decentralized manner provided each market maximizes selfishly its objective function, like in Germany for instance [13], or in a centralized manner where a central agent rules the market. Each market $i = 1, ..., N$ is composed of:

(i) Providers delivering energy to consumers characterized by their aggregated demand

(ii) Conventional energy producers characterized by their aggregated cost function

(iii) Renewable (more specifically wind) energy producers characterized by their aggregated production

Additionnally, we consider two time periods: t_f, defining the occurence of the forward market, and t_0, defining the occurence of the real-time or spot market. The forward market can take place the day before t_0, in this case it will be called day-ahead market, or it can take place months or years before t_0. In this latter case, it is called long-term market.

2.1 Description of the markets

2.1.1 Market i

It is defined by:

- d_i the end users' total demand of energy at time t_0
- w_i the energy produced at time t_0 by the market renewable energy producers. This information is available to the market at t_0 but not at t_f. Furthermore, we have: $w_i = \hat{w}_i - \epsilon_i$ where $\hat{w}_i$ is the forecast made at t_f of the quantity of renewable energy that market i producer will produce at t_0. ϵ_i is a random variable representing the forecast error, f_{ϵ_i} its density function, with support $[L_i; B_i]$ where $L_i \in \{-\infty\} \cup \mathbb{R}$ and $B_i \in \mathbb{R} \cup \{+\infty\}$ and F_{ϵ_i} (resp. $\overline{F}_{\epsilon_i} = 1 - F_{\epsilon_i}$) its direct (resp. complementary) cumulative distribution function. In the rest of the article, the forecast generating density function will coincide with a Gaussian distribution function centered in zero and of standard deviation σ_i. More general distribution functions can, of course, be considered
- $c_i^f(s_i^f) = a_i^f + b_i^f s_i^f$ (resp. $c_i^0(s_i^0) = a_i^0 + b_i^0 s_i^0$) the marginal cost function of conventional energy produced by market i and purchased at t_f (resp. at t_0). For convenience, they are supposed to be linear in the supply [10], [14]. We assume that $a_i^0 > a_i^f > 0$ and that $b_i^0 > b_i^f > 0$ guaranteeing that the marginal cost on the spot market remains larger than on the day-ahead market
- q_i^f (resp. q_i^0) market i demand of conventional energy on forward (resp. spot) markets

- s_i^f (resp. s_i^0) market i supply of conventional energy on forward (resp. spot) markets

The global market is characterized by the equilibrium between the supply and the demand: $q_{tot}^f(N) = \sum_{i=1,...,N} q_i^f = \sum_{i=1,...,N} s_i^f$ (resp. $q_{tot}^0(N) = \sum_{i=1,...,N} q_i^0 = \sum_{i=1,...,N} s_i^0$) which is the global quantity of conventional energy exchanged on forward (resp. spot) markets.

The amounts of energy purchased by market i at t_f and at t_0 are defined as follows: $q_i^f = \left(d_i - \hat{w}_i + r_i\right)_+$ and $q_i^0 = \left(d_i - w_i - q_i^f\right)_+$ where r_i is a reserve of energy purchased on forward (lower cost) market because of uncertainty of supply at t_0. Reserve r_i is determined by the energy supplier on market i for the consumers' demand d_i to be satisfied at t_0 at the lowest possible cost. Market i knows d_i and $\hat{w}_i$. Hence it is equivalent for him to determine q_i^f or r_i. The hypothesis that $q_i^f > 0$ holds as long as the demand exceeds the average wind capacity. In the rest of the article, we will assume that: $q_i^f \triangleq d_i - \hat{w}_i + r_i$.

As usual in industrial organization theory [24], we make the assumption that the prices p_i^f and p_i^0 paid by market i providers for the energy purchased at t_f and t_0 respectively equal the marginal costs[2]: $p_i^f \triangleq c_i^f(s_i^f)$ and $p_i^0 \triangleq c_i^0(s_i^0)$. As stated in the Introduction, the fundamental idea behind market coupling is to create an integrated energy market with uniform energy prices among the involved countries. Therefore, we assume that a clearing price is reached at t_f i.e., $p_i^f = p_j^f \triangleq p^f, \forall i, j = 1, ..., N, i \neq j$. Because the transfers are limited by the available transmission capacities, it will be harder to align the market prices at t_0: if there is equilibrium then $p_i^0 = p_j^0 \triangleq p^0, \forall i, j = 1, ..., N, i \neq j$; otherwise there exists at least one market $i \in \{1, ..., N\}$ in which the provider pays $p_i^0 \neq p_j^0$ for $j \in \{1, ..., N\}$ and $j \neq i$[3]. In the rest of the article, we will make the hypothesis that, at t_f, the markets are myopic and do not anticipate the potential congestion of the lines. This implies that, at t_f, they forecast that the prices will be aligned at t_0 on all the markets i.e., that no congestion occurs[4]. Markets might learn sequentially the other market forecast errors and then, infer their spot prices. However, such learning models are out of the scope of the present article.

[2]The coincidence of prices with marginal costs, even during periods of supply scarcity, can be justified provided a capacity market [17] is deployed in one of the coupled markets, guaranteeing adequate levels of capacity payment [2]. Alternative approaches for bid modeling can be found in [5].

[3]In case where $p_i^0 \neq p_j^0$, a congestion rent $CR = (p_i^0 - p_j^0)t_{j\to i}^0$ is paid to the transmission operator ; $t_{j\to i}^0$ represents the traded flow of energy from market j to market i on spot market. CR is: positive if the lower price market is exporting energy to the higher price market ; null if the interconnection line, binding market i to market j, is not congested and $p_i^0 = p_j^0 = p^0$; negative if the lower price market is importing energy from the higher price market.

[4]Under this assumption, the congestion rent is expected to be null.

2.2 Providers' total cost, producers' profits and social welfare

We define U_i, as the expected total cost, TC_i, that the provider has to pay for his end user energy consumption:

$$U_i = \mathbb{E}\Big[TC_i\Big] = q_i^f p^f + \mathbb{E}\Big[q_i^0 p_i^0\Big] \tag{1}$$

Then market i end users' utility is inferred according to the relation: $U_{i0} - U_i$ where $U_{i0} > 0$ contains their net utility.

We let Π_i be the profit of market i energy producer. It is defined as the difference between the price paid by all the markets for the puchase of conventional energy and the cost of the energy. We assume that all the supply is sold at each time. Then:

$$\Pi_i \quad = \quad s_i^f p^f + \mathbb{E}[s_i^0 p_i^0] - \int_0^{s_i^f} c_i^f(s)ds - \mathbb{E}\Big[\int_0^{s_i^0} c_i^0(s)ds\Big] \tag{2}$$

Finally, we define W_i, the welfare of market i, as the difference between the producer's profit and the provider's total cost:

$$W_i = \Pi_i - U_i$$

The social welfare is defined as the sum of the welfares of all the involved agents:

$$W = \sum_{i=1,\dots,N} W_i$$

2.3 Renewable energy modeling

For each market, its renewable wind energy production is a function of the number of wind farms and of their concentration which is characterized by their spatial distribution over market i country. To determine the renewable energy procurement for market i, we use Nair et al. model [19]. For market i, we introduce:

- α_i the average production of a single wind farm
- γ_i the number of wind farms
- $\theta_i \in [\frac{1}{2}; 1]$ (resp. $1 - \theta_i \in [0; \frac{1}{2}]$) a constant capturing the concentration (resp. the scattering) of the wind farm locations over market i country. The more (resp. the less) concentration, the more (resp. the less) correlation there is between the wind farm production

We suppose that, at t_f, α_i is the best forecast of wind energy procurement of a wind farm [19]. Then: $\hat{w}_i(\gamma_i) = \alpha_i \gamma_i$. The forecast error will depend on the wind penetration too, and we choose the coefficient θ_i so that $\epsilon_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\epsilon_i}$ where $\tilde{\epsilon_i}$ represents the forecast error for the production of a single wind farm and $\tilde{\sigma_i}$ its standard deviation. We propose the following interpretation for the scaling of θ_i: If the wind farms are co-located they will all produce the same quantity of energy at the same time i.e., their productions are strongly correlated. This is the case when $\theta_i = 1$. This implies in turn

that: $\epsilon_i = \gamma_i \tilde{\epsilon}_i$ and that: $\hat{w}_i = w_i + \gamma_i \tilde{\epsilon}_i$. On the contrary, if they are spatially distributed so that their productions are independent from one another i.e., uncorrelated, and under the assumption that the forecast errors are distributed according to Gaussian distribution functions, the Central Limit Theorem tells us that: $\sigma_i = \sqrt{\gamma_i}\tilde{\sigma}_i$ [19]. Therefore, the wind farm productions are independent from one another if, and only if, $\theta_i = \frac{1}{2}$. Note that in case of more general forecast error distribution functions, it can be interpreted as an approximation for γ_i large enough. Finally, in case where $\theta_i \in]\frac{1}{2}; 1[$, the wind farms are randomly located over the country and their spatial distribution is intermediate between perfect independence and co-location.

With these notations, we obtain $w_i(\gamma_i) = \hat{w}_i(\gamma_i) - \epsilon_i(\gamma_i) = \gamma_i \alpha_i - \gamma_i^{\theta_i} \tilde{\epsilon}_i$ and $\sigma_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\sigma}_i$.

In the next section, we will determine the optimal quantities of energy to be purchased by the markets on the forward market, $(q_i^f)_{i=1,\dots,N}$, or equivalently, their optimal reserves, $(r_i)_{i=1,\dots,N}$. The optimization programs can be centralized by a supervisor who determines the optimal reserves for all the markets or, it can be decentralized provided each market optimizes its reserve selfishly. Furthermore, depending on the actors who take the decision and on the timing of the game, it can be relevant to minimize the provider's total cost or, to maximize the market welfare. Indeed, if we consider the short term effect, the costs of the producers remain fixed and, in perfect concurrence, they will bid at their marginal cost, so that the energy providers, who buy energy to the producers, have most of the power decision and will try to minimize their own total costs. But, if we consider longer term effects, the producers can choose to change their costs, by investing in new technologies or by scaling their plant, in order to optimize their own profits, so that the decision power is shared between the producers and the providers. In this latter case, it is more appropriate to maximize the market welfare.

3 Providers' total cost and social welfare optimization

We describe the solving of the provider's total cost minimization in Subsection 3.1 and of the market welfare maximization in Subsection 3.2, both under decentralized and centralized management of the agents.

3.1 Optimization of the providers' total costs

We start by determining the analytical expressions of the coupling prices for forward and spot markets. Then substituting these values in the providers' total costs, we derive the optimal reserves under decentralized and centralized management.

3.1.1 Derivation of the coupling prices

We set: $A^f \triangleq \sum_{i=1,\dots,N} \frac{a_i^f}{b_i^f}$ and $B^f \triangleq \sum_{i=1,\dots,N} \frac{1}{b_i^f}$. Furthermore, we make the assumption that the marginal cost parameters at t_f are chosen so that $B^f \neq 0$.

Lemma 1. *The coupling price for the forward market is:* $p^f = \frac{\sum_{i=1,\dots,N} q_i^f + A^f}{B^f}$.

Proof of Lemma 1. Using the assumption of the supply and demand equilibrium guaranteed by the market rules, we have:

$$\begin{aligned} q_{tot}^f(N) &= \sum_{i=1,\dots,N} q_i^f = \sum_{i=1,\dots,N} s_i^f \\ &= \sum_{i=1,\dots,N} \frac{p_i^f - a_i^f}{b_i^f} \text{ under the assumption that } p_i^f = c_i^f \\ &= \sum_{i=1,\dots,N} \frac{p^f - a_i^f}{b_i^f} \text{ since the } N \text{ markets are coupled at } t_f \\ &= p^f \Big(\sum_{i=1,\dots,N} \frac{1}{b_i^f} \Big) - \sum_{i=1,\dots,N} \frac{a_i^f}{b_i^f} \end{aligned}$$

We infer from the following equations the forward price on the coupling zone: $p^f = \frac{\sum_{i=1,\dots,N} q_i^f + A^f}{B^f}$. □

We set: $A^0 \triangleq \sum_{i=1,\dots,N} \frac{a_i^0}{b_i^0}$ and $B^0 \triangleq \sum_{i=1,\dots,N} \frac{1}{b_i^0}$. Furthermore, we make the assumption that the marginal cost parameters at t_0 are chosen so that $B^0 \neq 0$. Using the same principle as in Lemma 1 proof, we infer the spot price on the coupling zone:

Lemma 2. *The N markets being coupled at time t_0, the coupling price for the spot market is:* $p^0 = \frac{\sum_{i=1,\dots,N} q_i^0 + A^0}{B^0}$.

3.1.2 Decentralized program

Each market i provider determines independently and simultaneously the quantity of energy to purchase q_i^f or equivalently its reserve r_i so as to minimize his procurement total cost:

$$\min_{r_i \geq 0} \qquad U_i = \mathbf{E}\Big[TC_i\Big] \tag{3}$$

Market i provider determines the best answer, $r_i^{BA}(\mathbf{r_{-i}})$, where $\mathbf{r_{-i}}$ is a $N-1$ dimensional vector containing the reserves of all the providers except market i provider, which minimizes its consumer total cost. The decentralized program output is a Nash equilibrium, $(r_i^{NE})_{i=1,\dots,N}$, defined by: $r_i^{NE} = r_i^{BA}(\mathbf{r_{-i}}), \forall i = 1, ..., N$.

Proposition 3. *There exists a unique Nash equilibrium solution of Program 3.*

$$\sum_{j=1,\ldots,N,j\neq i} \mathbb{E}\Big[(\epsilon_j - r_j)|\epsilon_j \geq r_j\Big] + A^0$$

Proof of Proposition 3. We let: $C_i^0 \triangleq \frac{j=1,\ldots,N,j\neq i}{B^0}$ and $C_i^f \triangleq \frac{\sum_{j=1,\ldots,N,j\neq i}(d_j - \hat{w}_j + r_j) + A^f}{B^f}$. The Lagrangian function associated to Program 3 is: $\mathcal{L}_i^U(r_i, \mu_1) = U_i - \mu r_i$ where $\mu \in \mathbb{R}_+$ is a Lagrange multiplier. According to the Karush-Kühn-Tucker (KKT) conditions, at the optimum in r_i: $\frac{\partial \mathcal{L}_i^U(r_i,\mu)}{\partial r_i} = 0$, $r_i = 0$ or $\mu = 0$.

We make the assumption that market i reserve is positive i.e., $r_i > 0$. Therefore: $\mu = 0$. Going back to the definition of the provider's total cost, as defined in Equation (1), it can be rewritten by substituting the coupling price expressions derived in Lemmas 1 and 2: $U_i = \Big(d_i - \hat{w}_i + r_i\Big)\frac{\sum_{j=1,\ldots,N}\Big(d_j - \hat{w}_j + r_j\Big) + A^f}{B^f} + \mathbb{E}\Big[(\epsilon_i - r_i)_+ \frac{\sum_{j=1,\ldots,N}(\epsilon_j - r_j)_+ + A^0}{B^0}\Big]$. Differentiating U_i with respect to r_i, we obtain:

$$\begin{aligned}
\frac{\partial U_i}{\partial r_i} &= \frac{\sum_{j=1,\ldots,N}(d_j - \hat{w}_j + r_j) + A^f}{B^f} + \frac{1}{B^f}(d_i - \hat{w}_i + r_i) \\
&+ C_i^0 \frac{\partial}{\partial r_i}\mathbb{E}\Big[(\epsilon_i - r_i)|\epsilon_i \geq r_i\Big] + \frac{1}{B^0}\frac{\partial}{\partial r_i}\mathbb{E}\Big[(\epsilon_i - r_i)^2|\epsilon_i \geq r_i\Big] \qquad (4)
\end{aligned}$$

But: $\frac{\partial}{\partial r_i}\mathbb{E}\Big[(\epsilon_i - r_i)|\epsilon_i \geq r_i\Big] = -\bar{F}_{\epsilon_i}(r_i)$ and

$$\begin{aligned}
\frac{\partial}{\partial r_i}\mathbb{E}[(\epsilon_i - r_i)^2|\epsilon_i \geq r_i] &= \frac{\partial}{\partial r_i}\int_{r_i}^{+\infty}(x_i - r_i)^2 f_{\epsilon_i}(x_i)dx_i \\
&= 2r_i\int_{r_i}^{+\infty} f_{\epsilon_i}(x_i)dx_i - 2\int_{r_i}^{+\infty} x_i f_{\epsilon_i}(x_i)dx_i \\
&= 2r_i\bar{F}_{\epsilon_i}(r_i) - 2\mathbb{E}\Big[\epsilon_i|\epsilon_i \geq r_i\Big]
\end{aligned}$$

By substitution in $\frac{\partial U_i}{\partial r_i}$ expression, we obtain the simplified expression: $\frac{\partial U_i}{\partial r_i} = \frac{2}{B^f}\Big(d_i - \hat{w}_i + r_i\Big) + C_i^f + \Big(\frac{2r_i}{B^0} - C_i^0\Big)\bar{F}_{\epsilon_i}(r_i) - \frac{2}{B^0}\mathbb{E}\Big[\epsilon_i|\epsilon_i \geq r_i\Big]$. Differentiating twice U_i with respect to r_i we obtain: $\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2}{B^f} + \Big(C_i^0 - \frac{2r_i}{B^0}\Big)f_{\epsilon_i}(r_i) + \frac{2}{B^0}\bar{F}_{\epsilon_i}(r_i) + \frac{2}{B^0}r_i f_{\epsilon_i}(r_i) = \frac{2}{B^f} + C_i^0 f_{\epsilon_i}(r_i) + \frac{2}{B^0}\bar{F}_{\epsilon_i}(r_i) \geq 0$. This proves that $\mathbf{r_{-i}}$ being fixed, function U_i is convex with respect to r_i. This implies that $\mathbf{r_{-i}}$ being fixed, the optimization Program 3 admits a unique best answer, for every market $i = 1, ..., N$. Therefore, it admits a unique Nash equilibrium. □

3.1.3 Centralized program

A supervisor determines the N market reserves $(r_i)_{i=1,...,N}$ minimizing the sum of the providers' total costs over the N markets:

$$\begin{aligned} \min_{(r_i)_{i=1,...,N}} \quad & U = \mathbf{E}\Big[\sum_{i=1,...,N} TC_i\Big] \\ \text{s.t.} \quad & r_i \geq 0, \forall i = 1, ..., N \end{aligned} \tag{5}$$

Proposition 4. *There exists a unique global optimum minimizing function U.*

Proof of Proposition 4. The Lagrangian function associated to Program 5 is: $\mathcal{L}^U\Big((r_i)_i, \tilde{\mu}\Big) = U - \sum_{i=1,...,N} \tilde{\mu}_i r_i$ where $\tilde{\mu} = (\tilde{\mu}_i)_{i=1,...,N} \in \mathbb{R}^N_+$ are Lagrange multipliers. According to KKT conditions, at the optimum in $(r_i)_i$: $\frac{\partial \mathcal{L}^U\Big((r_i)_i, \tilde{\mu}\Big)}{\partial r_j} = 0, \forall j = 1, ..., N$, $r_i = 0$ or $\tilde{\mu}_i = 0, \forall i = 1, ..., N$.

We make the assumption that all the reserves are positive i.e., $r_i > 0, \forall i = 1, ..., N$. Going back to the definition of U, we obtain: $U = \mathbb{E}\Big[\sum_{i=1,...,N} TC_i\Big] = \sum_{i=1,...,N} \mathbb{E}\Big[TC_i\Big]$ $= \sum_{i=1,...,N} U_i$. The differentiation of U with respect to r_i gives us:

$$\begin{aligned} \frac{\partial U}{\partial r_i} &= \frac{1}{B^f}\Big(d_i - \hat{w}_i + r_i\Big) + \sum_{j=1,...,N} \frac{d_j - \hat{w}_j + r_j}{B^f} + C_i^f \\ &- \bar{F}_{\epsilon_i}(r_i) \sum_{j=1,...,N, j\neq i} \mathbb{E}\Big[\frac{(\epsilon_j - r_j)}{B^0} | \epsilon_j \geq r_j\Big] + \Big(\frac{2r_i}{B^0} - C_i^0\Big)\bar{F}_{\epsilon_i}(r_i) \\ &- \frac{2}{B^0}\mathbb{E}\Big[\epsilon_i | \epsilon_i \geq r_i\Big] \end{aligned} \tag{6}$$

Differentiating twice U with respect to r_i we obtain:

$$\frac{\partial^2 U}{\partial r_i^2} = \frac{2}{B^f} + \Big(C_i^0 + \sum_{j=1,...,N, j\neq i} \frac{\mathbb{E}[(\epsilon_j - r_j)|\epsilon_j \geq r_j]}{B^0}\Big) f_{\epsilon_i}(r_i) + \frac{2}{B^0}\bar{F}_{\epsilon_i}(r_i) \geq 0$$

and for any $j = 1, ..., N, j \neq i$: $\frac{\partial^2 U}{\partial r_i \partial r_j} = \frac{2}{B^f} + \frac{2}{B^0}\bar{F}_{\epsilon_i}(r_i)\bar{F}_{\epsilon_j}(r_j) \geq 0$. Hence, the Hessian matrix associated to U is non-negative. This implies that function U is convex with respect to each of its component. Therefore, the minimization of U over $\mathbb{R}^N_+$ admits a unique global optimum. □

3.2 Optimization of the welfare

Substituting the coupling prices derived in Lemmas 1 and 2 in market i producer's profit Π_i defined in Equation (2), we obtain:

$$\begin{aligned}\Pi_i &= s_i^f \frac{\sum_{j=1,\dots,N}\Big(d_j - \hat{w}_j + r_j\Big) + A^f}{B^f} + s_i^0 \mathbb{E}\Big[\frac{\sum_{j=1,\dots,N}(\epsilon_j - r_j)_+ + A^0}{B^0}\Big] \\ &- a_i^f s_i^f - b_i^f \frac{(s_i^f)^2}{2} - a_i^0 s_i^0 - b_i^0 \frac{(s_i^0)^2}{2} \end{aligned} \tag{7}$$

3.2.1 Optimal conventional energy procurement

The conventional energy procurement can be optimized before the market takes place i.e., at t_f and t_0. It should be optimized so as to maximize the producer profit at t_f and at t_0. It is straightforward to observe, judging by Equation (7), that market i producer's profit at t_f (resp. t_0) is concave in s_i^f (resp. s_i^0) since it is a second order polynomial equation in s_i^f (resp. s_i^0) with a negative highest order coefficient: $\frac{-b_i^f}{2}$ (resp. $\frac{-b_i^0}{2}$).

Proposition 5. *Over market i, the supply of conventional energy at time t_f maximizing the producer's profit is:* $s_i^{f*} = \frac{1}{b_i^f}\Big\{\frac{\sum_{j=1,\dots,N}(d_j - \hat{w}_j + r_j) + A^f}{B^f} - a_i^f\Big\}$ *and, at time t_0:* $s_i^{0*} = \frac{1}{b_i^0}\Big\{\frac{\sum_{j=1,\dots,N}\mathbb{E}[(\epsilon_j - r_j)|\epsilon_j \geq r_j] + A^0}{B^0} - a_i^0\Big\}$.

Proof of Proposition 5. Before market takes place at t_f, the producer optimizes his conventional energy procurement so as to maximize his profit: Π_i. But, as already mentioned, Π_i is a second order polynomial equation in s_i^f with a negative highest order coefficient: $-\frac{b_i^f}{2}$. Therefore Π_i admits a unique maximum in s_i^f. It is obtained as solution of $\frac{\partial \Pi_i}{\partial s_i^f} = 0$. The producer uses the same principle to determine s_i^0 just before market occurs at t_0. □

3.2.2 Decentralized program

Each market i determines independently and simultaneously its reserve, r_i, so as to maximize its welfare:

$$\max_{r_i \geq 0} \quad W_i \tag{8}$$

Proposition 6. *Program 8 admits a Nash equilibrium if, and only if, there exists $r_i \geq 0$ such that $\frac{\partial U_i}{\partial r_i} = \frac{s_i^f}{B^f} - \frac{s_i^0}{B^0}\bar{F}_{\epsilon_i}(r_i)$ and $\Big(\frac{s_i^0}{B^0} - C_i^0\Big)f_{\epsilon_i}(r_i) \leq 2\Big(\frac{1}{B^f} + \frac{1}{B^0}\bar{F}_{\epsilon_i}(r_i)\Big)$. Furthermore, if $\Big(\frac{s_i^0}{B^0} - C_i^0\Big)f_{\epsilon_i}(r_i) \leq 2\Big(\frac{1}{B^f} + \frac{1}{B^0}\bar{F}_{\epsilon_i}(r_i)\Big), \forall r_i \geq 0$, it admits a unique Nash equilibrium.*

Proof of Proposition 6. We start by assuming that market i reserve is positive. To solve Program 8, we need to determine the zeros of the differentiate of W_i with respect to r_i:

$$\frac{\partial W_i}{\partial r_i} = -\frac{\partial U_i}{\partial r_i} + \frac{s_i^f}{B^f} - \frac{s_i^0}{B^0}\bar{F}_{\epsilon_i}(r_i) \tag{9}$$

Differentiating twice W_i with respect to r_i, we obtain: $\frac{\partial^2 W_i}{\partial r_i^2} = -\frac{2}{B^f} + \left(\frac{s_i^0}{B^0} - C_i^0\right) f_{\epsilon_i}(r_i) - \frac{2}{B^0}\bar{F}_{\epsilon_i}(r_i)$. Therefore W_i admits a minimum in r_i if, and only if, $\frac{\partial W_i}{\partial r_i} = 0$ and $\frac{\partial^2 W_i}{\partial r_i^2} \leq 0$. Furthermore, W_i is concave if, and only if, $\left(\frac{s_i^0}{B^0} - C_i^0\right) f_{\epsilon_i}(r_i) \leq 2\left(\frac{1}{B^f} + \frac{1}{B^0}\bar{F}_{\epsilon_i}(r_i)\right), \forall r_i \in \mathbb{R}_+$. If W_i is concave in r_i, it admits a unique maximum in r_i i.e., a unique best answer to $\mathbf{r_{-i}}$. Therefore, the resulting Nash equilibrium defined, for each market, as the best answer to the other markets' reserves, is unique. □

3.2.3 Centralized program

A supervisor determines the N market reserves $(r_i)_i$ maximizing the sum of the market welfares i.e., the social welfare:

$$\begin{aligned} \max_{(r_i)_i} \quad & W = \sum_{i=1,...,N} W_i \\ \text{s.t.} \quad & r_i \geq 0, \forall i = 1, ..., N \end{aligned} \tag{10}$$

Proposition 7. *Program 10 admits a global optimum if, and only if there exists* $(r_i)_i \in \mathbb{R}_+^N$ *such that* $\frac{\partial U}{\partial r_i} = \sum_{j=1,...,N} \left(\frac{s_j^f}{B^f} - \frac{s_j^0}{B^0}\right)\bar{F}_{\epsilon_i}(r_i)$ *and* $\sum_{j=1,...,N} \frac{s_j^0}{B^0} f_{\epsilon_i}(r_i) \leq \frac{\partial^2 U}{\partial r_i^2}$. *Furthermore, if* $\sum_{j=1,...,N} \frac{s_j^0}{B^0} f_{\epsilon_i}(r_i) \leq \frac{\partial^2 U}{\partial r_i^2}, \forall r_i \geq 0, \forall i = 1, ..., N$, *it admits a unique global optimum.*

Proof of Proposition 7. Assuming that all the reserves are positive, to solve Program 10, we need to determine the zeros of the differentiate of W with respect to $r_i, \forall i = 1, ..., N$:

$$\frac{\partial W}{\partial r_i} = -\frac{\partial U}{\partial r_i} + \sum_{j=1,...,N} \frac{s_j^f}{B^f} - \sum_{j=1,...,N} \frac{s_j^0}{B^0}\bar{F}_{\epsilon_i}(r_i), \forall i = 1, ..., N \tag{11}$$

Differentiating twice W with respect to r_i, we obtain:

$$\frac{\partial^2 W}{\partial r_i^2} = -\frac{\partial^2 U}{\partial r_i^2} + \sum_{j=1,...,N} \frac{s_j^0}{B^0} f_{\epsilon_i}(r_i)$$

and with respect to $r_j, j \neq i$: $\frac{\partial^2 W}{\partial r_i \partial r_j} = -\frac{\partial^2 U}{\partial r_i \partial r_j} \leq 0$. Therefore, W admits a minimum in $(r_i)_i$ if, and only if, $\frac{\partial W}{\partial r_i} = 0$ and $\frac{\partial^2 W_i}{\partial r_i^2} \leq 0, \forall i = 1, ..., N$. Futhermore, W is

concave in $(r_i)_i$ if, and only if, $\sum_{j=1,...,N} \frac{s_j^0}{B^0} f_{\epsilon_i}(r_i) \leq \frac{\partial^2 U}{\partial r_i^2}, \forall r_i \in \mathbb{R}_+, \forall i = 1, ..., N.$

□

3.3 Algorithm implementation

In practice, we distinguish two cases for the optimization of the reserve of market $i = 1, 2$:

> **Case** 1**:** market i provider has no a priori about the beliefs of market $j = 1, 2, j \neq i$ provider. In that case, market i provider will assume that market j provider has the same beliefs as him and he will optimize his reserve through a joint gradient descent algorithm with market j
>
> **Case** 2**:** market i provider knows that market j provider is making an error about his demand and offer parameters σ_i, d_i or $\hat{w}_i$. In that case, market i provider will optimize his reserve through a two step game where market j provider optimizes his reserve first and market i provider reacts with his true parameters. This second case will be described more formally in Subsections 4.1 and 4.2.

We now describe the algorithms used for each case.

In Case 1, the idea behind the gradient descent algorithm is to find the optimal reserves by following the direction of the gradient of the total costs for all the markets. Different variations of the gradient descent algorithm exist but we have chosen the Fletcher and Reeves one [4]. It is robust, converges quite fast and does not require to calculate the Hessian of the utilities.

We let: $\mathbf{U}_v = \begin{pmatrix} U_1 & U_2 \end{pmatrix}^T$ be the vector containing the total costs of markets 1 and 2 and $\mathbf{DU}_v = \begin{pmatrix} \frac{\partial U_1}{\partial r_1} & \frac{\partial U_2}{\partial r_2} \end{pmatrix}^T$ be the vector of the differentiates of the total costs with respect to the market reserves. In practice, we start with initial reserves $r_i^0 = 10$ for all $i = 1, ..., N$ and with initial gradient direction $d_i^1 = -\frac{\partial U_i}{\partial r_i}(r^0)$ for all $i = 1, ..., N$. Then, at each step $k \geq 1$, we define:

- $r^k = r^{k-1} + s_k d^k$ where s_k is chosen such that $U_i(r^k)$ is maximal
- $d^{k+1} = -\mathbf{DU}_v(r^k) + \frac{\|\mathbf{DU}_v(r^k)\|^2}{\|\mathbf{DU}_v(r^{k-1})\|^2} d^k$

Then, the sequence r^k converges towards the optimal reserves. This optimum can be a local optimum. We use a simple criterion to check if the sequence has converged: if $\|r^k - r^{k-1}\| \leq \varepsilon$ we stop the algorithm loop. In our case, we have fixed $\varepsilon = 10^{-4}$.

To choose the optimum s_k at each step k in the previous algorithm, we use a dichotomia algorithm. The objective is to find the point where the differentiate of $\mathbf{U}_v(r^{k-1} + sd^k)$ in s equals 0. We start off by locating a first point where this differentiate is negative. We start off with $h_1 = 1$ and we double the value i.e., $h_k = 2h_{k-1}$, until $\frac{\partial \mathbf{U}_v(r^{k-1}+sd^k)}{\partial s}|_{s=h_k} \leq 0$. Then, we select $x_{\min} = h_{k-1}$ and $x_{\max} = h_k$. We

know that the optimum is located between $x_{\min}$ and $x_{\max}$. Then, we divide this bounding interval by two, at each step, by calculating $\frac{\partial \mathbf{U}_v(r^{k-1}+sd^k)}{\partial s}|_{s=\frac{x_{\min}+x_{\max}}{2}}$ and update the values of $x_{\min}$ and $x_{\max}$ depending on the sign of the result. We stop when the difference between $x_{\min}$ and $x_{\max}$ is inferior to ε.

In Case 2, if market i knows that market j is mistaking regarding its demand and renewable energy procurement parameters: d_i, σ_i or $\hat{w}_i$, all happens as if market j would optimize its reserve first and market i would react. More precisely, i assumes that market j selects its optimal reserve $r_j^\star$ as the solution of Case 1, described above, with its biased beliefs about market i. Then market i reacts by selecting its optimal reserve with the reserve of j fixed to $r_j^\star$ and its demand and offers parameters fixed to their true values. In our case, we select the point where the differentiate of U_i in r_i equals 0, all other values being fixed.

To find this optimum, we use, once more, a dichotomia algorithm. We start off by locating a first point where this differentiate is negative. We start with $h_1 = 1$ and, at each iteration k, we double the value i.e., $h_k = 2h_{k-1}$, until $\frac{\partial U_i(r_i,r_j^\star)}{\partial r_i}|_{r_i=h_k} \leq 0$. Then we select $x_{\min} = h_{k-1}$ and $x_{\max} = h_k$. We know that the optimum is located between $x_{\min}$ and $x_{\max}$. Then, we divide this bounding interval by two, at each step, by calculating $\frac{\partial U_i(r_i,r_j^\star)}{\partial r_i}|_{r_i=\frac{x_{\min}+x_{\max}}{2}}$ and update the values of $x_{\min}$ and $x_{\max}$ depending on the sign of the result. We stop when the difference between $x_{\min}$ and $x_{\max}$ is inferior to ε.

3.4 Numerical illustrations

3.4.1 Analytical expressions of the forecast errors

Expressions of the providers' total costs, of the market welfares and of their differentiates cannot be used in the forms obtained in Subsections 3.1 and 3.2 to perform numerical simulations since they contain terms such as $\mathbb{E}\Big[\epsilon_i|\epsilon_i \geq r_i\Big]$, $\mathbb{E}\Big[(\epsilon_i - r_i)|\epsilon_i \geq r_i\Big]$ and $\mathbb{E}\Big[(\epsilon_i - r_i)^2|\epsilon_i \geq r_i\Big]$. However, since the forecast errors ϵ_i are distributed according to Gaussian distribution functions centered in 0 and of standard deviation σ_i, it is possible to express the above conditional expectations as functions of the incomplete gamma function. We recall the expression of the incomplete gamma function with lower bound: $P(a,x) = \frac{1}{\Gamma(a)} \int_x^{+\infty} t^{a-1} \exp(-t)dt$ where $x \in \mathbb{R}_+$ is the non-negative lower bound of the integral, $a \in \mathbb{R}_+$ and $\Gamma(a)$ is the Gamma function evaluated in a. After a change of variables, the above mentioned conditional expectations can be rewritten:

$$\mathbb{E}\Big[\epsilon_i|\epsilon_i \geq r_i\Big] = \tfrac{\sigma_i}{\sqrt{2\pi}}\Gamma(1)P(1, \tfrac{r_i^2}{2\sigma_i^2})$$

$$\mathbb{E}\Big[(\epsilon_i - r_i)|\epsilon_i \geq r_i\Big] = \tfrac{\sigma_i}{\sqrt{2\pi}}\Gamma(1)P(1, \tfrac{r_i^2}{2\sigma_i^2}) - r_i\bar{F}_{\epsilon_i}(r_i)$$

$$\mathbf{E}\Big[(\epsilon_i - r_i)^2|\epsilon_i \geq r_i\Big] = \tfrac{\sigma_i^2}{\sqrt{\pi}}\Gamma(\tfrac{3}{2})P(\tfrac{3}{2}, \tfrac{r_i^2}{2\sigma_i^2}) - \tfrac{\sigma_i}{\sqrt{2\pi}}\Gamma(1)P(1, \tfrac{r_i^2}{2\sigma_i^2}) + r_i^2$$

By substitution in the providers' total costs, in the market welfares and in their differentiates, we obtain expressions that are computationally tractable.

Numerical simulations are run for $N = 3$, $\alpha = 1$, $a_i^f = \frac{1}{N}, b_i^f = 10^{-2}\ a_i^f, a_i^0 = 10\ a_i^f, b_i^0 = a_i^f, \forall i = 1, ..., N$ and homogeneous demands: $d_i = 100, \forall i = 1, ..., N$.

3.4.2 Impact of the intermittent source number and concentration and of their production variability on the utilities and on the welfares

Assuming that all the $N = 3$ markets have identical number of wind farms and concentration, we capture the impact of these parameters on the optimal reserves, on the sum of the providers' total costs U and on the social welfare W in Figures 1 (a) and (b). We observe in Figure 1 (a) that, at the optimum, the reserve purchased by each market increases as a function of the number of wind farms hold by the market and of their concentration. Going back to the forecast error definition introduced in Subsection 2.3, we check analytically that the larger the number of wind farms γ_i and their concentration θ_i are, the higher the uncertainty associated with wind energy procurement. Therefore, the increase of γ_i and θ_i forces the market to increase its level of reserve at time t_f. Moreover, juding by Figure 1 (a), we observe that reserves are larger under welfare maximization than under total cost minimization, for the same set of parameters.

As already mentioned, the forecast error relies on the wind farm concentration, θ_i. Therefore, depending on θ_i, market i might over-estimate or, on the contrary, under-estimate its renewable energy production. In case of over-estimation, more conventional energy has to be bought at time t_0 causing the total procurement cost to increase. On the contrary, in case of under-estimation, the market realizes at time t_0 that too much energy has been bought at time t_f, which represents a waste of money. In both cases, the total cost U_i (resp. welfare W_i) should be increasing (resp. decreasing) as functions of θ_i. This coincides with the observations in Figure 2 (a). The same variations are observed for the sum of the providers' total costs U (resp. the social welfare W) in Figure 1 (b). The impact of γ_i on U and W is less straightforward. Three trends can be observed in Figure 1 (b):

- If $\theta_i < 0.8$, U (resp. W) is decreasing (resp. increasing) as a function of γ_i
- If $\theta_i > 0.9$, U (resp. W) is increasing (resp. decreasing) as a function of γ_i
- If $0.8 \leq \theta_i \leq 0.9$, U (resp. W) is constant in γ_i

An increase of the number of wind farms and hence, of average renewable procurement, leads to a global decrease of the quantity of conventional energy which has to be bought by each market, on the pool markets. This is a plausible explanation for the welfare increase as a function of γ_i.

To better understand the impact of γ_i on each market, we concentrate on its specific effect on each market in Figure 2 (b), for spatially independent ($\theta = \frac{1}{2}$) and co-located ($\theta = 1$) wind farms. If renewable sources are located over the country so that their productions are independent of one another, we observe that U_i (resp. W_i) is a decreasing (resp. increasing) function of γ_i. An increase of γ_i causes a decrease of conventional energy to be purchased by market i on the pool markets and hence a decrease of the clearing price which, in turn, leads to a decrease of the other markets total cost $U_j, j \neq i$ and an increase of the welfare $W_j, j \neq i$ of the other markets. On the

contrary, if wind farms are nearly co-located, the total cost U_i (resp. the welfare W_i) of market i is an increasing (resp. decreasing) function of the number of wind farms γ_i: the forecast error being so large, due to a large θ, and still increasing in the number of wind farms, it has the dominant effect over the cost reduction caused by the renewable production increase. On the other markets, the increase of $U_j, j \neq i$ and decrease of $W_j, j \neq i$ are consequences of variations of the clearing prices and renewable energy procurement.

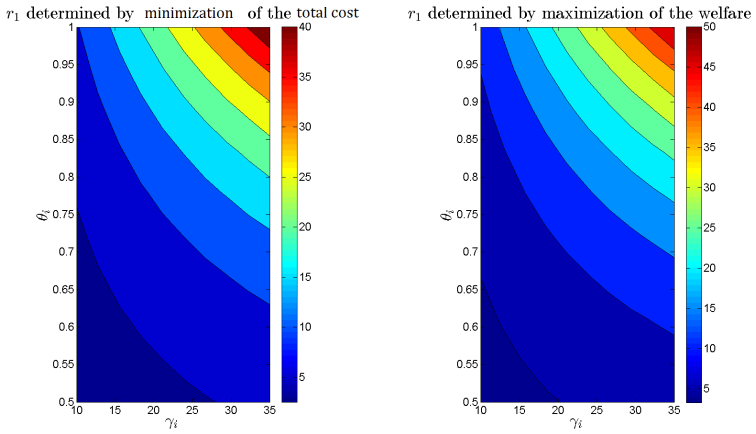
3.4.3 Renewable energy penetration

In this subsection, we consider a single scenario for wind energy procurement over each market using the relation introduced in Subsection 2.3: $w_i = \alpha_i \gamma_i - \epsilon_i$. We generate a single realization for the forecast error ϵ_i according to the Gaussian density function centered in 0 and of standard deviation σ_i. We assume that all the markets have the same parameters i.e., $\tilde{\sigma}_i = 1.1, \gamma_i = 20, \theta_i = 0.65$.

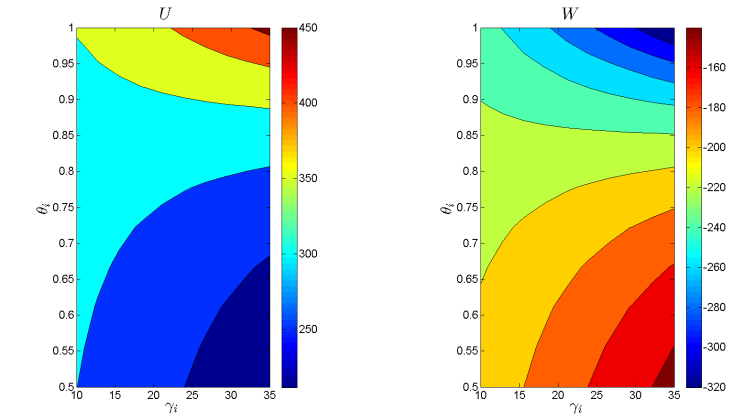
Renewable energy penetration is measured as the ratio between the wind energy production over the sum of conventional supplies at times t_0, t_f and renewable production i.e., $\sum_{i=1,\dots,N} \Big(q_i^0 + q_i^f + w_i \Big)$.

We suppose to be in the case where the amount of energy purchased by the markets at t_0 is positive i.e., $d_i - w_i - q_i^f > 0$ and that the $N = 3$ markets share identical parameters: $\tilde{\sigma}, \gamma, \theta$. The assumption of positivity of q_i^0 implies that the renewable energy penetration ratio can be simplified to give: $\dfrac{\sum_{i=1,\dots,N} (\hat{w}_i - \epsilon_i)}{\sum_{i=1,\dots,N} d_i}$. The expectation of the ratio taken with respect to $(\epsilon_i)_i$ is: $\dfrac{\sum_{i=1,\dots,N} \hat{w}_i}{\sum_{i=1,\dots,N} d_i} = \frac{\alpha\gamma}{d}$ since $\mathbb{E}[\epsilon_i] = 0$ by assumption and its variance: $\dfrac{N\sigma_i^2}{\Big(\sum_{i=1,\dots,N} d_i \Big)^2}$ since the $(\epsilon_i)_i$ are independent from one another by assumption. It is straightforward to infer the ratio standard deviation: $\Big\{ - \dfrac{\sqrt{N}\sigma_i}{\sum_{i=1,\dots,N} d_i} ; + \dfrac{\sqrt{N}\sigma_i}{\sum_{i=1,\dots,N} d_i} \Big\} \Leftrightarrow \Big\{ - \frac{\gamma^\theta \tilde{\sigma}}{\sqrt{N} d} ; \frac{\gamma^\theta \tilde{\sigma}}{\sqrt{N} d} \Big\}$.

We check in Figures 3 (a), (b), (c) that the sampled renewable penetration rate varies around the expected average renewable penetration rate and that the standard deviation values framed it by higher and by inferior values. Furthermore, the number of wind farms makes the expected penetration rate increase. Their concentration and the forecast error standard deviation largely impact the variation of this rate around its expected value.

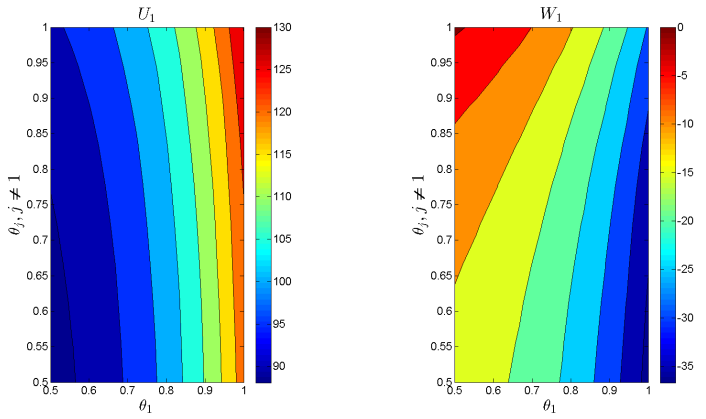


(a)

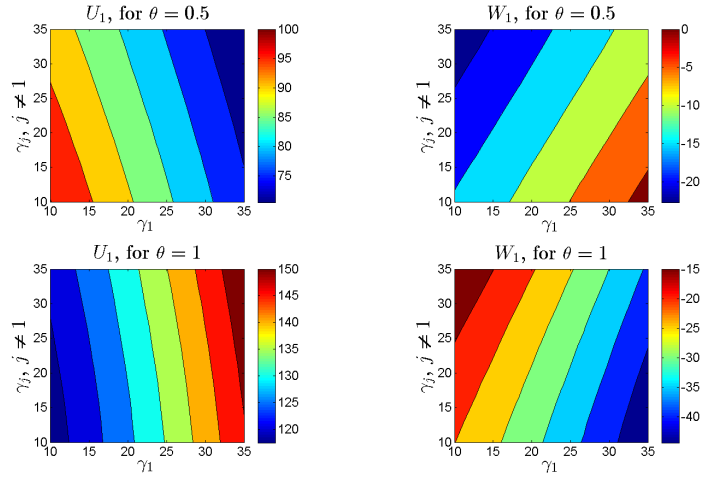


(b)

Figure 1: For these plots, we have assumed that there are $N = 3$ markets sharing the same parameters (γ_i, θ_i) and centrally managed. We have set: $\tilde{\sigma}_i = 1, 1, \forall i = 1, 2, 3$. In (a), we have represented the optimal reserves as functions of the number of wind farms and of their concentration. Whereas in (b), we have captured the number of wind farms and their concentration impact on the social welfare W and on the sum of the providers' utilities U.

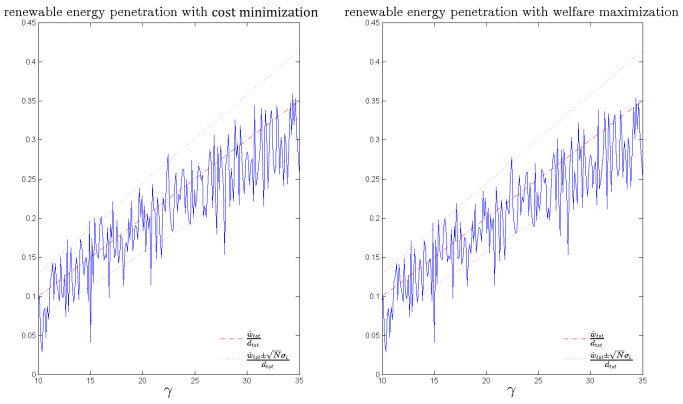


(a)

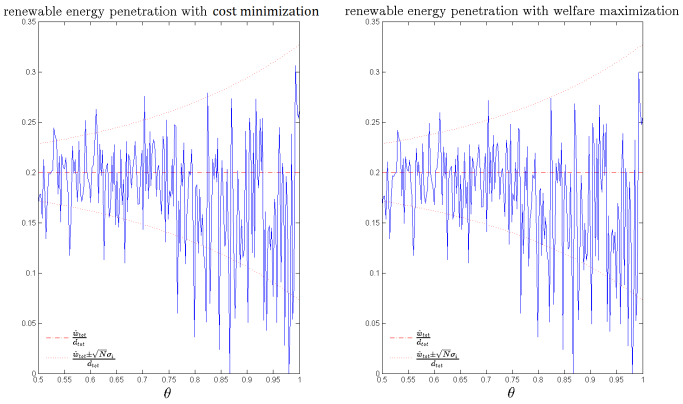


(b)

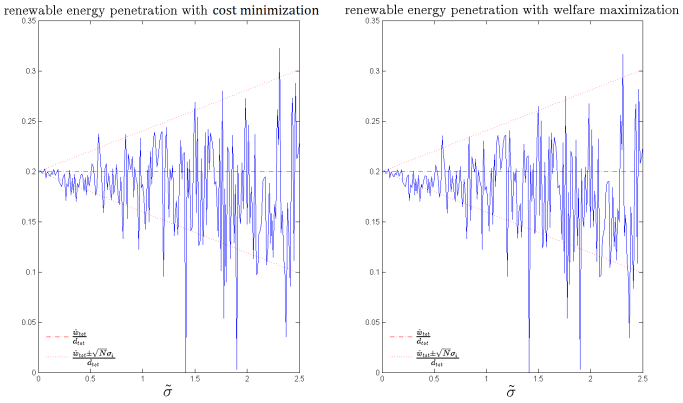
Figure 2: For these plots, we have assumed that there are $N = 3$ markets sharing the same forecast error standard deviation $\tilde{\sigma}_i = 1.1, \forall i = 1, 2, 3$ and centrally managed. Additionnally, the number of wind farms and their concentration are identical on markets 2 and 3 i.e.: $\gamma_2 = \gamma_3, \theta_2 = \theta_3$. We have represented the welfare W_1 and the total cost U_1 of market 1 as functions of the number of wind farms in (a) and of their concentration in (b), on each market.



(a)



(b)



(c)

Figure 3: We have represented the expectation of the renewable penetration rate with an interval of variation corresponding to the associated standard deviation, and a sampled renewable penetration rate. The following conventions are used: $\hat{w}_{tot} \triangleq \sum_i \hat{w}_i$ and $d_{tot} \triangleq \sum_i d_i$.

4 The price of information

In this section, we restrict to the case $N = 2$ markets.

According to Equations (4), to compute their optimal reserves in case of decentralized total cost minimization, the provider on each market needs to know the optimal reserves and the standard deviations of the forecast errors on all the other markets. In Section 3, we have assumed that, on each market, reserves and forecast error standard deviations were public knowledge. However, the providers may have incentives to hide these informations provided the resulting biases, introduced in the other providers' optimal reserves, enable them to decrease their total cost [20].

We introduce a Principal, who can be assimilated with a server, centralizing all the information regarding the producers' forecast error standard deviations. In practice, the providers report the forecast error standard deviations of their producer, to the Principal, who makes them public knowledge. We let $\bar{\sigma}_i \in \mathbb{R}_+$ be the report made by market i provider, to the Principal, regarding his producer's forecast error standard deviation. This report can be biased implying that: $\bar{\sigma}_i \neq \sigma_i$ or unbiased in which case: $\bar{\sigma}_i = \sigma_i$.

The market provider then optimizes his reserve using as input his information regarding the other market producer's forecast error standard deviation. If the other market producer's forecast error standard deviation is private, the provider will need to use the report that the other market provider has made to the Principal.

We distinguish between two cases. First, in Subsection 4.1, market 2 has full access to the information i.e., he knows σ_1, σ_2 whereas market 1 has only access to its own producer's forecast error standard deviation: σ_1. In this first case, the information is asymmetric. The second case, described in Subsection 4.2, coincides with symmetric private information i.e., on each market, the provider has only access to his own producer's forecast error standard deviation and not to the other producers' ones.

In many industries, it has become common practice to sell and buy access to the private information of one's competitors or of targeted consumers. Such information might be collected through surveys, sensors deployed on a communication network, measurements performed in electricity production parks, etc. These investigations are usually performed by a certification operator who certifies the delivered information according to a certain confidence level.

The accuracy of the information depends on the importance of the deployed investigation means: if a survey is used, more accurate information can be gathered provided a larger sample is questionned or provided more relevant questions are added to the survey ; in case of decentralized measurements, more sensors can be deployed over the communication network to measure the load of the links where the traffic flows of a rival provider are routed, etc. Of course, these improvements should increase the information price. Usually, it is the certification operator who sets the price depending on the accuracy of his information. The aim of Subsection 4.3 will be to determine the optimal price of information on the other market producer's forecast error standard deviation as a function of the expected confidence level.

4.1 Information asymmetry

First, we assume that market 1 provider has access to his own producer's standard deviation σ_1 but needs to forecast the other market producer's standard deviation: σ_2 while market 2 has access to the whole information.

Decentralized Reserve Optimization with Information Asymmetry

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Agents: Market 1 and market 2 providers

Information:

- σ_1 known by market 1 and by market 2
- σ_2 known by market 2 solely
- $\bar{\sigma}_2$ market 2's reported standard deviation on its producer's forecast error

(i) Market 1 provider determines $r_1(\sigma_1, \bar{\sigma}_2)$ as solution of Program 3 (resp. 8) in $\sigma_1, \bar{\sigma}_2$ assuming that market 2's optimal reserve is $r_2(\bar{\sigma}_2, \sigma_1)$ obtained solution of Program 3 (resp. 8) in $\bar{\sigma}_2, \sigma_1$.

(ii) Simultaneously and independently, market 2 provider determines $r_2(\sigma_1, \sigma_2)$ as solution of Program 3 (resp. 8) in σ_1, σ_2 assuming that market 2's optimal reserve is $r_1(\sigma_1, \bar{\sigma}_2)$ obtained solution of Program 3 (resp. 8) in $\sigma_1, \bar{\sigma}_2$.

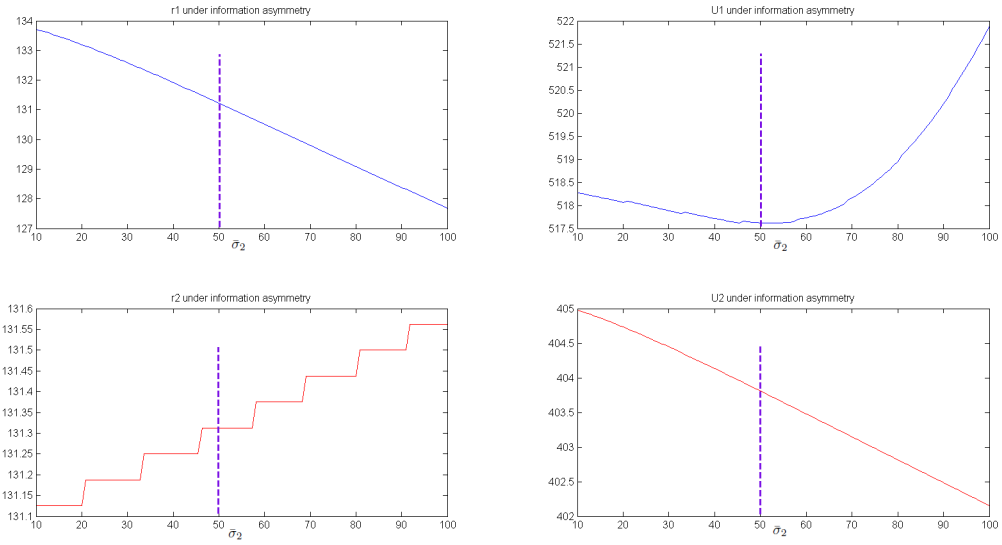

Figure 4: Evolution of the reserve and utilities as functions of market 2 producer's reported standard deviation.

In Figure 4, we have represented the optimal reserves and utilities for both producers. The true forecast error standard deviations are: $\sigma_1 = \sigma_2 = 50$. We observe

that market 1 provider has incentives to know the true standard deviation of market 2 producer since his total cost is minimal in this value. However, market 2 provider has no incentives to report his true standard deviation since his total cost is decreasing in the standard deviation. Therefore, he will most probably exaggerate his forecast error standard deviation when reported to the Principal.

4.2 Private information

Decentralized Reserve Optimization with Private Information

Agents: Market 1 and market 2 providers

Information:

- σ_1 known by market 1 solely
- σ_2 known by market 2 solely
- $\bar{\sigma}_1$ market 1 provider's reported standard deviation on his producer's forecast error
- $\bar{\sigma}_2$ market 2 provider's reported standard deviation on his producer's forecast error

(i) Market 1 provider determines $r_1(\sigma_1, \bar{\sigma}_2)$ as solution of Program 3 (resp. 8) in $\sigma_1, \bar{\sigma}_2$ assuming that market 2's optimal reserve is $r_2(\bar{\sigma}_2, \bar{\sigma}_1)$ obtained solution of Program 3 (resp. 8) in $\bar{\sigma}_2, \bar{\sigma}_1$.

(ii) Simultaneously and independently, market 2 provider determines $r_2(\bar{\sigma}_1, \sigma_2)$ as solution of Program 3 (resp. 8) in $\bar{\sigma}_1, \sigma_2$ assuming that market 1's optimal reserve is $r_1(\bar{\sigma}_1, \bar{\sigma}_2)$ obtained solution of Program 3 (resp. 8) in $\bar{\sigma}_1, \bar{\sigma}_2$.

In Figure 5, we have represented each market producer's total cost, the sum of the market producers' total costs and the difference of the market producers' total costs. The forecast error standard deviations which stand for private information are: $\sigma_1 = \sigma_2 = 90$. We observe that on each market, the provider's total cost is minimum when the producer knows the other market producers' true standard deviation and biases by lower value his own standard deviation when reported to the certification operator. However, if both producers agree to cooperate they minimize the sum of their total costs by revealing their private information. Furthermore, the resulting minimum is lower than the sum of the minimum obtained when each market provider minimizes selfishly his total cost by deciding wich information to reveal.

4.3 The price of information

We take the point of view of market i producer.

To obtain an estimate of market $j, j \neq i$ producer's forecast error standard deviation, the certification operator needs to perform a sequence of n observations: $\epsilon_j(k), k = 1, ..., n$ of market j producer forecast error. We let $\hat{\sigma}_j$ be an estimate of market j producer forecast error standard deviation, obtained through a second order moment estimation. According to Agard [1], we have a probability $\mathcal{C}(\delta) \neq 0$, also called confidence level, that, with n large enough, $\hat{\sigma}_j = \sigma_j$ belongs to the interval $\Big[- \frac{\delta\sigma_j}{\sqrt{n}} \frac{1}{2} \Big\{ \frac{\sqrt{\pi}\Gamma(\frac{5}{2})}{\Gamma^2(\frac{3}{2})} -$

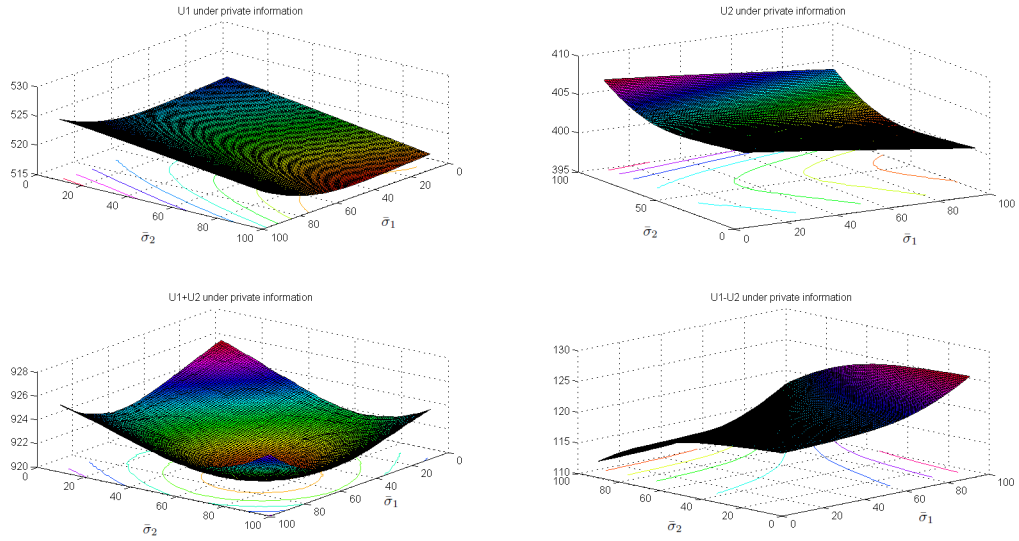

Figure 5: From left to right and top to bottom, we have represented Market 1 provider's total cost, Market 2 provider's total cost, the sum of the market providers' total costs and the difference of their total costs as functions of their reported forecast error standard deviation. The demand on each market is defined as: $d_1 = 100$ and $d_2 = 40$.

$1\Big\}^{\frac{1}{2}}; \frac{\delta\sigma_j}{\sqrt{n}}\frac{1}{2}\Big\{\frac{\sqrt{\pi}\Gamma(\frac{5}{2})}{\Gamma^2(\frac{3}{2})} - 1\Big\}^{\frac{1}{2}}\Big]$. $\mathcal{C}(\delta)$ coincides the probability that the random variable distributed according to a Gaussian density function centered in 0 and of standard deviation 1 belongs to the interval $[-\delta; +\delta]$ [1]. Therefore, it can be rewritten as follows: $\mathcal{C}(\delta) = F_{\mathcal{N}(0;1)}(\delta) - F_{\mathcal{N}(0;1)}(-\delta)$ where $F_{\mathcal{N}(0;1)}$ is the cumulative distribution function associated to the Gaussian distribution centered in 0 and of standard deviation 1.

It is straightforward to observe that the increase of $\mathcal{C}(\delta)$ makes this interval increase but that the accuracy of the forecast decreases which implies in turn that the information price should decrease. Therefore, for a confidence level $\mathcal{C}(\delta)$, we define the information price as: $\Phi(\delta, \bar{\sigma}_j) \geq 0$ and $\phi(\delta, \bar{\sigma}_j) \to 0$ when $\delta \to +\infty$.

Proposition 8. *Market j producer having reported $\bar{\sigma}_j$ as forecast error standard deviation and the confidence level being set to $\mathcal{C}(\delta)$, the price of the information is:* $\Phi(\delta, \bar{\sigma}_j) = \frac{1}{B^0}\frac{\exp(-\frac{r_j^2}{2\tilde{\sigma}_j^2})}{\sqrt{2\pi}}[1 + \frac{r_j^2}{2\sqrt{2\pi}}\frac{1}{\tilde{\sigma}_j^2}]\Big(\max\{\bar{\sigma}_j; \hat{\sigma}_j\} - \min\{\bar{\sigma}_j; \hat{\sigma}_j\}\Big)$ *where* $\tilde{\sigma}_j \in [\min\{\bar{\sigma}_j; \hat{\sigma}_j\}; \max\{\bar{\sigma}_j; \hat{\sigma}_j\}]$ *is such that:* $\frac{1}{B^0}\frac{\exp(-\frac{r_j^2}{2\tilde{\sigma}_j^2})}{\sqrt{2\pi}}[1 + \frac{r_j^2}{2\sqrt{2\pi}}\frac{1}{\tilde{\sigma}_j^2}] = \frac{U_i(\hat{\sigma}_j) - U_i(\bar{\sigma}_j)}{\hat{\sigma}_j - \bar{\sigma}_j}$ *and* $\hat{\sigma}_j = \frac{\delta\sigma_j}{\sqrt{n}}\frac{1}{2}\Big\{\frac{\sqrt{\pi}\Gamma(\frac{5}{2})}{\Gamma^2(\frac{3}{2})} - 1\Big\}^{\frac{1}{2}}$.

Proof of Proposition 8. We introduce the opportunity cost for market i producer

to buy information. It is defined as the difference between the benefit resulting from the knowledge of the private standard deviation of the other market producer and the expected price of information depending on the required confidence level $\mathcal{C}(\delta)$. Therefore, the opportunity cost for market i producer to buy information is:

$\underbrace{\Big\{U_i(\bar{\sigma}_j) - U_i(\hat{\sigma}_j)\Big\}}_{\text{Benefit resulting from the knowledge of } \sigma_j} - \Phi(\delta, \bar{\sigma}_j)$. We observe that the producer will buy information provided his opportunity cost remains non-negative and that the certification operator will propose the highest admissible price. Hence, we infer the analytic expression for the optimal expected information price: $\Phi(\delta, \bar{\sigma}_j) = \Big\{U_i(\bar{\sigma}_j) - U_i(\hat{\sigma}_j)\Big\}$. Using the Finite Increment Theorem, we infer that there exists $\tilde{\sigma}_j \in [\min\{\bar{\sigma}_j; \hat{\sigma}_j\}; \max\{\bar{\sigma}_j; \hat{\sigma}_j\}]$ such that $\frac{\partial U_i(\sigma_j)}{\partial \sigma_j}|_{\sigma_j = \tilde{\sigma}_j} = \frac{U_i(\hat{\sigma}_j) - U_i(\bar{\sigma}_j)}{\hat{\sigma}_j - \bar{\sigma}_j}$. But, using the definition of the incomplete gamma function, U_i can be rewritten: $U_i = \Big(d_i - \hat{w}_i + r_i\Big) \frac{\sum_j (d_j - \hat{w}_j + r_j) + A^f}{B^f} +$ $\frac{1}{B^0} \sum_{j \neq i} \Big[\frac{\sigma_j}{\sqrt{2\pi}} \exp(-\frac{r_j^2}{2\sigma_j^2}) - r_j \bar{F}_{\epsilon_j}(r_j)\Big] \Big[\frac{\sigma_i}{\sqrt{2\pi}} \exp(-\frac{r_i^2}{2\sigma_i^2}) - r_i \bar{F}_{\epsilon_i}(r_i)\Big] + \frac{A^0}{B^0} \Big[\frac{\sigma_i}{\sqrt{2\pi}}$ $\exp(-\frac{r_i^2}{2\sigma_i^2}) - r_i \bar{F}_{\epsilon_i}(r_i)\Big] + \frac{1}{B^0} \Big[\frac{\sigma_i^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) P(\frac{3}{2}, \frac{r_i^2}{2\sigma_i^2}) - 2 r_i \frac{\sigma_i}{\sqrt{2\pi}} \Gamma(1) P(1, \frac{r_i^2}{2\sigma_i^2}) + r_i^2\Big]$.

Assuming that the information price is paid a posteriori i.e., after the estimate $\hat{\sigma}_j$ has been delivered to market i provider, we obtain: $\frac{\partial U_i}{\partial \sigma_j} = \frac{1}{B^0} \frac{\exp(-\frac{r_j^2}{2\sigma_j^2})}{\sqrt{2\pi}} [1 + \frac{r_j^2}{\sqrt{2\pi}} \frac{1}{\sigma_j^2}]$ which differentiate with respect to σ_j is always increasing. Therefore, the linear coefficient of the information price is increasing in $\tilde{\sigma}_j$. □

5 Conclusion

In this article, we aim at studying the impact of renewable energy on $N \in \mathbb{N}^*$ coupled electricity markets. More precisely, we study how the scaling of the relative concentration of the intermittent sources being aggregated, impacts the coupled markets optimal reserves, optimal conventional energy productions, prices on day ahead and spot markets. In a context of incomplete information on the values of the reserves and renewable energy productions, we show, using numerical simulations, that markets have incentives to buy information regarding the other markets' variability of renewable production. The optimal price at which this information should be certified is derived analytically. It depends on the associated confidence level.

In extensions of our article, it might be interesting to take into account the available transmission capacity constraints and also, not only the concentration of the wind farms but also their spatial locations using appropriate databases. However, this would limit the analytical derivations and require the extensive use of simulations. Another point that needs to be mentioned concerns the wind energy procurement which relies heavily on the density which generates the forecast error. Alternative approaches, based on stochastic programming, were wind energy procurement could be associated with

a random variable generated according to an unknown density function might be envisaged [12], [23]. Monte Carlo sampling could be of interest to provide an accurate estimation of the stochastic programming problem. Futhermore, the opportunity to associate accurate probabilities to the scenario occurence would enable the improvement of the algorithm performance [23] ; potentially using online observations. Finally, the existence of polynomial time algorithms providing robust solutions to uncertain linear programming problems has been proven under some assumptions about the uncertainty set geometry [3] ; extensions to our problem would be relevant for practical applications.

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