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Iterative linear cuts strengthening the second-order cone relaxation of the distribution system optimal power flow problem

Seddik Yassine Abdelouadoud

Robin Girard

François-Pascal Neirac

Center for Energy and Processes

Mines Paristech

Sophia-Antipolis, France

seddik.abdelouadoud@mines-paristech.fr

Thierry Guiot

Renewable Energy Center

CSTB

Sophia-Antipolis, France

Abstract—We present a novel iterative algorithm to solve the distribution system optimal power flow problem over a radial network. Our methodology makes use of a widely studied second order cone relaxation applied to the branch flow model of a radial network. Several types of conditions have been established under which this relaxation is exact and we focus here on the situations where this is not the case. To overcome this difficulty, we propose to add increasingly tight linear cuts to the second-order cone problem until a physically meaningful solution is obtained. We apply this technique to a sample system taken from the literature and compare the results with a traditional nonlinear solver.

Index Terms-- Distributed power generation, power system planning, smart grids

I. INTRODUCTION

With the aim to increase the sustainability of the electric power system, the share of renewable energies in the production mix is scheduled to increase in the future. For example, the European Union has set goals for its member states in order to attain a 20% share of renewable energy in its final energy consumption by 2020, and some countries have taken even more ambitious stances. This target will be partially met by integrating significant amounts of dispersed renewable energy generators (mainly photovoltaic (PV) and wind power) to the distribution grid. These developments will have a considerable impact on the design and operation of the electric system, both at the national and local level and so new tools will be needed to assist in the planning and operation of at least the distribution network. Indeed, as the current passive distribution network turns into an Active Distribution Network (ADN) [1] with the introduction of partially and totally controllable generation and storage means, planning studies based solely on power flows for extreme load conditions will not be adapted anymore. Considering the similarities between the current transmission network and the future ADN, it is a

safe bet to assume that the Optimal Power Flow, a tool first introduced in 1962 by Carpentier [2] and now widely used for the planning and operation of the transmission network, will prove useful for this purpose. Consequently, the adaptation of the OPF concept and resolution algorithms to the distribution network has been the subject of numerous publications in the last decade such as [3], [4], [5], [6]. Parallel to these efforts, a promising path has been taken by, for example, [7] and [8], to show that the OPF problem can be cast as a Quadratically Constrained Quadratic Program (QCQP). The QCQP class of problem has already been widely studied, as many instances of engineering problems, such as principal component analysis and the combinatorial max-cut problem, are included in it. General QCQP are non convex problems and thus cannot be solved in polynomial time. To deal with this, the standard practice in the OPF literature is to relax the problem to a convex conic program that can either be cast as a Second-Order Cone Program (SOCP) or Semi-Definite Program (SDP). On the application of such techniques to radial networks, we can cite the work of [9] and [10], who prove that the SDP relaxation over tree networks is exact under certain condition, or the work of [11]-[13], who took the SOCP route applied to the branch flow model and proved that the relaxation is exact if there are no upper bounds on loads or voltage magnitude and certain conditions on the objective function are respected. In these instances, it can therefore be shown that the OPF over radial network can be solved in polynomial time. However, as has been shown in [14], such approaches may fail to produce physically meaningful solutions in realistic conditions. Such conditions are the focus of our work here, as we are proposing a methodology based on adding increasingly tight linear cuts to the initial SOCP relaxation based on the branch flow model until a satisfactory solution is obtained.

The paper is organized as follows: first we introduce the problem formulation, then we present the methodology and we finish by applying it to a case study.

II. PROBLEM FORMULATION

A. Objective function

In this paper, we are particularly interested in situations described in detail in our previous work [14] and [15] where distributed generation and storage are integrated at the distribution system level, not only to alleviate network constraints through active and reactive power control, but also to provide services at the national level, such as energy and ancillary services provision or energy time-shift in the case of storage. As the time-coupling constraints can create extremely large problems, we have proposed to separate them into master problems dealing with the multistage scheduling of active powers and single-stage slave problems ensuring the respect of network constraints, while minimizing the deviation from the master problem. We focus here on the solution of the slave problem where, therefore, the objective function is of the following form:

$$\min \sum_{k \in K} (P_{c,k}^{MP,t} - P_{c,k}^t)^2 \quad (1)$$

, where $P_{c,k}^{MP,t}$ is the set of controllable active powers resulting from the master problem solution and are the corresponding control variable in the slave problem.

B. Network and apparent power constraints

As discussed earlier, we adopt here the branch flow model first introduced in [17] and [18]. Henceforth, we assume that the storage devices and distributed energy units are connected to the grid through an Advanced Power Electronic Interface (APEI), as defined in [19], and that the substation transformer is equipped with an On-Load Tap Changer (OLTC), for which we use a simplified continuous model. Consequently, the network and apparent power equations are cast in the following form for all nodes j :

$$P_j^t - \sum_{kk \in KK_j, kl \in KL_j} R_{kl} \times \frac{P_{kk}^{t^2} + Q_{kk}^{t^2}}{U_{kk}^t} - \sum_{kk \in KK_j} P_{kk}^t = P_{NetLoad,j}^t \quad (2)$$

$$Q_j^t - \sum_{kk \in KK_j, kl \in KL_j} X_{kl} \times \frac{P_{kk}^{t^2} + Q_{kk}^{t^2}}{U_{kk}^t} - \sum_{kk \in KK_j} Q_{kk}^t = Q_{NetLoad,j}^t \quad (3)$$

$$U_j^{t^2} + 2 \left(P_j^t \times R_j + Q_j^t \times X_j - \frac{U_{f(j)}^t}{2} \right) \times U_j^t + Z_j^2 \times S_j^{t^2} = 0 \quad (4)$$

$$U_j^t = V_j^{t^2}, \quad Z_l^2 = R_l^2 + X_l^2, \quad S_j^{t^2} = P_j^{t^2} + Q_j^{t^2} \quad (5)$$

$$P_{NetLoad,j}^t = P_{Load,j}^t - P_{c,j}^t \quad (6)$$

$$Q_{NetLoad,j}^t = Q_{Load,j}^t - Q_{c,j}^t \quad (7)$$

$$V^{\min} \leq V_j^t \leq V^{\max}, \quad V_{OLTC}^{\min} \leq V_{OLTC}^t \leq V_{OLTC}^{\max} \quad (8)$$

$$P_{c,j}^{t^2} + Q_{c,j}^{t^2} \leq S_{c,j}^{\max^2}, \quad (9)$$

, where P_j^t and Q_j^t are, respectively, the active and reactive power entering node j from the upstream node $f(j)$, V_j^t and V_{OLTC}^t the voltage magnitude, respectively, at node j and downstream of the substation transformer, during time step t . $Q_{Load,j}^t$ is the reactive load, $Q_{c,j}^t$ are the algebraic reactive powers injected by the APEIs, of which $S_{c,j}^{\max}$ is the maximal apparent powers. V^{\min} and V^{\max} define the range of permissible voltages in the network considered, while V_{OLTC}^{\min} and V_{OLTC}^{\max} mark the boundaries of the attainable voltage downstream of the substation. Eq. (1)-(9) define the OPF problem that we propose to study.

C. SOCP relaxation

As explained earlier, we inspire ourselves from the work of, for example, [11] to implement the SOCP relaxation of the OPF problem. First, we define an intermediary variable I_j^t for all nodes j :

$$I_j^t = \frac{P_j^{t^2} + Q_j^{t^2}}{U_j^t} \quad (9)$$

We then replace $\frac{P_j^{t^2} + Q_j^{t^2}}{U_j^t}$ by I_j^t in Eq. (2) and (3). In the

resulting formulation, the non convexity of the problem stems only from the equality in Eq. (9) which we relax by keeping only the following inequality:

$$I_j^t \geq \frac{P_j^{t^2} + Q_j^{t^2}}{U_j^t} \quad (10)$$

We consequently obtain a new problem consisting of Eq. (10) and (1) through (8) that we will subsequently call R-OPF.

III. METHODOLOGY

A. Guiding principle

Our methodology is based on the fact that, in order to obtain a physically meaningful solution, the equality in Eq. (9) needs to be verified. Therefore, we add linear cuts to R-OPF that take the following form for all nodes:

$$I_j^t \leq a_j \times P_j^t + b_j \times Q_j^t + c_j \times U_j^t + d_j \quad (11)$$

These cuts are only valid only on a certain domain, which can be apprehended in the following way:

$$\frac{P_j^{t^2} + Q_j^{t^2}}{U_j^t} \leq a_j \times P_j^t + b_j \times Q_j^t + c_j \times U_j^t + d_j \quad (12)$$

Dropping either the active or reactive power variable to fit in a three-dimensional representation, such cuts and their domain of validity can be graphically represented in the following way :

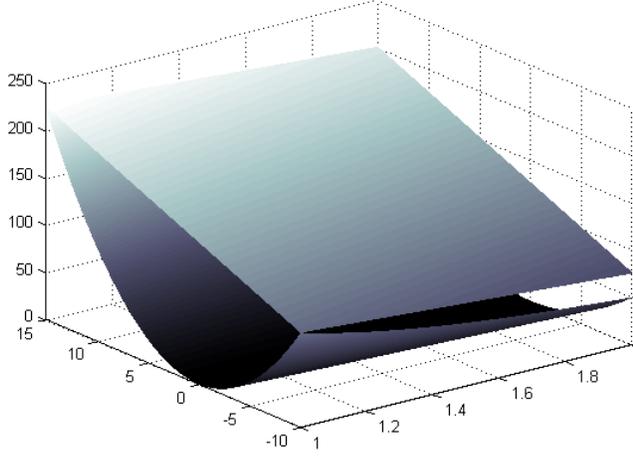


Figure 1 : Graphic concepts of the linear cuts, with arbitrary units

Our methodology then basically consists in progressively restraining the span of the validity domain around the solution of the previous iteration until a stopping criterion of the following form is reached:

$$\frac{I_j^t \times U_j^t - (P_j^{t^2} + Q_j^{t^2})}{I_j^t \times U_j^t} \leq \varepsilon \quad (13)$$

B. Calculation of the linear cut defining parameters

If we consider that the domain of validity is of the form $U_j^t \in [U_j^{\min}, U_j^{\max}]$, $P_j^t \in [P_j^{\min}, P_j^{\max}]$, $Q_j^t \in [Q_j^{\min}, Q_j^{\max}]$ we calculate the defining parameters of the linear cuts so that the barycenter of the cutting plane is the lowest possible. This is obtained by solving the following linear problem in the variables a_j, b_j, c_j and d_j :

$$\min a_j \left(\frac{P_j^{\min} + P_j^{\max}}{2} \right) + b_j \left(\frac{Q_j^{\min} + Q_j^{\max}}{2} \right) + c_j \left(\frac{U_j^{\min} + U_j^{\max}}{2} \right) + d_j \quad (14)$$

To which we add constraints of the same form than Eq. (12) for each extremity of the validity domain.

C. Tightening of the validity domain

To identify the solution obtain at the k-th iteration, we add a second subscript to each variable in the following manner

$U_{j,k}^t$. Then we define the domain of validity by the equations hereafter :

$$U_{j,k+1}^{\min} = U_{j,k}^t \times (1 - \alpha), \quad U_{j,k+1}^{\max} = U_{j,k}^t \times (1 + \alpha) \quad (15)$$

$$P_{j,k+1}^{\min} = P_{j,k}^t \times (1 - \alpha), \quad P_{j,k+1}^{\max} = P_{j,k}^t \times (1 + \alpha) \quad (16)$$

$$Q_{j,k+1}^{\min} = Q_{j,k}^t \times (1 - \alpha), \quad Q_{j,k+1}^{\max} = Q_{j,k}^t \times (1 + \alpha) \quad (17)$$

$$\alpha = \exp\left(\frac{-k}{\beta}\right) \quad (18)$$

Where β is a parameter that governs the speed of the tightening of the validity domain.

IV. CASE STUDY

A. Definition of the parameters

We apply this methodology to a case study consisting in the 69-bus network described in [17], with residential-type loads connected at each node and scaled so that the annual maximal load is equal to the loads used in [17]. Reactive loads are defined, so that the power factor remains constant and equal to the one used in [17]. Storage units are deployed at 13 nodes in increasing order of driving-point impedance magnitude. The maximal apparent power of the storage units is chosen to be equal to the maximal annual active load of the node where it is connected. We select the most critical time steps in the sense that we defined in [16] to ensure that our algorithm is suitable even in extreme loading or storage power injection conditions. The results are obtained through the use of the commercial solver IBM ILOG CPLEX.

B. Convergence

For this example, we set ε to 10e-5 to enforce a high precision of the result and we present hereafter the rate of convergence for a β parameter varying between 6 and 12. Lower values of this parameter led to infeasible problems in this example.

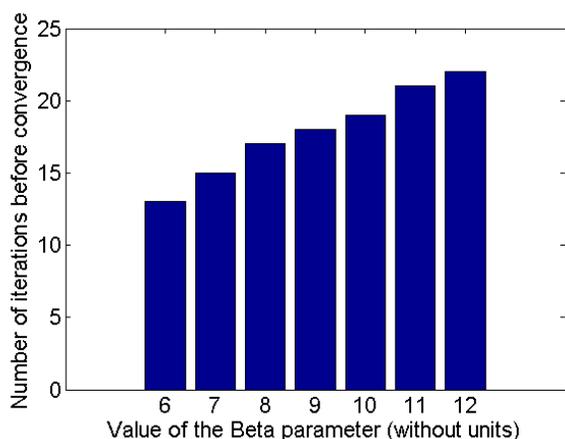
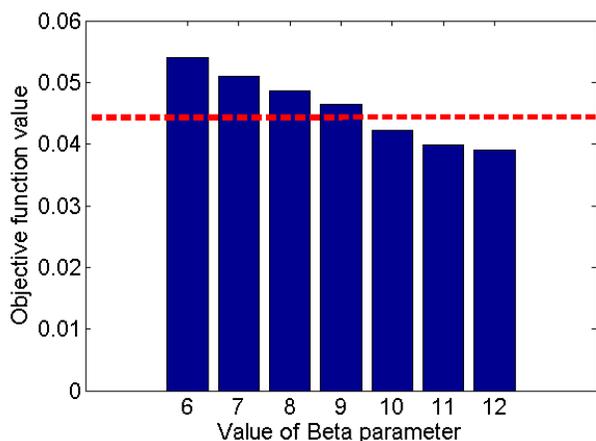


Figure 2 : Rate of convergence as a function of Beta

We then compare the final value of the objective function with the value obtained from the nonlinear solver IPOPT [19].



We can thus observe that the higher the value of β , the better the result will be. Moreover, we show that this methodology is suitable to obtain comparable or even better results than off-the-shelf nonlinear solvers, in this particular case study.

I. CONCLUSION

We have presented a novel iterative algorithm to solve the radial distribution system optimal power flow that relies heavily on recent breakthroughs in this field, especially those concerning the use of a second-order cone algorithm. We have then detailed the methodology employed and the parameters that influence it. We finished by applying to a specific case study, which showed that this approach is promising, especially in terms of rate of convergence and optimality. Further work has to be done that will concern testing the methodology on broader sets of network, understanding in a more systematic ways the relationships between the parameters governing the rate of convergence and the optimality of the resulting solution and linking it to

the previous work done the establishment of a criticality criterion by time steps.

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