



**HAL**  
open science

# A criticality criterion to decrease the computational burden in multistage distribution system optimal power flow

Abdelouadoud Seddik Yassine, Robin Girard, François-Pascal Neirac, Guiot Thierry

## ► To cite this version:

Abdelouadoud Seddik Yassine, Robin Girard, François-Pascal Neirac, Guiot Thierry. A criticality criterion to decrease the computational burden in multistage distribution system optimal power flow. PowerTech 2013, Jun 2013, Grenoble, France. pp.1-6 - Article number 13882321, 10.1109/PTC.2013.6652282 . hal-00932778

**HAL Id: hal-00932778**

**<https://minesparis-psl.hal.science/hal-00932778>**

Submitted on 26 May 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A Criticality Criterion to decrease the Computational Burden in Multistage Distribution System Optimal Power Flow

Abdelouadoud Seddik Yassine  
Robin Girard  
François-Pascal Neirac  
Center For Energy and Process  
Mines Paristech  
Sophia-Antipolis, France  
seddik.abdelouadoud@mines-paristech.fr

Guiot Thierry  
Renewable Energy Center  
CSTB  
Sophia-Antipolis, France  
thierry.guiot@cstb.fr

**Abstract**—As the penetration of distributed generation and storage means in the distribution system is expected to increase, new tools for its planning and operation will be needed and optimal power flow calculations will certainly play a prominent role. However, obstacles have to be overcome before these can be deployed, among which their computational burden is of particular concern. Consequently, we introduce here the use of a criticality criterion aimed at detecting for which time steps the voltage constraints need to be evaluated. We apply the methodology to a distribution system extracted from the literature and discuss the influence of various parameters on the validity of the methodology and the computational gains expected.

**Index Terms**— Distributed power generation, power system planning, smart grids

## I. CONTEXT

As the interest for distributed generation and storage grows, so does the need for new tools to assist in the planning of the distribution network. Indeed, as the current passive network transforms into an Active Distribution Network (ADN) [1] with the introduction of partially and totally controllable generation and storage means, planning studies based solely on power flows for extreme load conditions will not be adapted anymore. Considering the similarities between the current transmission network and the future ADN, it is a safe bet to assume that the Optimal Power Flow, a tool first introduced in 1962 by Carpentier [2] and now widely used for the planning and operation of the transmission network, will prove useful for this purpose. Consequently, the adaptation of the OPF concept and resolution algorithms to the distribution network has been the subject of numerous publications in the last decade such as [3], [4], [5], [6]. Among the structural differences between transmission and distribution network, the radial topology and higher R/X ratio of line impedances are

often identified as the main obstacles to the direct use of transmission network OPF algorithms to distribution systems as explained in [7]. With this in mind, an OPF for a distribution system will still be a highly dimensional nonlinear nonconvex programming problem. Thus, in a planning study for which OPFs have to be solved for each hour of the year and multiple planning options evaluated, the computational burden can become dissuasive. And so, heuristics that allow to decrease the computational burden while controlling the loss of optimality can have practical applications, as demonstrated in [8]. In the work presented here, we make use of the characteristics of the problem studied to propose a methodology aiming at evaluating the voltage constraints only when they are binding, thus drastically reducing the computational cost of the resolution.

## II. METHODOLOGY

### A. Problem studied

First, let us define the general form of the OPF problem for which the methodology presented here applies. It consists in finding, for each time steps,  $P$ ,  $Q$ ,  $V$  and  $\alpha$  that verify:

$$\arg \min_P f(P) \quad (1)$$

$$DF(V, P, Q, \alpha) = 0 \quad (2)$$

$$h(V, \alpha) \geq 0 \quad (3)$$

$$k(P, Q) = 0 \quad (4)$$

$$m(P, Q) \geq 0 \quad (5)$$

where  $P$  and  $Q$  are the vectors of, respectively, controllable active and reactive power algebraic injections,  $V$  is the vector of node voltage magnitude,  $\alpha$  is the tap setting of the

substation transformer on-load tap changer (OLTC),  $f$  represents the objective function,  $DF$  the set of DistFlow network equations used in, for example, [9],  $h$  the inequality voltage constraints and  $k$  and  $m$  represent the other equality and inequality constraints implemented in the problem (e.g. limit on apparent powers, reserve provision requirements or limits on storage means energy capacity). We emphasize here the fact that the methodology proposed is applicable only if the reactive power injections have no influence on the objective function.

### B. Problem simplification

In many instances, the problem complexity can be greatly reduced by ignoring (2) and (3). Indeed, for example, if the objective function consists in the sum of generation piecewise linear costs and (5) represents bounds on active and reactive power, the problem defined by (1), (4) and (5) is a simple linear programming problem. Moreover, we aim at taking advantage of the fact that voltage limit violation occurrence are infrequent phenomena, as evidenced by [10]. This finding supports the assumption that (2) and (3) will be binding constraints only during a relatively small number of time steps. To support this claim, we present the results of a simulation undertaken on a 69-bus medium-voltage distribution system (see [11] for the detailed characteristics of the network). Residential-type loads are simulated for each bus and scaled so that the maximal annual active load is equal to the one used in [11] and the power factor remains constant.

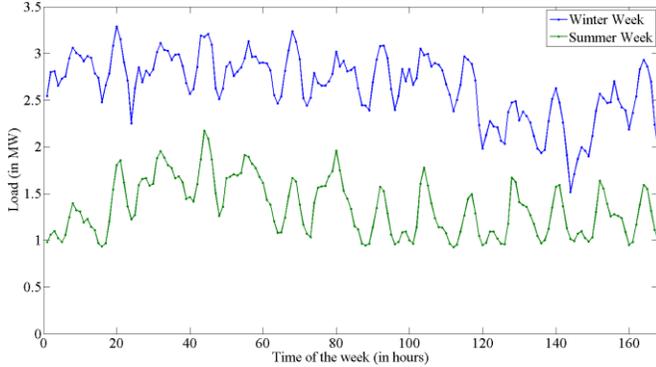


Figure 1 : Total active power load for a winter and a summer week

Then, 17 buses are equipped with photovoltaic and energy storage systems in increasing order of driving-point impedance magnitude, with varying penetration levels (the penetration level at a given bus is defined by the ratio between the maximum power of the storage and photovoltaic systems and the annual maximal load at this bus). The storage systems have a 5 hour discharge time and their charge and discharge efficiency are both equal to 0.95. The objective function is defined as a measure of market transaction cost. In the graph below, we observe the number of time steps where the voltage constraints are active or binding, as a function of the penetration level.

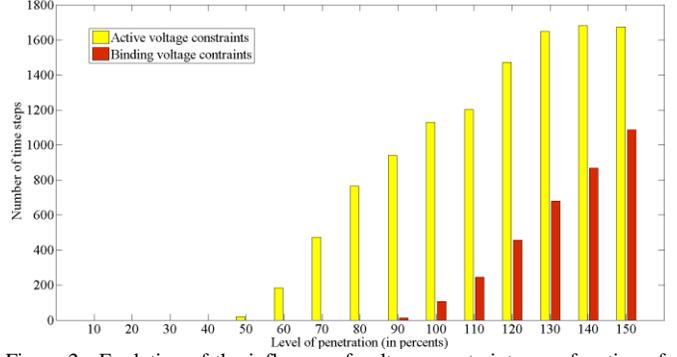


Figure 2 : Evolution of the influence of voltage constraints as a function of penetration level

Consequently, we propose to separate the initial problem into a master problem composed of (1), (4) and (5) that is easier to solve and a slave problem consisting in finding  $P, Q, V, \theta$  and  $\alpha$  that verify:

$$\arg \min_P \sum (P_{master} - P)^2, \quad (7)$$

while satisfying the constraints (2), (3), (4) and (5). The slave problem is still a highly dimensional nonlinear nonconvex programming problem. We will now introduce a procedure that will allow us to solve it only when it is necessary, i.e. when its result has an impact on the objective function of the initial problem.

### C. Criticality Criterion

#### 1) Guiding principle

For the remainder of this paper, we will adopt the following convention: a time step will be deemed critical if (3) is a binding constraint, semi-critical if it is active but not binding, and noncritical if it is inactive. On a more practical level, the noncritical time steps are those for which the active powers resulting from the master problem directly verify (3), the semi-critical are those for which the reactive power and/or tap settings but not the active powers have to be adjusted and the critical time steps are those for which the active powers have to be modified. In order to solve the slave problem only when necessary, we need a reliable and computationally effective method to separate the critical time steps from the others. To achieve this, we calculate a relative criticality criterion aimed at ranking the time steps so that the critical time steps are first, followed by the semi-critical and non-critical ones. First, let us set aside the non-critical time steps as these can be identified with a mere power flow to concentrate on separating the semi-critical from the critical ones. Essentially, this means finding a measure of the ability of the reactive powers and OLTC to correct the voltage violations incurred from the behavior of the loads, distributed generation and storage systems resulting from the master problem solution.

#### 2) Voltage violation and OLTC influence

In order to take into account the influence of the on-load tap changer, we define  $A$  as the set of permissible tap-settings and  $V_{\alpha}^{OLTC,t}$  the substation downstream voltage at time step  $t$

corresponding to the tap position  $\alpha$ . The first step of the criticality criterion calculation is to obtain a power flow for a chosen initial tap setting  $\alpha_0$  and then expand the result to all possible tap settings for each bus  $k$  by making the following approximation:

$$V_\alpha^{k,t} \approx V_{\alpha_0}^{k,t} + V_\alpha^{OLTC,t} - V_{\alpha_0}^{OLTC,t} \quad (8)$$

We then obtain an estimation of the voltage limit violation during time step  $t$  at bus  $k$  for the tap setting  $\alpha$  in the following way:

$$\Delta V_\alpha^{k,t} = \begin{cases} V_\alpha^{k,t} - V^{\max}, & \text{if } V_\alpha^{k,t} > V^{\max} \\ 0, & \text{if } V^{\min} \leq V_\alpha^{k,t} \leq V^{\max} \\ V^{\min} - V_\alpha^{k,t}, & \text{if } V_\alpha^{k,t} < V^{\min} \end{cases} \quad (9)$$

This gives us an estimation of the absolute voltage violation to be compensated at each bus and for each permissible tap-setting.

### 3) Reactive power voltage violation compensation capability

We base our reasoning on the concept of Voltage Change Potential introduced in [12] and aimed at evaluating the local voltage variations caused by the connection of a given distributed energy resource. It is based on a linearization of the network equations, under the assumption that the actual voltage change is small compared to the pre-existing voltage, and is defined as follows:

$$\Delta V = \frac{X \times Q + R \times P}{V}, \quad (10)$$

where  $X$  is the driving-point reactance,  $R$  the driving-point resistance,  $Q$  the reactive power injected at the considered bus,  $P$  the active power and  $V$  the pre-existing voltage.

For our purpose, we define the Reactive Voltage Change Potential during time step  $t$  at bus  $k$  as follows:

$$\Delta V_Q^{k,t} = \frac{X^k \times Q^{k,t}}{V^{k,t}} \quad (11)$$

$\Delta V_Q^{k,t}$  is effectively a measure of the ability of a given reactive power source to modify the voltage at its bus.

### 4) Formal definition of the criterion

We now have at our disposal a measure of the voltage violations taking into account the role of the OLTC and an estimation of the reactive power local compensation capability. We can combine them in order to define a local compensated voltage violation in the following way:

$$\Delta V_{\alpha,Q}^{k,t} = \Delta V_\alpha^{k,t} - \Delta V_Q^{k,t} \quad (12)$$

As we need a single measurement by time step in order to establish our criticality criterion, we define it as the minimum

of the L1-norm of the compensated voltage violation, while respecting constraints on reactive power availability and permissible tap-settings. This can be formally expressed as:

$$C^t = \min_{Q,\alpha} \sum_{k \in K} |\Delta V_{\alpha,Q}^{k,t}| \quad (13)$$

$$0 \leq Q^{k,t} \leq Q_{\max}^{k,t} \quad (14)$$

$$\alpha \in A \quad (15)$$

Where  $K$  is the set of network buses and  $Q_{\max}^{k,t}$  is the available reactive power at bus  $k$  during time step  $t$ .

### 5) Calculation

As  $\alpha$  is a discrete variable, we can define, for each  $\alpha \in A$ :

$$C_\alpha^t = \min_Q \sum_{k \in K} |\Delta V_{\alpha,Q}^{k,t}|, \quad (16)$$

while respecting (14). We can observe that, for a fixed  $\alpha$ , the terms in the sum are independent from one another. Thus, we also have :

$$C_\alpha^t = \sum_{k \in K} \min_Q |\Delta V_{\alpha,Q}^{k,t}| \quad (17)$$

where :

$$\min_Q |\Delta V_{\alpha,Q}^{k,t}| = \begin{cases} \Delta V_{\alpha,Q_{\max}}^{k,t} & \text{if } \Delta V_{\alpha,Q_{\max}}^{k,t} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

We can thus easily calculate  $C_\alpha^t$  for each  $\alpha \in A$  and then obtain  $C^t$  by choosing the minimum value of  $C_\alpha^t$ .

## III. APPLICATION

### A. Case Study

#### 1) Criticality criterion calculation

We apply this methodology to the case study introduced in paragraph B. The master problem is solved using IBM ILOG CPLEX and the slave problem is solved using IPOPT [13], both through their Matlab interface. Below are two graphs presenting the ranking obtained with the criticality criterion for the various types of time steps. In the first simulation considered, 17 buses have been equipped with photovoltaic generators and storage systems, and the level of penetration has been set at a 100%, while in the second, 22 buses have been equipped with a level of penetration of 110%.

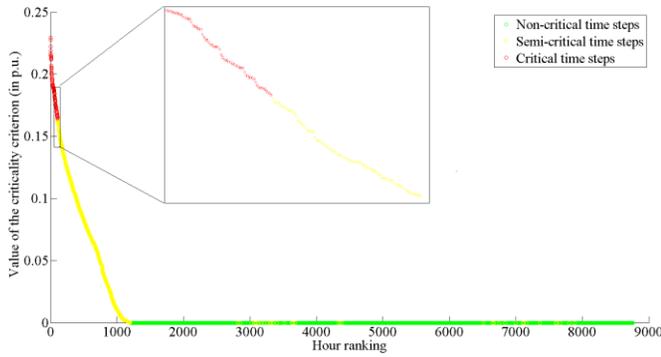


Figure 3 : Criticality criterion ranking, with 17 nodes equipped and a level of penetration of 100%

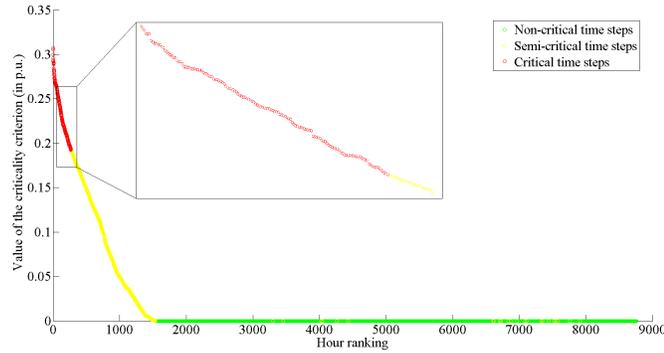


Figure 4: Criticality criterion ranking, with 22 nodes equipped and a level of penetration of 110%

We can observe that the criticality criterion fulfills its goal, as we obtained a strict separation between the critical time steps and the semi-critical ones.

### 2) Computational time spared

As the stated goal of this methodology is to decrease the computational burden associated with such calculations, we now present an evaluation of the computational time spared for various levels of penetration and in the case where 17 buses have been equipped. These results have been obtained on a desktop computer with a 2.4 GHz processor and 6 GB of RAM.

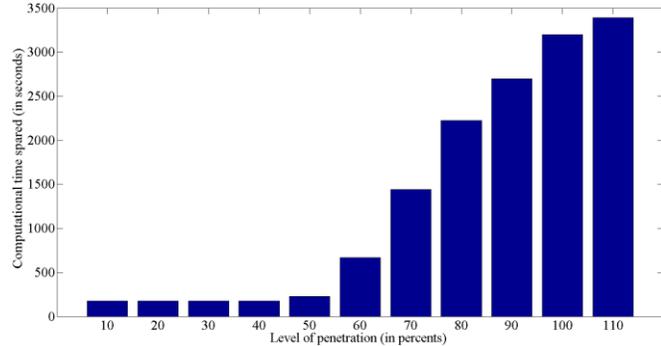


Figure 5 : Computational time spared as a function of the level of penetration

We observe that the computational gain obtained increase substantially starting from a level of penetration of 60%, which also corresponds to the occurrence of a significant and

growing number of semi-critical time steps (see fig.1). This is to be expected, as the slave problem takes typically 0.02 seconds to be solved for non-critical time steps, while it takes between 2.5 and 3 seconds for a semi-critical time step.

### 3) Voltage control impact

A possible application of the methodology introduced is to evaluate the deoptimization introduced to the master problem scheduling by the voltage control implemented. First, let us observe the solution from the slave problem for a critical time step.

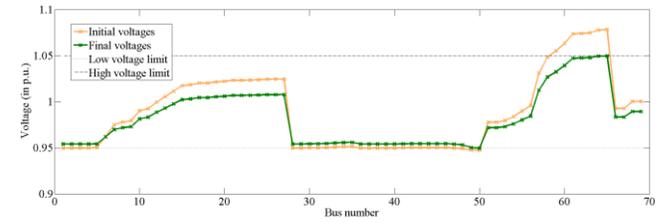


Figure 6 : Bus voltages before and after solving the slave problem

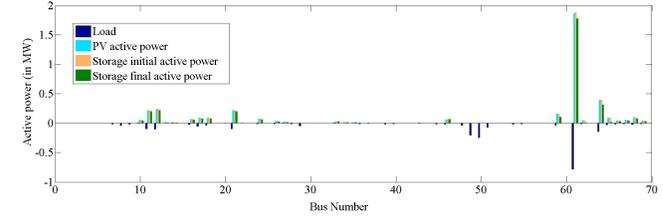


Figure 7 : Active powers resulting from the slave problem solution

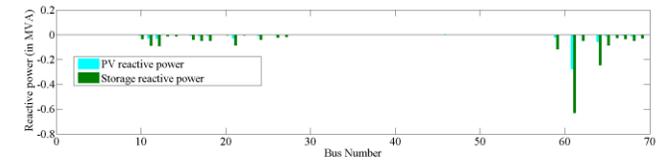


Figure 8 : Reactive powers resulting from the slave problem solution

We can see that the active power injections from the storage systems are effectively reduced to cope with overvoltage occurrences. This has two effects: the first one is the decrease in voltage due to lower active power injections and the second one is the freeing up of reactive power capabilities that allows for reactive power consumption, further reducing the voltage.

To evaluate the impact of the voltage control, we first compute the gains obtained in terms of scheduling costs through the deployment of photovoltaic generators associated with storage systems, for various scenarios. These are expressed in percentage of the cost of scheduling when only the loads are present. Then we compute the cost associated with voltage control in terms of deoptimization of the scheduling, and in this case, the results are also expressed in percentage of the cost of scheduling when only the loads are present.

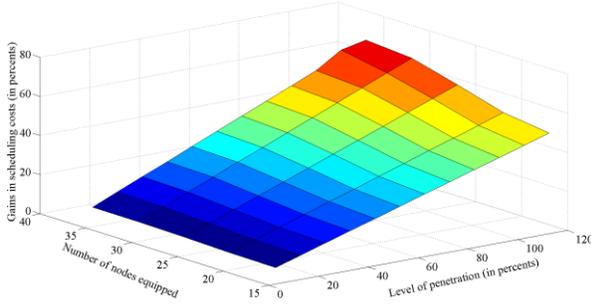


Figure 9 : Gains obtained by the deployment of storage and PV systems, for various levels of penetration and numbers of nodes equipped

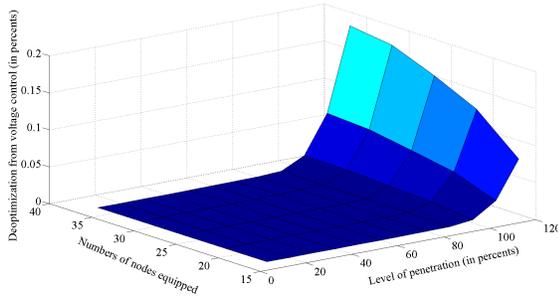


Figure 10 : Deoptimization resulting from voltage control, for various levels of penetration and numbers of nodes equipped

### B. Limitation of the criticality criterion

We have presented results derived from the use of the criticality criterion in cases where the separation obtained was satisfactory. However, this is not always the case, especially when the level of penetration is increased substantially. For example, the ranking obtained when 22 nodes are equipped and the level of penetration is 140 percents is as follows:

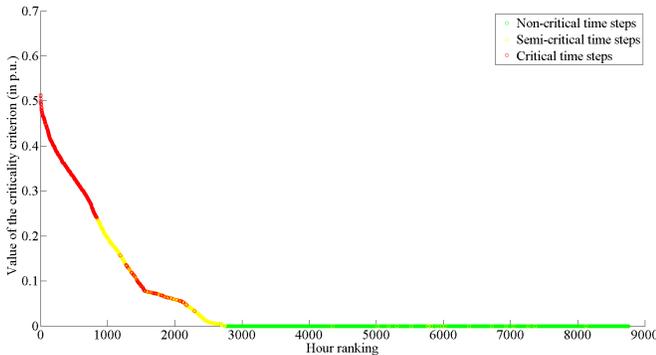


Figure 11 : Criticality criterion ranking, with 22 nodes equipped and a level of penetration of 140%

The ranking obtained is not satisfactory, as a significant number of critical time steps have lower criticality criterion values than some semi-critical time steps (hereafter named missed critical time steps). This is to be expected, in the sense that the criticality criterion, in essence, aims at describing the behaviors of heterogeneous buses with only one value. Thus, when the behaviors, for a given time steps, of various buses

starts to diverge significantly (for example, a high load is present on a bus without storage while a high level of power is injected from another bus), we cannot expect the criticality criterion to be efficient. In order to investigate the limits to the efficiency of the criticality criterion, we define the maximal difference of active voltage change potential, as a proxy measurement of the divergence in behaviors between various buses. It is calculated in the following way:

$$\Delta V_P^{k,t} = \frac{R^k \times P^{k,t}}{V^{k,t}} \quad (19)$$

$$\Delta V_P^{\max} = \max_{t \in T} \left( \max_{k \in K} \Delta V_P^{k,t} - \min_{k \in K} \Delta V_P^{k,t} \right) \quad (20)$$

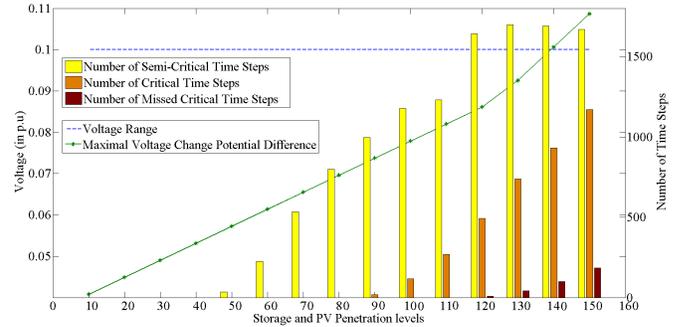


Figure 12 : Evolution of criticality criterion efficiency and maximum difference of active voltage change potential, as a function of the level of penetration

We observe here that the criticality criterion remains efficient until a level of penetration of 130 percents is attained. At this level, the maximum voltage change potential difference is close to the span of the range of permissible voltages (chosen here to be between 0.95 p.u. and 1.05 p.u.). This observation gives us a hint that we may be able to define a domain of applicability for the criticality criterion through the mere calculation of the maximal difference of active voltage change potential. To investigate this claim, we have completed various sets of simulation while modifying several relevant parameters: the R/X ratio of the lines, the storage discharge time and the number of nodes equipped. For each set of simulation, the level of penetration varies between 10 and 150 percents. The results are presented below.

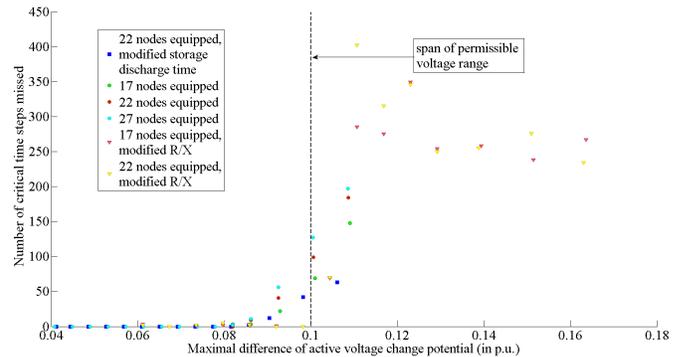


Figure 13 : Number of critical time steps missed, as a function of maximal difference of voltage change potential, for several simulation setups

We observe that the number of critical time steps missed remains relatively low (under 60) compared to the total number of time steps (8760) when the maximal difference of active voltage change potential is below the span of the permissible voltage range. As the original goal stated is to find the minimal total cost associated with the operation of such a system, not taking into account certain critical time steps will have the consequence to under evaluate this cost. Thus, we compute the difference between the total cost taking into account all the critical time steps and the total cost obtained through the use of the criticality criterion, and define this as the loss of precision incurred by the implementation of the criticality criterion. It is expressed in percentage of the total cost and is presented below as a function of the maximal difference of active voltage change potential, with varying levels of penetration and numbers of nodes equipped.

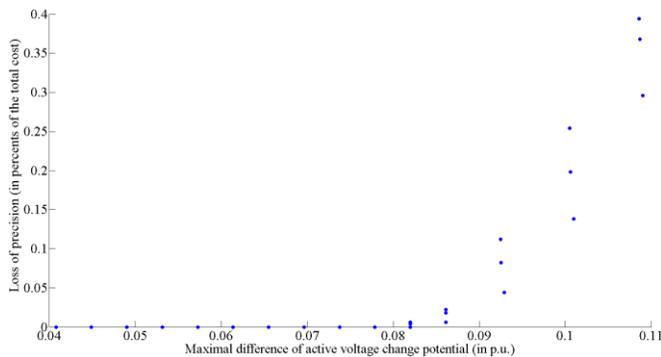


Figure 14 : Loss of precision as a function of the maximal active voltage change potential

We can observe that the loss of precision that can be attributed to the use of the criticality criterion remains fairly low, even when the maximum difference of active voltage change potential surpasses the span of the permissible voltage range. This can also be used to determine a domain of applicability for the criticality criterion dependent on the level of precision required by the considered application.

#### IV. CONCLUSION

We have presented a methodology aimed at significantly reducing the computational burden associated with the resolution of optimal power flows for distribution systems, in order to render it more practical for planning purposes. We have verified its validity and the computational gains obtained for a practical case study. Afterwards, we have defined an indirect measurement of the domain of validity with a negligible computational cost and have tested it against several extensions of the aforementioned case study. Finally, we have examined the relationship between the loss of precision incurred from the use of the criticality criterion and the position in the domain of validity. Further research has to be done to try and characterize this relationship in order to be

able to determine *a priori* the loss of precision –or an upper bound for it. This will be done by applying this methodology to representative practical distribution networks, with realistic loading conditions and testing several options for the objective function. Another area of improvement lies in the resolution of the slave problem, which could be sped up by drawing on its specificities, especially the fact that it is close to being a quadratically-constrained quadratic programming problem. Finally, this methodology will be integrated in a comprehensive distribution system planning framework adapted to the development of distributed generation and storage

#### References:

- [1] F. Pilo, G. Celli, S. Mocci and G. G. Soma, “Active distribution network evolution in different regulatory environments”, in *Proc. 2010 7th Mediterranean Conference and Exhibition on Power Generation, Transmission, Distribution and Energy Conversion*, pp. 1-8.
- [2] Carpentier, J., “Contribution à l’étude du dispatching économique”. *Bull. Soc. Française Electriciens*, vol. 8, pp.431–447, Aug. 1962
- [3] A. Gabash and P. Li, “Active-Reactive Optimal Power Flow in Distribution Networks With Embedded Generation and Battery Storage”, *IEEE Tran. Power Systems*, vol. 27, pp. 2026-2035, Mar. 2012.
- [4] A. R. Ahmadi and T. C. Green, “Optimal power flow for autonomous regional active network management system”, in *proc. 2009 IEEE Power Energy Society General Meeting*, pp. 1-7.
- [5] K. K. Swarnkar, S. Wadhvani and A. K. Wadhvani, “Optimal power flow of large distribution system solution for combined economic emission dispatch problem using partial swarm optimization”, in *proc. 2009 International Conference on Power Systems*, pp 1-5
- [6] M. J. Dolan, E. M. Davidson, G. W. Ault, F. Coffele, I. Kockar and J. R. McDonald, “Using optimal power flow for management of power flows in active distribution networks within thermal constraints”, in *proc. 2009 Universities Power Engineering Conference (UPEC)*, pp. 1-5.
- [7] S. Bruno, S. Lamonaca, G. Rotondo, U. Stecchi and M. La Scala, “Unbalanced three-phase optimal power flow for smart grids”, *IEEE Trans. Industrial Electronics*, vol. 58, pp. 4504-4513, 2011.
- [8] S. Paudyal, C. A. Canizares and K. Bhattacharya, “Three-phase distribution OPF in smart grids: Optimality versus computational burden”, in *proc. 2011 IEEE PES Innovative Smart Grid Technologies Conference Europe*, pp. 1-7
- [9] A. Dukpa, B. Venkatesh and L. Chang, “An accurate voltage solution method of radial distribution system”, *Can. J. Elect. Comput. Eng.*, vol. 34, Winter/Spring 2009
- [10] J. Widén, E. Wäckelgård, J. Paatero, and P. Lund, “Impacts of distributed photovoltaics on network voltages: Stochastic simulations of three Swedish low-voltage distribution grids”, *Electric Power Systems Research*, vol. 80, pp. 1562-1571, Dec. 2010.
- [11] M. E. Baran and F. F. Wu, “Optimal capacitor placement on radial distribution system”, *IEEE Trans. Power Delivery*, vol. 4, pp. 725-734, Jan 1989.
- [12] A. E. Auld, J. Brouwer and G. S. Samuelsen, “Analysis and visualization method for understanding the voltage effect of distributed energy resources on the electric power system”, *Electric Power Systems Research*, vol. 82, pp. 44–53, 2012.
- [13] Wächter and L. T. Biegler, “On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming”, *Mathematical Programming*, vol. 106, pp. 25-57, Mar. 2006