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1 **MAPPING ROADSIDE NITROGEN DIOXIDE**
2 **CONCENTRATIONS USING NON-STATIONARY**
3 **KRIGING**

4
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15
16 **Abstract**

17 Atmospheric nitrogen dioxide (NO₂) concentrations around a major road in Alsace (France) are estimated
18 on a fine grid using measurements given by passive samplers and a geostatistical approach. Data are
19 referenced to a local coordinate system where (x, y) are respectively the distance from and along the road.
20 They show a strong non-stationarity which does not allow ordinary kriging to be used in the estimation.
21 Therefore a trend is modelled by a combination of exponential and polynomial functions. Experimental
22 residuals are then computed as the differences between measurements and the trend. The idea is to
23 interpolate the residuals at the nodes of the grid, applying kriging methods, and to add them to the trend
24 estimate. Since their variance is not stationary either, an intermediary step is required. It consists in
25 modelling the standard deviation of the residuals as a function of the drift and normalizing the residuals
26 by this model. This defines a new regionalized variable which can be estimated in the framework of
27 stationary geostatistics. Two possible kriging systems are tested, depending on the fitted variogram
28 model: in the first one, a pure nugget effect (white noise) is used, in which case the best linear estimator
29 of NO₂ concentration is the trend model; in the second one, a structured exponential variogram is
30 adjusted. This case study shows that non-stationarity may not only characterize the raw variable but can

31 also affect the variance of a phenomenon. It illustrates the interest of modelling it so as to improve the
32 experimental variogram, fit an acceptable variogram model and compute the variance of the estimation
33 error even if the estimator is reduced to a simple regression function.

34

35 *Keywords:* geostatistics, non-stationarity, trend estimation, standard deviation model, universal kriging

36

37

38 **1. Introduction**

39

40 Air pollution mapping is a valuable tool for assessing population long-term exposure and informing the
41 public about the spatial distribution of outdoor concentrations. At the regulatory level, calculation of
42 concentration maps is prompted by the European daughter directives on ambient air quality assessment
43 (1999/30/ EC, 2000/69/ EC, 2002/3/ EC).

44 In France geostatistical methods have been receiving particular attention for several years and are now
45 commonly used by the French air quality monitoring organizations. They are mostly applied for mapping
46 background concentrations at the city or regional scale so as to fulfill the regulatory recommendations.
47 Roadside pollution is equally a matter of concern as regards people exposure and has been the subject of
48 many investigations (Gilbert, 2003, Kodama, 2001, Roorda-Knape, 1999). However, the difficulty in such
49 cases is that concentration usually shows a strong drift (it decreases with distance from the source), which
50 makes the classical methods of stationary geostatistics like ordinary kriging or cokriging unsuitable.

51 Deletraz and Dabos (2001) tested different interpolating methods in order to map the environmental
52 impact (expressed as NO₂ deposition) of a road in a mountainous region and opted for a multiple
53 regression estimator using the inverse distance from the road and the logarithm of roughness as
54 explicative variables. The regression model explains 87% of the deposition variability. Unlike the
55 geostatistical method explored hereafter, it does not take the residual variability into account. Briggs et al.
56 (2000) also proposed a regression mapping technique to model the spatial patterns of traffic-related
57 pollution. The developed model is a linear combination of variables derived from traffic counts, land
58 cover and altitude.

59 Several studies show the relevance of geostatistics to treat non-stationary problems in the vicinity of
60 chemical or contaminating sources. Figueira et al. (1999) applied kriging with external drift for estimating
61 soil salinity in a coastal land strip. The logarithm of the distance from the coast was used as an auxiliary
62 variable. The results were compared with those of ordinary kriging. They attested the efficiency of the

63 external drift. Saito and Goovaerts (2001) applied different techniques based on kriging to estimate lead
64 soil concentrations around a former smelter in Dallas. In particular they modelled a trend by a linear
65 combination of $\log(d)$ and $\Delta\theta$, d being the distance from the smelter and $\Delta\theta$ the deviation from the main
66 wind direction. Then they worked with residuals obtained from the difference between measurements and
67 the trend estimates.

68

69 In this methodology-oriented study we propose to perform kriging of residuals from a global trend model
70 in order to estimate NO_2 roadside concentrations in the Thur Vosges Valley in Alsace, France.

71

72 Concentration data are derived from passive diffusion tube measurements performed 3 meters above the
73 ground at 39 sites around a major road, (Fig. 1-A). The aim of the measurement campaign, which took
74 place in summer 2001, was to assess the environmental impact of traffic. Therefore the samplers were
75 located in places supposed to be strictly under the influence of road emissions. Most tubes were
76 positioned on transects and situated 1 m, 2 m, 50 m, 200 m, 400 m far from the road. Similar
77 measurements were also conducted in winter. However, the present article focuses on summer average
78 concentrations, calculated from 6 fortnightly records. Results for winter are mentioned only if they
79 present significant differences with summer. The road is 20 km long. To simplify its width has been set
80 to zero as it is not precisely known.

81

82 Among all the auxiliary variables (land use, emission inventory, population density ...) distance to the
83 road and elevation appear to be explanatory of NO_2 concentration, as indicates a Factorial Component
84 Analysis. Now, the elevation is strongly correlated with distance along the road (curvilinear abscissa)
85 (see Fig.1-B). Consequently NO_2 concentrations are expressed in a coordinate system where (x, y) are
86 distances from and along the road, respectively (Fig. 1-C). By this way elevation is implicitly taken into
87 account.

88

89 Data show a clear tendency of NO_2 concentration to decrease with distance from the road, indicating the
90 non-stationary nature of the pollution phenomenon. In that context a mapping methodology based on
91 estimating the trend is suggested.

92 In what follows, concentration measurements are considered as the realization of a random function
93 $\text{NO}_2(x,y)$. The estimation involves four stages.

- 94 1. At any point (x,y), NO₂ is written as the sum of a trend m(x,y) providing the general pattern of
 95 pollution along and across the road, and a random residual: $NO_2(x,y)=m(x,y)+R(x,y)$
 96 Therefore this first step consists in adjusting a trend model on the concentration data.
- 97 2. As the variance of the residuals depends on their value (non stationary variance), those residuals have
 98 to be normalized so that classical kriging methods can be applied. For this purpose their standard
 99 deviation is modelled by a function $\sigma(x,y)$.
- 100 3. The experimental variogram of the normalized residuals $NR(x,y)=R(x,y)/\sigma(x,y)$ is calculated to
 101 describe the spatial variability of concentration fluctuations around the trend and a variogram model is
 102 fitted.
- 103 4. A simple kriging system is solved to estimate NR(x,y) at the nodes of a 250m x 250m mesh grid.
 104 Finally the estimator of NO₂(x,y) is built :

$$NO_2^*(x, y) = \sigma(x, y)NR^*(x, y) + m(x, y)$$

105
 106 After presenting each stage, we conclude by a discussion where :

- 107 - the results are interpreted;
 108 - other possible kriging systems are considered, depending on the confidence given to the trend
 109 estimate.

110

111 2. Modelling the trend

112

113 As a preamble, let us define the notion of non-stationarity : a regionalized phenomenon, namely a
 114 physical phenomenon stretching out in space and represented by a function Z, is stationary if its mean
 115 $m(p)$ and its covariance $cov[Z(p),Z(p+h)]$ are independent of the location of point p.

116 Fig. 2-A represents the projection of concentrations on a plane perpendicular to the road. Two main
 117 observations can be made: 1) NO₂ concentration decreases sharply with distance from the road, indicating
 118 the non-stationary nature of the phenomenon; 2) there is no significant dissymmetry between the left and
 119 right sides of the road. The first remark involves trend modelling, the second one allows us to handle the
 120 left and right sides together, increasing the total number of available data in a class and hence, the
 121 accuracy of the model.

122 A difficult task is now to give a proper analytical form to the trend.

123

124 • For a given $y = y_j$, i.e. when concentration variations are examined perpendicularly to the road,
 125 optimality in the least square sense is obtained in the frame of a bi-exponential function of x :

$$m(x, y_j) = a(y_j) e^{-\frac{|x|}{s_1}} + b(y_j) e^{-\frac{|x|}{s_2}} \quad [1]$$

126 with $(s_1, s_2) = (18, 550)$. For distances from the road greater than 1600 meters the drift can be considered
 127 as zero. The average concentrations and their interpolated values by function (1) are represented in Fig. 2-
 128 B.

129 In winter $(s_1, s_2) = (14, 940)$. Concentrations are distributed following a different pattern, and become
 130 zero at more than 3000 meters from the road. Such a difference with summer can be explained by
 131 climatology, changes in the emissions and photochemical consumption of nitrogen oxide in summer.
 132 Such observations are consistent with what Monn et al. (1997) already noticed.

133

134 • For a given $x = x_i$, i.e. when concentration variations are examined parallel to the road, optimality in
 135 the least square sense is obtained in the frame of a linear function of y (Fig. 2-C) :

$$m(x_i, y) = c(x_i) y + d(x_i) \quad [2]$$

136

137 • Combining formulas (1) and (2) leads to the trend model :

$$m(x, y) = c_0 + (c_1 y + c_2) e^{-\frac{|x|}{s_1}} + (c_3 y + c_4) e^{-\frac{|x|}{s_2}} \quad [3]$$

138

139 The constant c_0 is introduced so that the trend is not necessarily zero for distances from the road greater
 140 than 1600 meters ; the factors s_1 and s_2 are fixed once and for all; the coefficients c_0 to c_4 are obtained by
 141 a global regression on the 39 measurements. The trend is represented in three dimensions (Fig. 3-A) and
 142 by isovalues (Fig 3-B).

143 Results are different for each season. For exemple, c_0 is equal to $0.3 \mu\text{g m}^{-3}$ in summer and $3.5 \mu\text{g m}^{-3}$ in
 144 winter.

145

146 Most measurements are located at distances to the road less than 400 meters. Beyond that, the trend
 147 model is not conditioned by experimental data and only determined by the mathematical properties of the
 148 exponential functions. To prevent improper extrapolation, the use of $m(x, y)$ is restricted to x values below
 149 500 meters on both sides of the road ($|x| < 500$ m).

150

151 3. Non-stationarity of the variance

152

153 Let u be the coordinate vector (x,y) . Using the trend model (3) the residuals $R(u)$ are calculated at the
154 measurement points :

$$R(u) = \text{NO}_2(u) - m(u) \quad [4]$$

155 Those residuals are plotted as a function of $m(u)$ in Fig. 4-A. As expected they fluctuate around 0 but the
156 amplitude of the fluctuations increases with $m(u)$, suggesting that the variance of the residuals is not
157 stationary. Therefore this variance has to be modelled too. The trend values calculated at the sampling
158 points are gathered in six different classes and for each class the standard deviation of the residuals is
159 computed. A power function of $m(u)$ is then adjusted (Fig. 4-B) :

$$\sigma(u) = d m(u)^p \quad [5]$$

160 The parameters d and p are equal to 0.261 and 0.879 respectively ($d=0.015$ and $p=1.57$ in winter).

161

162 4. Variograms

163

164 To get stationarity the residuals are normalized :

$$\text{NR}(u) = \frac{R(u)}{\sigma(u)} \quad [6]$$

165 Such an operation is possible because $\sigma(u)$ is strictly positive in the domain under study.

166 Is this normalization (6) really necessary ? The transverse semivariogram (i.e. perpendicular to the road)
167 of $R(u)$ is plotted in Fig. 5-A : the large fluctuations are mainly explained by the contrast of variability
168 between the measurements close to the road and the other concentration measurements. If the non-
169 stationarity of the variance is ignored and the residuals are not normalized then the variance assigned to
170 the first set of measurements is too low and the variance assigned to the second one is too high.
171 Normalization makes it possible to homogenize residuals and is essential to go further in the analysis.

172 When calculated on $\text{NR}(u)$, the transverse variogram is significantly improved (Fig. 5-B). The
173 longitudinal variogram (i.e. along the road, Fig. 5-C) has a cyclic aspect due to the alternation of higher
174 (about $50 \mu\text{g m}^{-3}$) and lower (about $40 \mu\text{g m}^{-3}$) values for samplers located at distances less than 2 meters
175 from the road. The housing type may explain such concentration variations but additional information
176 need to be collected to confirm such an hypothesis. The first point of the longitudinal variogram is
177 calculated with only ten pairs of data whereas the second one is computed with 118 pairs. Whether or not

178 the first point is taken into account and the transversal or longitudinal variogram is assigned with a
 179 structure, two approaches are examined :

- 180 - the variogram is considered as not being structured in any of the directions (white noise) : it is
 181 modelled by a pure nugget effect equal to the variance of NR(u) : C(0)=1.13.
- 182 - a structure is detected and described by the sum of a nugget effect and an exponential model with a
 183 geometrical anisotropy of ranges 1450 m in the x-direction and 10000 m in the y-direction
 184 (respectively 80 m and 2200 m in winter).

185

186 5. Estimation

187

188 Whatever the approach, let C(h) be the covariance of NR(u). It is linked to the stationary variogram by
 189 the relationship :

$$190 \quad C(h) = C(0) - \gamma(h)$$

191 The normalized residuals are estimated at the nodes $u_0 = (x_0, y_0)$ of a grid by a linear combination of the n
 192 = 39 concentration measurements :

$$NR^*(u_0) = \sum_{\alpha=1}^n \lambda_{\alpha} NR(u_{\alpha}) \quad [7-a]$$

193 The kriging weights λ_{α} are the solution of a simple kriging system :

$$\sum_{\alpha=1}^n \lambda_{\alpha} C_{\alpha\beta} = C_{\beta 0} \quad \forall \beta : 1, n \quad [7-b]$$

194 where $C_{\alpha\beta}$ (resp. $C_{\beta 0}$) stands for the covariance between measurements taken in u_{α} and u_{β} (resp. u_{β} and
 195 u_0).

196 The estimator of $NO_2(u_0)$ is obtained setting :

$$NO_2^*(u_0) = \sigma(u_0) NR^*(u_0) + m(u_0) \quad [7-c]$$

197 where $m(u_0)$ and $\sigma(u_0)$ are given by (3) and (5).

198 The variance of the estimation error is :

$$\sigma_{NO_2}^2(u_0) = \sigma^2(u_0) (C(0) - \sum_{\alpha=1}^n \lambda_{\alpha} C_{\alpha 0}) \quad [7-d]$$

199 When the covariance C(h) is described by a pure nugget effect, the weights minimizing (7-d) are all zero
 200 and the estimation is reduced to :

$$\text{NO}_2^*(\mathbf{u}_0) = \mathbf{m}(\mathbf{u}_0) \quad [8-a]$$

201 This result is actually well-known : kriging with a nugget covariance is strictly equivalent to estimating
202 the trend. Here the trend has been obtained by a mean-square regression. At each node of the grid, the
203 estimation variance is then :

$$\sigma_{\text{NO}_2}^2(\mathbf{u}_0) = \sigma^2(\mathbf{u}_0) C(0) \quad [8-b]$$

204 As $C(0)$ is close to 1, σ_{NO_2} is nearly equal to the non-normalized residues standard deviation model σ
205 given by (5).

206 Fig. 6-A shows the estimation of concentrations using an exponential covariance. Comparing it with the
207 trend representation (Fig. 3-B) makes it evident how the isolines get distorted to fit the experimental
208 values. Such a comparison points up the advantage of kriging over a mean-square regression and the
209 interest of modelling the variogram when possible. The estimation standard deviation for that kriging is
210 displayed in Fig. 6-B. Compared to the one associated with regression (Fig. 6-C) it is lower around the
211 data points.

212

213 **6. Conclusion and discussion**

214

215 Through the analysis of a case example, a methodology is proposed for elaborating concentration maps in
216 presence of a spatial trend. Though the limited number of data did not facilitate the interpretation of the
217 variogram (see section 4), our aim was to point out the interest of modelling non-stationarity when it
218 affects a variable and its variance. Here we discuss the choices we made for calculating the variogram and
219 defining a model.

220

221 **6.1 Interpretation of the results**

222 The variogram $\gamma(h)$ is computed with residuals derived from a trend estimate, which necessarily
223 introduces a bias (Matheron, 1970). As a consequence, the real underlying variogram, assuming it is
224 structured, can be either masked or deformed, depending on the representativity of the data. Quantifying
225 the impact of the number of samplers on the quality of the trend estimate is not easy. However the
226 estimation standard deviation map for a nugget covariance model may be a useful tool for evaluating
227 kriging results (Fig. 6-C). On the road axis the standard deviation is $8 \mu\text{g m}^{-3}$ at $y=5$ km. Then it decreases
228 sharply till it reaches 1 at $y = 20$ km and $x = \pm 500$ m. If the standard deviation is not to be larger than 3.5
229 $\mu\text{g m}^{-3}$ for example, then only the area where this condition is met should be selected for representing

230 concentrations, whereas estimates near the road are ignored (hatched region in Fig. 6-C). This
231 interpretation of the kriging standard deviation map is strengthened by sensitivity tests consisting in
232 estimating $m(x, y)$ with restricted data sets. According to such tests, the trend estimate was sensitive to
233 data located less than 2 m from the road. For longer distances the coefficients c_0 to c_4 of formula (3) did
234 not change significantly if 4 or 5 measurements were removed randomly. The poorer quality of estimation
235 near the road is to be expected since concentrations, and hence the estimation variances, are higher there.
236 To improve it, more measurements are required.

237

238 The kriging standard deviation map for an exponential covariance model (Fig. 6-B) can be studied in the
239 same way. Regions respecting the quality criterion, expressed as a maximal value for kriging standard
240 deviation, can be selected as previously.

241

242 **6.2 Using the IRF-k theory ?**

243 A drawback of the proposed methodology is that the final NO_2 concentration estimate closely depends on
244 the mean-square trend estimate. To weaken this dependency relationship a more elaborated kriging
245 system might be used. Actually two different systems could be envisaged :

- 246 - universal kriging of NO_2 (Appendix 1) : an affine function of the trend $m(u)$ is reestimated;
- 247 - a more general universal kriging of NO_2 (Appendix 2): all the coefficients of the trend are
248 reestimated.

249 Whatever the chosen method, all the systems proceed from the non stationary approach called Universal
250 Kriging (Matheron, 1969) and do not solve the problem of the bias of the variogram which has been
251 previously mentioned.

252 The theory of the Intrinsic Random Functions of order k (IRF- k) (Matheron, 1971) was developed to
253 answer that problem. A synthetic description can be found in Chilès and Delfiner (1999). To summarize,
254 we can say that this theory implies fitting most part of the phenomenon by all possible linear combination
255 of polynomials having a given degree k , the variability of the residuals being modelled by a Generalized
256 Covariance. This method is useful when polynomials can explain the large-scale spatial behaviour of the
257 phenomenon under study. Is it the case here? Fig. 7 represents the averaged concentrations projected on a
258 plane perpendicular to the road, and their regression by polynomials. To obtain an acceptable
259 approximation, we need to use at least all the degrees of x up to 4. This implies modelling the trend using
260 a term in yx^4 relevant of an IRF-5. As the set of polynomials must be complete, in order to ensure the

261 regularity of the kriging system, the trend must then be developed on 21 monomials with as much
 262 multiplicative coefficients to be calculated. This is not reasonable.

263

264 Universal kriging seems therefore to be an effective way of handling the problem of double non-
 265 stationarity in mean and variance. It is based on hypotheses consistent with the spatial pollution pattern.

266 Besides this methodology does not involve complex mathematical developments and is suitable for
 267 regular use, in compliance with regulatory requirements.

268

269 **Appendix 1 : Universal Kriging of NO₂**

270 A linear system is built where a linear function of the trend $m(u)$ is locally estimated by kriging. $NO_2(u)$
 271 is consequently expressed as :

$$NO_2(u) = \sigma(u) NR(u) + a_0 m(u) + a_1 \quad [A1-1]$$

272 where a_0 and a_1 are unknown and $m(u)$ is the trend previously obtained by least square regression. The
 273 specificity of the present application is a non stationary covariance :

$$Cov(NO_2(u), NO_2(v)) = \sigma(u) \sigma(v) C(v-u) \quad [A1-2]$$

274

275 At any node u_0 of the estimation grid, the estimator is a linear combination of measurements taken at the
 276 points u_α :

$$NO_2^*(u_0) = \sum_{\alpha=1}^n \lambda_\alpha NO_2(u_\alpha) \quad [A1-3]$$

277 The weights λ_α are solutions of the following system :

$$\sum_{\alpha=1}^n \lambda_\alpha \sigma(u_\alpha) \sigma(u_\beta) C_{\alpha\beta} + \mu_0 + \mu_1 m(u_\beta) = \sigma(u_\beta) \sigma(u_0) C_{\beta 0} \quad \forall \beta : 1, n \quad [A1-4]$$

$$\sum_{\alpha=1}^n \lambda_\alpha = 1 \quad [A1-5]$$

$$\sum_{\alpha=1}^n \lambda_\alpha m(u_\alpha) = m(u_0) \quad [A1-6]$$

278 The estimation error is defined as the difference $NO_2^*(u_0) - NO_2(u_0)$. Its variance is given by :

$$\sigma_{NO_2}^2(u_0) = \sigma^2(u_0) C(0) - \sum_{\alpha=1}^n \lambda_\alpha \sigma(u_\alpha) \sigma(u_0) C_{\alpha 0} - \mu_0 - \mu_1 m(u_0) \quad [A1-7]$$

279 Coefficients μ_0 et μ_1 are Lagrange parameters imposed by the universality conditions (A1-5) and (A1-6).
 280 Equations (A1-4) to (A1-6) make a classical Universal Kriging system which has the distinctive feature of
 281 using a non-stationary covariance $\sigma(u)\sigma(u+h)C(h)$. This system is reversible and has a unique solution
 282 since the covariance $\sigma(u)\sigma(u+h)C(h)$ is positive definite like $C(h)$. The data are NO_2 measurements (and
 283 not the normalized residuals NR). It must be useful to use a moving neighborhood with at least 10
 284 measurements around the node u_0 where the estimator is calculated.

285

286 **Appendix 2 : more general Universal Kriging of NO_2**

287 A linear system is built where the coefficients c_0 to c_4 of the trend $m(u)$ are locally estimated by kriging.

288 If we put :

$$289 \{X^l, l=1 \text{ to } 5\} = \{1, e^{-\frac{|x|}{s_1}}, e^{-\frac{|x|}{s_2}}, y e^{-\frac{|x|}{s_1}}, y e^{-\frac{|x|}{s_2}}\}$$

290 the kriging system becomes :

$$\sum_{\alpha=1}^n \lambda_{\alpha} \sigma(u_{\alpha}) \sigma(u_{\beta}) C_{\alpha\beta} + \sum_{l=1}^5 \mu_l X^l(u_{\beta}) = \sigma(u_{\beta}) \sigma(u_0) C_{\beta 0} \quad \forall \beta : 1, n \quad [\text{A2-1}]$$

$$\sum_{\alpha=1}^n \lambda_{\alpha} X^l(u_{\alpha}) = X^l(u_0) \quad \forall l : 1, 5 \quad [\text{A2-2}]$$

291 and the variance of the error is :

$$\sigma_{\text{NO}_2}^2(u_0) = \sigma^2(u_0) C(0) - \sum_{\alpha=1}^n \lambda_{\alpha} \sigma(u_{\alpha}) \sigma(u_0) C_{\alpha 0} - \sum_{l=1}^5 \mu_l X^l(u_0) \quad [\text{A2-2}]$$

292 Those expressions use five Lagrange multipliers μ_l . As previously, it is advised to use a moving
 293 neighborhood. The selected data must not lie on a line.

294

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303

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356 **Figure captions**

357

358 Fig. 1 : (A) Road representation in the original geographical system. The red points stand for the 39
359 measurement points, 33 of which lie on transepts T1 to T6.

360 (B) Correlation cloud between altitude and measurement distance along the road (curvilinear abscissa).

361 (C) Location of measurements in the local reference system.

362

363

364 Fig. 2 : (A) Projection of the measurement data onto a plane perpendicular to the road (only data located
365 on transepts have been projected). Black points represent the average concentrations for different distance
366 classes ($|x|=1\text{m}, 2\text{m}, 50\text{m}, 200\text{m}$ and 400m apart from the road).

367 (B) Average concentrations for the considered distance classes and adjustment of a combination of two
368 exponential functions.

369 (C) Concentration variations along the road (i.e. along y) for each $|x|$ -distance class and their
370 approximation by linear functions.

371

372

373 Fig. 3 : (A, B) Graphic representation of the trend $m(x,y)$ in perspective (A) and in isolines (B). The red
374 and black isolines are $10 \mu\text{g m}^{-3}$ and $2 \mu\text{g m}^{-3}$ distant from each other, respectively.

375

376

377 Fig. 4 : (A) Residuals $R(u) = \text{NO}_2(u) - m(u)$ as a function of the trend $m(u)$. $u = (x,y)$ is the coordinate
378 vector in the local reference frame. $R(u)$ fluctuations around 0 grow with $m(u)$. (B) $R(u)$ standard
379 deviations for each $m(u)$ value class and adjustment of a model written as $d m(u)^p$, with $d=0.261$ and
380 $p=0.879$.

381

382

383 Fig. 5 : (A) Experimental variogram of the residuals $R(u)$ in the transverse direction before
384 standardization. (B) Experimental variogram of the standardized residuals $R(u)$ in the transverse
385 direction. The number of pairs participating to the calculation of each point are indicated on the graph.
386 Two types of models have been fitted: a pure nugget model (in blue) and a nugget+exponential

387 anisotropic model with a range of 1.45 km along x and 10 km along y. (C) Variogram of the standardized
388 residuals along the road.

389

390

391 Fig. 6 : (A) Nitrogen dioxide concentration obtained by simple kriging of the standardized residuals,
392 using a structured variogram model. Size of the data points is proportional to the measurement values. (B)
393 Kriging standard deviation for the above calculated map. (C) Kriging standard deviation for a pure nugget
394 variogram model. It is almost equal to $\sigma(x,y)$ and can be regarded as the error standard deviation for the
395 trend estimation. In the red hatched area the standard deviation is greater than 3.5.

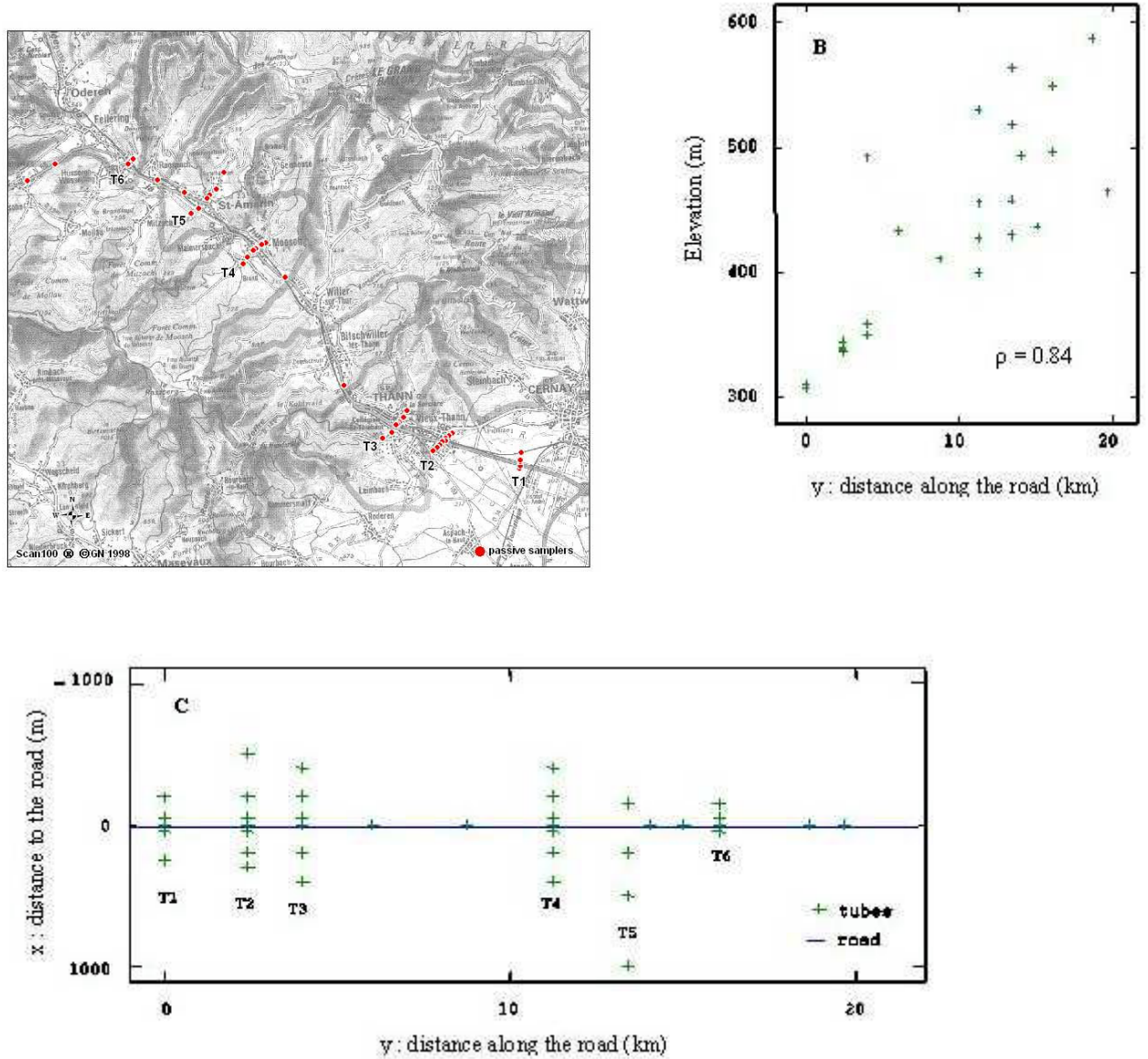
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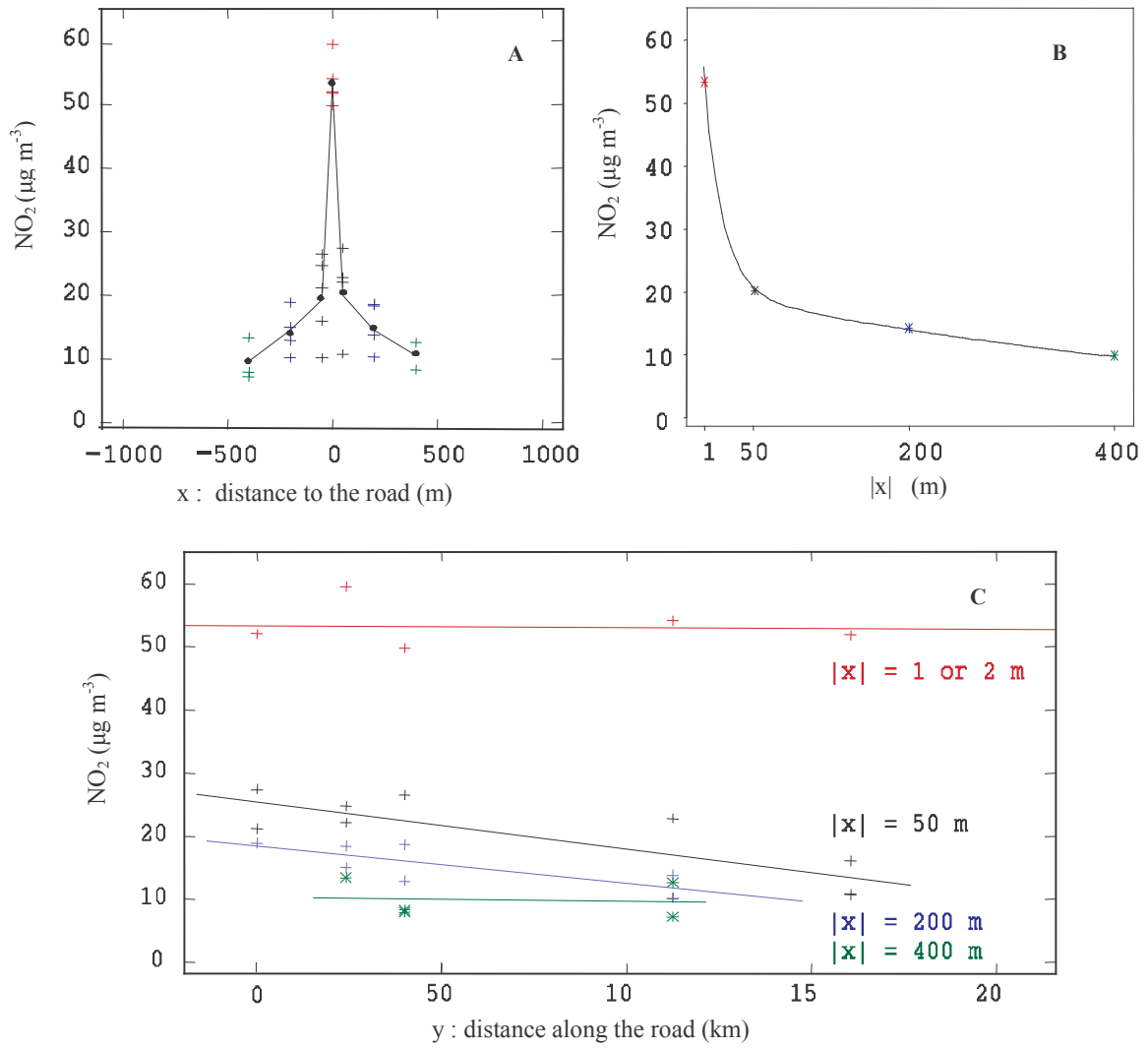
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398 Fig. 7 : NO₂ concentrations across the road (green dashed curve) and their approximation by 2nd to 4th
399 order polynomial functions (solid curves).

400

A

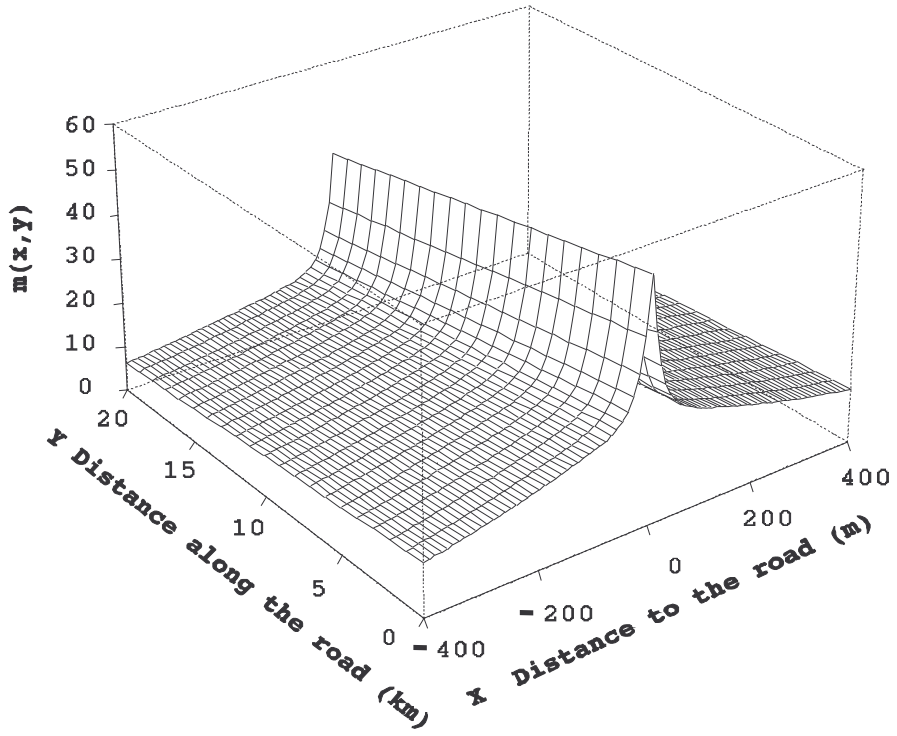




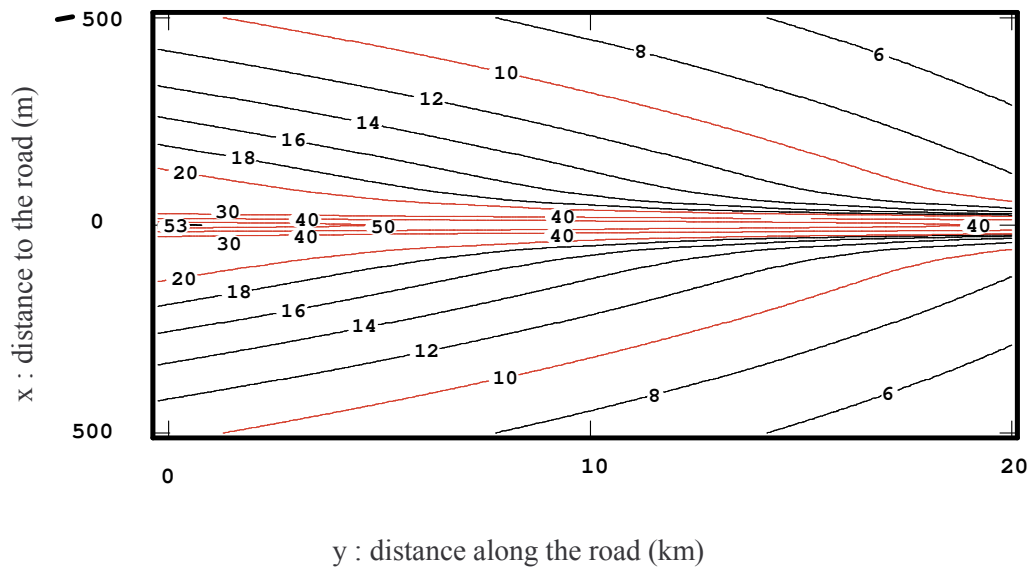
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409 Fig. 2

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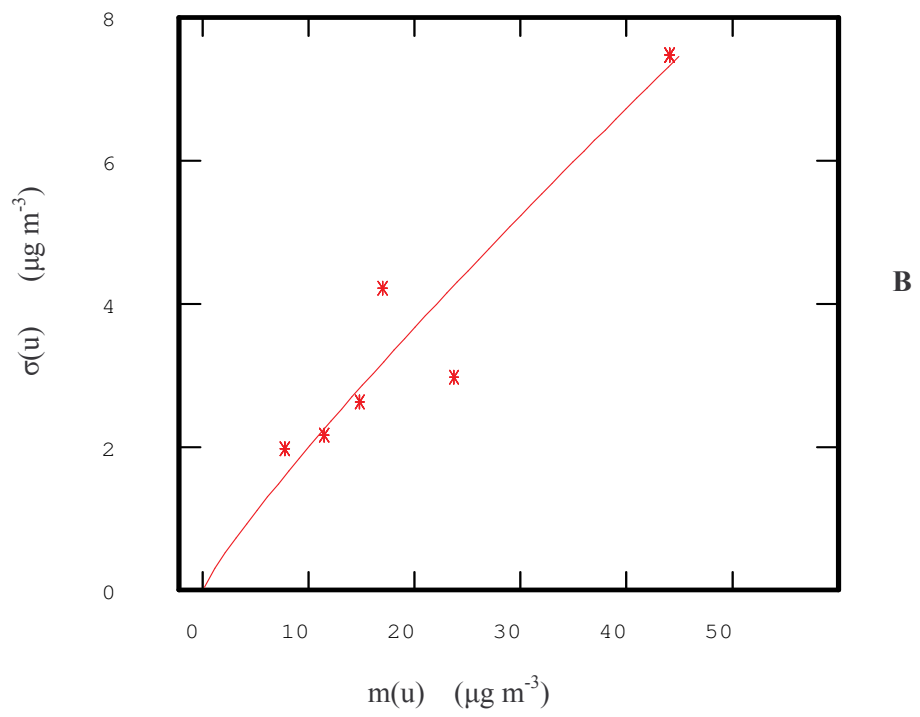
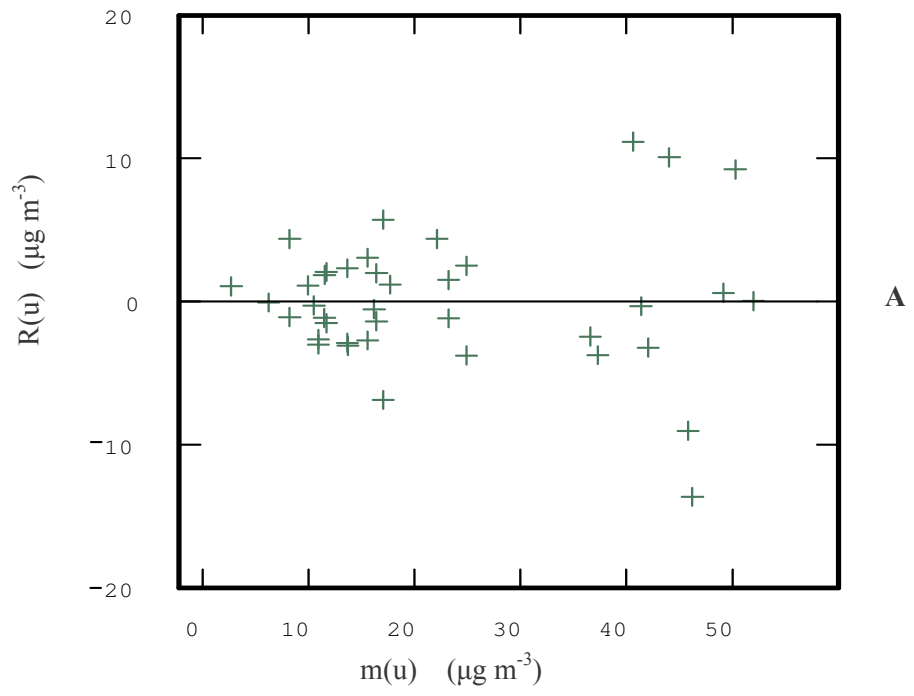
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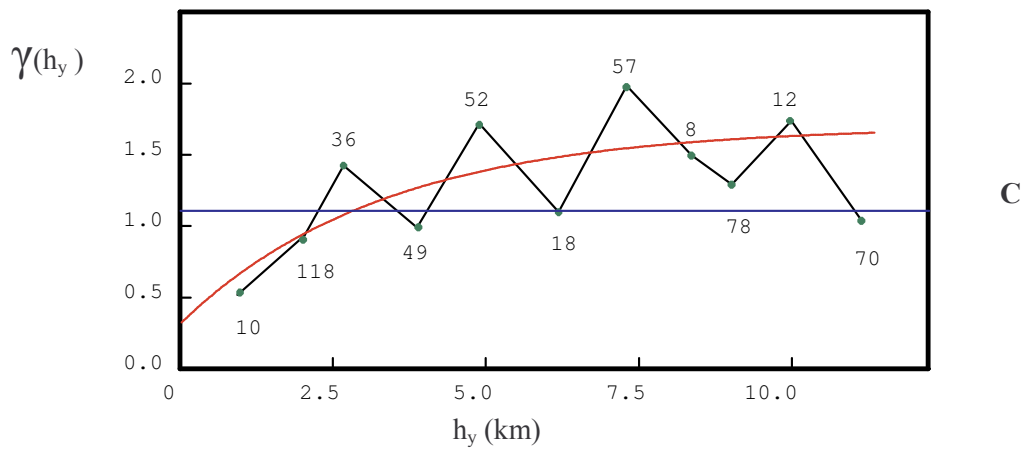
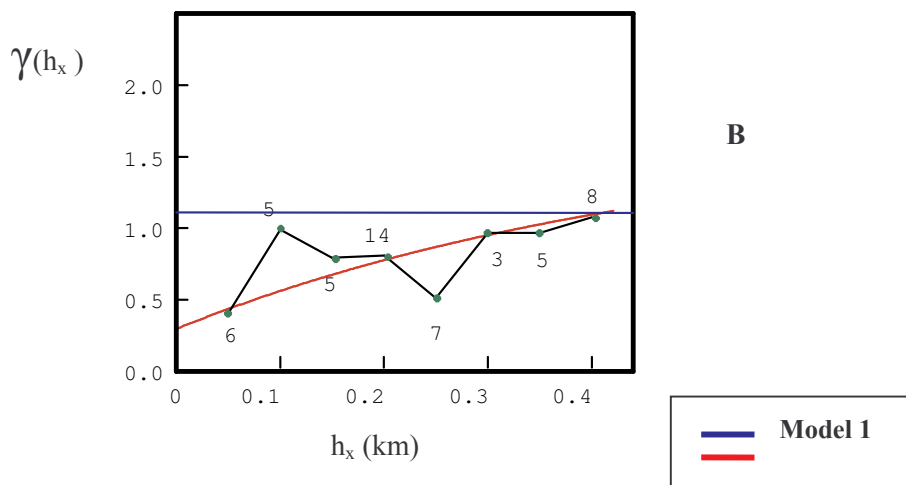
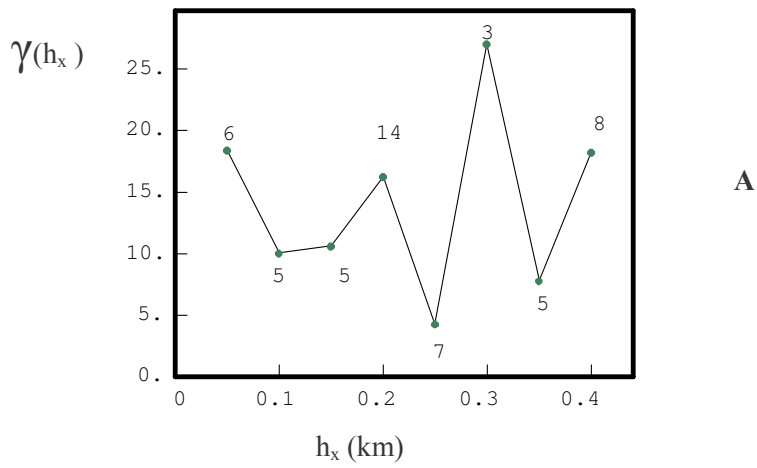
411 Fig. 3

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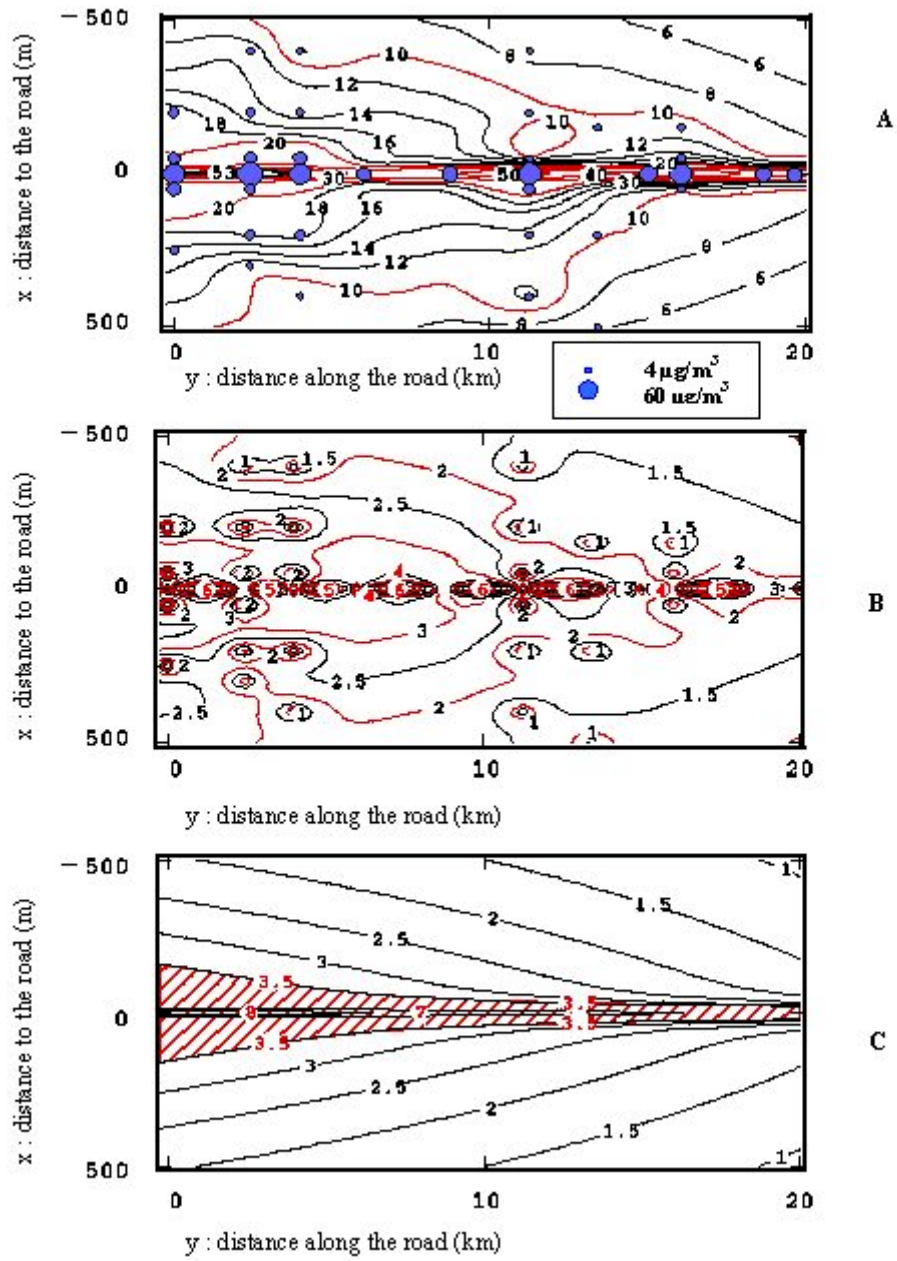
413 Fig. 4

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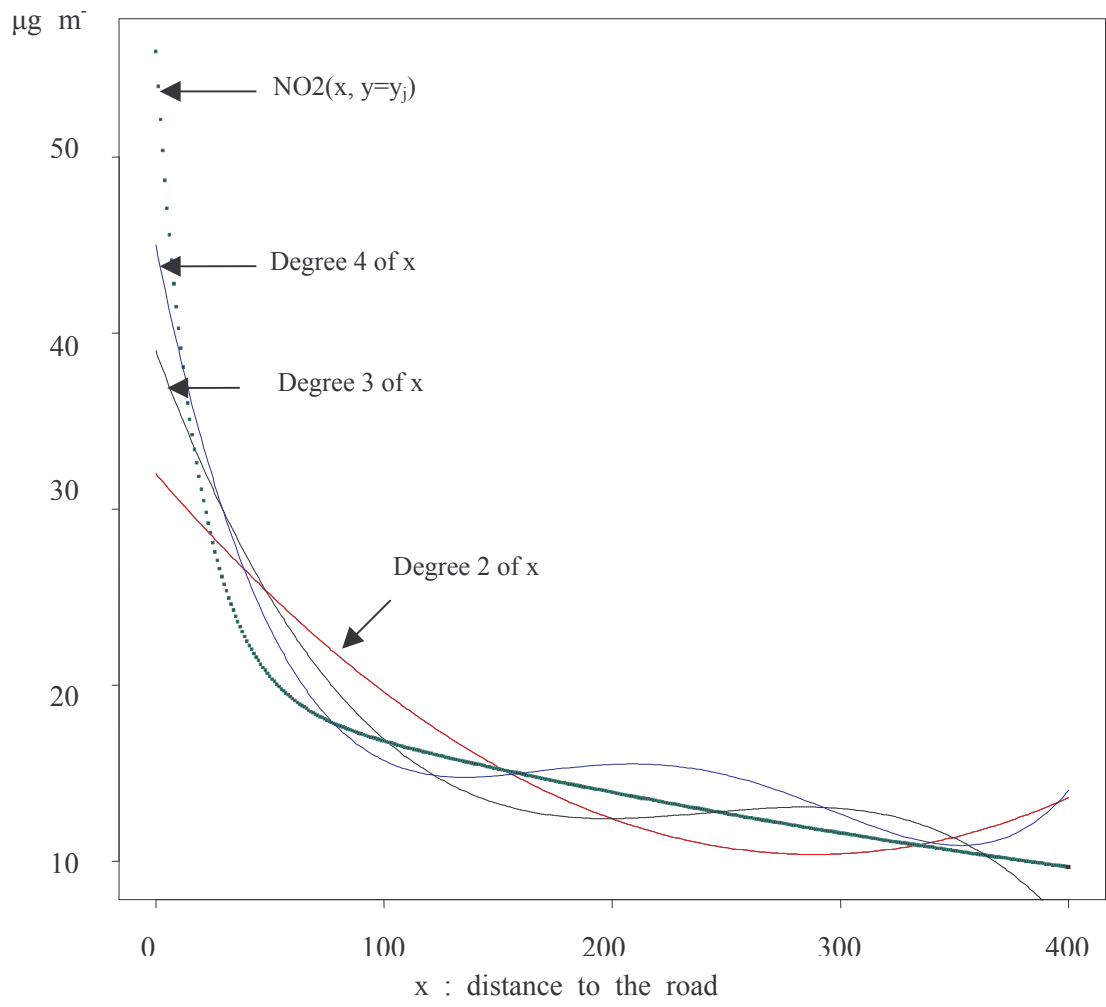
415 Fig. 5

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417 Fig. 6

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419 Fig. 7

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