Polyèdres et compilation

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- Find large grain data and task parallelism
  includes medium and fine grain parallelism

Still holding today for manycore and GPU computing!

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- But exact analyses when possible

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Polyhedral School of Fontainebleau... vs Polytope Model

- Summarization/abstraction vs exact information
- No restrictions on input code

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- And many more transformations:
  Control simplification, constant propagation, partial evaluation, induction variable substitution, scalarization, inlining, outlining, invariant generation, property proof, memory footprint, dead code elimination...

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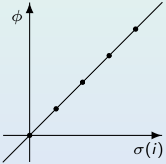
- Code synthesis

Simplified notations

- Identifiers i, a, Locations l or (a, ϕ), Values
- Environment: $\rho : Id \rightarrow Loc$
- Memory state: $\sigma : Loc \rightarrow Val$, $\sigma(i) = 0$
- Preconditions: $P(\sigma) \subset \Sigma$,
  `for(i=0; i<n; i++) {...}` $\longrightarrow \{\sigma \mid 0 \leq \sigma(i) < \sigma(n)\}$
- Transformers: $T(\sigma, \sigma') \subset \Sigma \times \Sigma$,
  `i++;` $\longrightarrow \{(\sigma, \sigma') | \sigma'(i) = \sigma(i) + 1\}$
- Array regions: $R : \Sigma \rightarrow Loc$,
  `a[i]` $\longrightarrow \sigma \rightarrow \{(a, \phi) \mid \phi = \sigma(i)\}$
- Statements $S, S_1, S_2 \in \Sigma \rightarrow \Sigma$
  function call, sequence, test, loop, CFG...
- Programs Π: a statement

Convex Array Region for a Reference



$W_{S_1}(\sigma) = \{(a, \phi) \mid 0 \leq \phi < 5\}$

```
for(i=0; i<5; i++)
```

$W_{S_2}(\sigma) = \{(a, \phi) \mid \phi = \sigma(i) \; [\; \wedge 0 \leq \sigma(i) < 5 \;] \; \}$

a[i]=0.;

Convex Array Regions of Statement S

Property of a written region W_S

$$\forall \sigma \quad \forall I \notin W_S(\sigma), \quad \sigma(I) = (S(\sigma))(I) \tag{1}$$

Note: The property holds for any over-approximation $\overline{W_S}$ of W_S.

Property of a read region R_S

$$\forall \sigma \quad \forall \sigma' \tag{2}$$

$$\forall I \in R_S(\sigma), \sigma(I) = \sigma'(I) \Rightarrow \begin{cases} R_S(\sigma) = R_S(\sigma') \\ W_S(\sigma) = W_S(\sigma') \\ \forall I \in W_S(\sigma), \big(S(\sigma)\big)(I) = \big(S(\sigma')\big)(I) \end{cases}$$

Note: The property holds for any over-approximation $\overline{R_S}$ of R_S in the left-hand side, but not for other over-approximations.

Conditions to exchange two statements S_1 and S_2

Evaluation of $S_1; S_2$: $\sigma \xrightarrow{S_1} \sigma_1 \xrightarrow{S_2} \sigma_{12}$

Assumptions:

$$\forall \sigma, \quad W_{S_1}(\sigma) \cap R_{S_2}(\sigma_1) = \varnothing \tag{3}$$

$$\forall \sigma, \quad W_{S_1}(\sigma) \cap W_{S_2}(\sigma_1) = \varnothing \tag{4}$$

Final state σ_{12}:

$$(3)\ \forall l \in R_{S_2}(\sigma_1), \quad l \notin W_{S_1}(\sigma) \stackrel{(1)}{\Longrightarrow} \sigma_1(l) = \sigma(l)$$

$$\stackrel{(2)}{\Longrightarrow} \left\{ \begin{array}{l} R_{S_2}(\sigma_1) = R_{S_2}(\sigma) \\ W_{S_2}(\sigma_1) = W_{S_2}(\sigma) \\ \forall l \in W_{S_2}(\sigma), \quad \sigma_{12}(l) = \sigma_2(l) \end{array} \right.$$

$$(4)\ \forall l \in W_{S_1}, \quad l \notin W_{S_2} \Longrightarrow \sigma_{12}(l) = \sigma_1(l)$$

$$\forall l \notin W_{S_1} \cup W_{S_2}, \quad \sigma(l) = \sigma_1(l) = \sigma_{12}(l)$$

Conditions to exchange two statements S_1 and S_2

Evaluation of $S_2; S_1$: $\sigma \xrightarrow{S_2} \sigma_2 \xrightarrow{S_1} \sigma_{21}$

Assumptions:

$$\forall \sigma, \quad W_{S_2}(\sigma) \cap R_{S_1}(\sigma_2) = \varnothing \tag{5}$$

$$\forall \sigma, \quad W_{S_2}(\sigma) \cap W_{S_1}(\sigma_2) = \varnothing \tag{6}$$

Final state σ_{21}:

$$(5) \forall I \in R_{S_1}(\sigma_2), \quad I \notin W_{S_2}(\sigma) \stackrel{(1)}{\Longrightarrow} \sigma_2(I) = \sigma(I)$$

$$\stackrel{(2)}{\Longrightarrow} \begin{cases} R_{S_1}(\sigma_2) = R_{S_1}(\sigma) \\ W_{S_1}(\sigma_2) = W_{S_1}(\sigma) \\ \forall I \in W_{S_1}(\sigma_2), \quad \sigma_{21}(I) = \sigma_1(I) \end{cases}$$

$$(6)\ \forall I \in W_{S_2}(\sigma), \quad I \notin W_{S_1}(\sigma_2) \Rightarrow \sigma_{21}(I) = \sigma_2(I)$$

$$\forall I \notin W_{S_1}(\sigma_2), \cup W_{S_2}(\sigma) \quad \sigma(I) = \sigma_2(I) = \sigma_{21}(I)$$

Bernstein's Conditions to Exchange S_1 and S_2

- Necessary condition:

$$\forall\sigma \quad let\ \sigma_1 = S_1(\sigma),\ \sigma_2 = S_2(\sigma)$$

$$\left.\begin{array}{l} W_{S_1}(\sigma) \cap R_{S_2}(\sigma_1) = \varnothing \\ W_{S_1}(\sigma) \cap W_{S_2}(\sigma_1) = \varnothing \\ W_{S_2}(\sigma) \cap R_{S_1}(\sigma_2) = \varnothing \\ W_{S_2}(\sigma) \cap W_{S_1}(\sigma_2) = \varnothing \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} W_{S_1}(\sigma) \cap R_{S_2}(\sigma) = \varnothing \\ W_{S_1}(\sigma) \cap W_{S_2}(\sigma) = \varnothing \\ W_{S_2}(\sigma) \cap R_{S_1}(\sigma) = \varnothing \end{array}\right.$$

according to the two previous slides.

Starting from Bernstein's conditions

- Let's assume: $\forall\sigma\ W_{S_1}(\sigma) \cap R_{S_2}(\sigma) = \varnothing$
- This implies by (1): $\forall\sigma\ \forall I \in R_{S_2} \quad (S_1(\sigma))(I) = \sigma(I)$
- Hence by (2): $\forall\sigma\ R_{S_2}(\sigma_1) = R_{S_2}(\sigma) \wedge W_{S_2}(\sigma_1) = W_{S_2}(\sigma)$

- In the same way: $\forall\sigma\ W_{S_2}(\sigma) \cap R_{S_1}(\sigma) = \varnothing$
- Implies: $R_{S_1}(\sigma_2) = R_{S_1}(\sigma) \wedge W_{S_1}(\sigma_2) = W_{S_1}(\sigma)$

So Bernstein's conditions are sufficient to prove:

$$\forall\sigma \quad (S_1; S_2)(\sigma) = (S_2; S_1)(\sigma)$$

Coarse grain parallelization of a loop

- Let's assume convex array regions R_B and W_B for the loop body
- Let P_B be the body precondition and $T^+_{B,B}$ the inter-iteration transformer
- Direct parallelization of a loop using convex array regions with Bernstein's conditions for the iterations of the body B:

  $\forall v \in Id \quad \forall \sigma, \sigma' \in P_B \; s.t. \; T^+_{B,B}(\sigma, \sigma')$

  $R_{B,v}(\sigma) \cap W_{B,v}(\sigma') = \varnothing$

  $R_{B,v}(\sigma') \cap W_{B,v}(\sigma) = \varnothing$

  $W_{B,v}(\sigma) \cap W_{B,v}(\sigma') = \varnothing$
- Note: $T^+_{B,B}(\sigma, \sigma') \Rightarrow \sigma(i) < \sigma'(i)$ where i is the loop index
- Each iteration can be interchanged with any other one.
- No dependence graph, no restrictions on loop body, no restriction on control, no restriction on references, no restriction on loop bounds...

Coarse Grain Parallelization of a Loop with Privatization

- Beyond Bernstein's conditions, use IN_B and OUT_B array regions instead of R_B and W_B regions
- Insure non-interference for interleaved execution: privatization or expansion for locations in $W_B - OUT_b$
- OUT_B can be over-approximated with $\overline{OUT_B}$ because it is used to decide the parallelization
- W_B cannot be overapproximated
- Must be combined with reduction detection

Fusion of Loops L_1 and L_2 with delay d

- `for(i1...)  S1; for(i2...)  S2`
- initial schedule:

$$S_1^0 \rightarrow S_1^1 \rightarrow S_1^2 \rightarrow S_1^3 \rightarrow S_1^4$$

$$S_2^0 \rightarrow S_2^1 \rightarrow S_2^2 \rightarrow S_2^3 \rightarrow S_2^4 \rightarrow$$

- new schedule:

$$S_1^0 \rightarrow S_1^1 \rightarrow S_2^0 \rightarrow S_1^2 \rightarrow S_2^1 \rightarrow S_1^3 \rightarrow S_2^2 \rightarrow S_1^4$$

- `for(...)  S1; for(...)  {S1;S2} for(...)  S2`

Fusion of Loops L_1 and L_2 with delay d: Legality

- Assumes convex array regions R_1 and W_1 for body B_1 of loop L_1 with index i_1, R_2 and W_2 for body B_2 of loop L_2 with index i_2
- Permutation of the last iterations of L_1 and the first iterations of L_2 with a delay d:

$$\forall \sigma_1 \ \forall \sigma_2 \quad P_1(\sigma_1) \wedge P_2(\sigma_2) \wedge T_{12}(\sigma_1, \sigma_2) \wedge \sigma_1(i_1) > \sigma_2(i_2) + d$$

$$\begin{array}{c} R_1(\sigma_1) \cap W_2(\sigma_2) = \varnothing \\ R_2(\sigma_2) \cap W_1(\sigma_1) = \varnothing \\ W_1(\sigma_1) \cap W_2(\sigma_2) = \varnothing \end{array}$$

- P_1, P_2, T_{12}, R_1, W_1, R_2, W_2 can be all over-approximated
- Check emptiness of convex sets for a polyhedral instantiation
- No restrictions on B_1 nor B_2 nor the loop index identifiers or ranges

Fusion of Loops L_1 and L_2 with delay d: Profitability

- Reduce memory loads:

$$\left(\bigcup_{\sigma_1 \in P_1} R_1(\sigma_1)\right) \bigcap \bigcup_{\sigma_2 \in P_2 \cap T_{1,2}(\sigma_1)} R_2(\sigma_2) \neq \varnothing$$

- Avoid intermediate store and reloads:

$$\left(\bigcup_{\sigma_1 \in P_1} W_1(\sigma_1)\right) \bigcap \bigcup_{\sigma_2 \in P_2 \cap T_{1,2}(\sigma_1)} R_2(\sigma_2) \neq \varnothing$$

- With minimal cache size:

$$\left| \left(\bigcup_{\sigma_1 \in P_1} (R_1(\sigma_1) \cup W_1(\sigma_1))\right) \bigcup \bigcup_{\sigma_2 \in P_2 \cap T_{1,2}(\sigma_1)} (R_2(\sigma_2) \cup W_2(\sigma_2)) \right|$$

Array privatization

- An array a is privatizable in a loop l with body B if

$$\forall \sigma \in P_B, \quad IN_{B,a}(\sigma) \;=\; OUT_{B,a}(\sigma) \;=\; \varnothing$$

- $IN_{B,a}$ is the set of elements of a whose input values are used in B. For a sequence `S1;S2`:

$$IN_{S_1;S_2} = IN_{S_1} \cup \Big(\big(IN_{S_2} \circ T_{S_1} \big) - W_{S_1} \Big)$$

- $OUT_{B,a}(\sigma)$ is the set of elements of a whose output values are used by the continuation of B executed in memory state σ. For a sequence `S1;S2`:

$$OUT_{S_1} = (OUT_{S_1;S_2} - W_{S_2} \circ T_{S_1}) \cup (W_{S_1} \cap IN_{S_2} \circ T_{S_1})$$

Examples of IN and OUT regions

- Source code for function foo

```
void foo(int n, int i, int a[n], int b[n]) {
  a[i] = a[i]+1;
  i++;
  b[i] = a[i]; }
```

- Source code for main:

```
int main() {
  int a[100], b[100], i;
  foo(100, i, a, b);
  printf("%d\n", b[0]); }
```

- *R*, *W*, *IN* and *OUT* array regions for call site to foo:

```
//  <a[PHI1]-R-EXACT-{i<=PHI1, PHI1<=i+1}>
//  <a[PHI1]-W-EXACT-{PHI1==i}>
//  <b[PHI1]-W-EXACT-{PHI1==i+1}>

//  <a[PHI1]-IN-EXACT-{i<=PHI1, PHI1<=i+1}>

//  <b[PHI1]-OUT-EXACT-{PHI1==0, PHI1==i+1}>

   foo(100, i, a, b);
```

Properties of IN regions

- If two states σ and σ' assign the same values to the locations in IN_S, statement S produces the same trace with σ and σ':

$$\forall \sigma \quad \forall \sigma' \tag{7}$$

$$\forall l \in IN_S(\sigma), \sigma(l) = \sigma'(l) \Rightarrow \begin{cases} R_S(\sigma) = R_S(\sigma') \\ W_S(\sigma) = W_S(\sigma') \\ IN_S(\sigma) = IN_S(\sigma') \\ \forall l \in W_S(\sigma), (S(\sigma))(l) = (S(\sigma'))(l) \end{cases}$$

- Almost identical to property for R regions
- But also $\forall \sigma \quad \forall \sigma'$:

$$\forall l \notin \bigcup_{\sigma \in P_S} \Big(R_S(\sigma) - IN_S(\sigma) \Big), \ \sigma(l) = \sigma'(l) \Rightarrow Equivalent_s(\sigma, \sigma')$$

Properties of OUT regions

- The values of variables written by S but not used later do not matter:

$$\begin{array}{l} \forall\sigma, \forall\sigma', \forall l \notin \bigcup_{\sigma\in P_S} \Big(W_S(\sigma) - OUT_S(\sigma)\Big), \\ (S(\sigma))(l) = (S(\sigma'))(l) \Rightarrow Equivalent_C(\sigma,\sigma') \end{array} \tag{8}$$

- In other words, statement S can be substituted by statement S' in Program Π if they only differ by writing different values in memory locations that are not read by the continuation

Scalarization

- Replace a set of array references by references to a local scalar:
  `a[j]=0; for(i...) { ... a[j]=a[j]*b[i];...}`
  $\rightarrow$ `s=0; for(i...) {... s*=b[i];... } a[j]=s;`
- Let B and i be a loop body and index, and W_B the write region function
- Sufficient condition: each loop iteration accesses only one array element
- Let
  $f : Val \rightarrow \mathcal{P}(\Phi)$ $s.t.$ $f(v) = \{\phi \mid \exists \sigma : \ \sigma(i) = v \wedge (a, \phi) \in W_B(\sigma)\}$
- If f is a mapping $Val \rightarrow \Phi$, array a can be replaced by a scalar.
- Initialization and exportation according to IN_B and OUT_B.

Statement Isolation

- Goal: replace S by a new statement S' executable with a different memory M':
  `i=i+1;`
  $\rightarrow$ {`int j; j=i;` **j=j+1;** `i=j;`}
- Let S be a statement with regions R_S, W_S, IN_S and OUT_s.
- Declare new variables $new(l)$ for $l \in \cup_{\sigma \in P_S}(R_S(\sigma) \cup W_S(\sigma))$
- Copy in: $\forall l \in IN_S(\sigma)\ M'[new(l)] = M[l]$
- Substitute all references to l by references to $new(l)$ in S
- Copy out: $\forall l \in OUT_S(\sigma)\ M[l] = M'[new(l)]$

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- Substitute all references to I by references to $new(I)$ in S
- Copy out: $\forall I \in OUT_S(\sigma)\ M[I] = M'[new(I)]$
- Copy out fails because of over-approximations of OUT_S!
- Copy in: $\forall I \in \overline{(IN_S(\sigma))} \cup \overline{(OUT_S(\sigma))}\ M'[new(I)] = M[I]$
- related to outlining and privatization and localization

Induction variable substitution

- Substitute k by its value, function of the loop index i:

  k=0; for(i=0;...) {k+=2; b[k] = ...}

  for(i=0;...) { b[2*i+2] = ...}

- Variable k can be substituted in statement S with precondition P_S within a loop of index i if P_S defines a mapping from $\sigma(i)$ to $\sigma(k)$:

$$v \rightarrow \{v' | \exists \sigma \in P_S \; \sigma(i) = v \wedge \sigma(k) = v'\}$$

Constant Propagation

- Replace references by constants:

  if(j==3) a[2*j+1]=0;

  if(j==3) a[7]=0;

- An expression e can be substituted under precondition P if:

$$|\{v \in Val | \exists \sigma \in P \ v = \mathcal{E}(e, \sigma)\}| = 1$$

- Simplify expressions:

  if(i+j==n) a[i+j]=0;

  if(i+j==n) a[n]=0;

Dependence Test for Allen&Kennedy Parallelization

- If you insist on:
  - using an algorithm with restricted applicability
  - reducing locality with loop distribution
- Use array regions to deal at least with procedure calls
- Dependence system for two regions of array a in statements S_1 and S_2 in a loop nest $\vec{\imath}$:

$$\begin{aligned}\{(\sigma_1,\sigma_2) \quad | \quad & \sigma_1(\vec{\imath}) \prec \sigma_2(\vec{\imath}) \wedge T_{S_1,S_2}(\sigma_1,\sigma_2) \wedge P_{S_1}(\sigma_1) \wedge P_{S_2}(\sigma_2) \\ & \wedge R^a_{S_1}(\sigma_1) \cap W^a_{S_2}(\sigma_2)\} = \varnothing \end{aligned} \tag{9}$$

- Useful for tiling, which includes all unimodular loop transformations

Dead code elimination

- Remove unused definitions:

  `int foo(int i) {int j=i+1; i=2; int k=i+1; return j;}`

  $\rightarrow$ `int foo(int i) {int j=i+1; return j;}`

- Useless? See some automatically generated code
- Useless? See some manually maintained code ☺
- Any statement S with no OUT_S region?
- Possible, but not efficient with the current semantics of OUT regions in PIPS

Code synthesis

Time-out!

- Declarations
- Control
- Communications
- Copy operations

Conclusion: simple polyhedral conditions in a compiler

- Difficulties hidden in a few analyses, available with PIPS:
  T, P, W, R, IN, OUT
- Legality of many program transformations can be checked with analyses:
  mapping, function, empty set,...
- Yes, quite often:
  Control simplification, constant propagation, partial evaluation, induction variable substitution, privatization, scalarization, coarse grain loop parallelization, loop fusion, statement isolation,...
- But not always: graph algorithms are useful too
  Dead code elimination,... wih OUT *regions?*

Conclusion: what might go wrong with polyhedra?

- The analysis you need is not available in PIPS:
  re-use existing analyses to implement it
- Its accuracy is not sufficient:
  implement a dynamic analysis (a.k.a. instrumentation)
- The worst case complexity is exponential:
  exceptions are necessary for recovery
- Monotonicity of results on space, time or magnitude exceptions:
  more work needed, exploit parallelism within PIPS
- Possible recomputation of analyses after each transformation:
  more work needed, composite transformations...

Conclusion: see what is available in PIPS!

- Many more program transformations
- Pointer analyses are improving
- Try PIPS with no installation cost: http://paws.pips4u.org
  On-going work... Do not overload our PIPS server ☺
- Or install it: http://pips4u.org
- Or install Par4all: http://www.par4all.org
- Or simply talk to us!

Questions?