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# HOW DID FUKUSHIMA-DAIICHI CORE MELTDOWN CHANGE THE PROBABILITY OF NUCLEAR ACCIDENTS?

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## Abstract

How to predict the probability of a nuclear accident using past observations? What increase in probability the Fukushima Dai-ichi event does entail? Many models and approaches can be used to answer these questions. Poisson regression as well as Bayesian updating are good candidates. However, they fail to address these issues properly because the independence assumption in which they are based on is violated. We propose a Poisson Exponentially Weighted Moving Average (PEWMA) based in a state-space time series approach to overcome this critical drawback. We find an increase in the risk of a core meltdown accident for the next year in the world by a factor of ten owing to the new major accident that took place in Japan in 2011.

*Keywords:* nuclear risk, nuclear safety, time series count data

## 1 Introduction

The triple core meltdown at the Fukushima Dai-ichi power plant on March 11, 2011 is the worst nuclear accident since the catastrophe at Chernobyl in 1986. It has aroused an old age debate: Is nuclear power safe? Critics claim that nuclear power entails a latent threat to society and we are very likely to witness an accident in the near future. Proponents say that the conditions that provoked the Fukushima disaster were unlikely, assert that new reactors can successfully face extreme conditions and conclude that the probability of a nuclear accident is very low.

Two months after the Fukushima Dai-ichi meltdown, a French newspaper published an article coauthored by a French engineer and an economist<sup>1</sup>. They both argued that the risk of a nuclear accident in Europe in the next thirty years is not unlikely but on the contrary, it is a certainty. They claimed that in France the risk is near to 50% and more than 100% in Europe.

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<sup>1</sup> Benjamin Dessus and Bernard Laponche, June 3, 2011, available at <http://www.liberation.fr/politiques/01012341150-accident-nucleaire-une-certitude-statistique>

Their striking result comes from dividing the number of reactor explosions (one in Chernobyl and 3 in Fukushima Dai-ichi) over cumulated experience (14.000 reactor-years) and multiplying this ratio by the number of reactors and 30 years. So if we take 58 operative reactors in France, we get 0.49 and if we consider 143 reactors in Europe we obtain 1.22; hence, their conclusion that a nuclear accident is a certainty in the European Union.

Their methodology is essentially flawed since the figures they found are larger than 1. On the other hand, the results found by nuclear probabilistic risk assessments (PRA, hereafter) have been quite optimistic with respect to what has been observed until now. According to EPRI (2008), the probability of a core meltdown damage type of accident in the US is of the order of  $2.0E-5$ . This means one expected event per 50.000 reactor years. As of today, one accounts 14500 operating years of nuclear reactors and 10 core meltdowns (see section 2). This implies an observed frequency of 1 accident per 1450 reactor years.

What seems to be clear from this debate is that assessing properly the probability of a nuclear accident with available data is key to clarify why there are such discrepancies and to shed light in the risks that nuclear power will entail on tomorrow.

Therefore, the main objective of this paper is to discuss different statistical approaches to estimate the probability of nuclear accidents. Our results suggest, that indeed there has been a sustained decrease in the frequency of core meltdowns as testified by PRAs made in different periods of time, but this feature challenges the assumptions that underline the Poisson model that is typically used to estimate the probability of accidents.

To deal both with the discrete and temporal nature of the data, we propose to use a Poisson Exponentially Weighted Moving Average (PEWMA) that is based in a state-space time series framework developed by Harvey and Fernandes (1989) to estimate core melt's arrival rate.

The remainder of this paper is structured as follows. The next section outlines the economic literature about nuclear risk assessment. Section three investigates how to estimate the probability of a nuclear accident using successively a frequentist and a Bayesian approach. Section four presents the PEWMA model and its results. Section five concludes.

## 2 Literature review

It is possible to distinguish two approaches to assess the probability of nuclear accidents: PRA models and statistical analysis.

PRA models describe how nuclear reactor systems will respond to different initiating events that can induce a core meltdown after a sequence of successive failures. This methodology estimates the core damage frequency (CDF, hereafter)<sup>2</sup> based on observed and assumed probability distributions for the different parameters included in the model.

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<sup>2</sup> PRA can be done at 3 levels. The first one considers internal or external initiating events followed by a series of technical and human failures that challenge the plant operation and computes the CDF as a final outcome. Level 2 evaluates how the containment structures will react after an accident, at this level the Large Early Release Frequency (LERF) is computed. The last level determines the frequencies of human fatalities and environmental contamination.

The use of PRAs in commercial reactors is a common practice in the U.S, because they are a key input for the risk-based nuclear safety regulation approach (Kadak and Matsuo (2007)). The Nuclear Regulatory Commission (NRC, hereafter) has been doing PRA studies since 1975, when the so called WASH-1400 was carried out. This first study estimated a CDF equal to  $5E-05$  and suggested an upper bound of  $3E-04$ .

The lessons from Three Mile Island accident and the ensuing improvements in PRA techniques allowed the NRC to perform a PRA for 5 units in 1990; the average CDF found was  $8.91E-05$ .

It was not until 1997, that the NRC published the Individual Plant Examination Program NUREG-1560, which contains the PRA results, including CDFs, for all the 108 commercial nuclear power plants in the U.S.

According to EPRI (2008) report, this metric of safety has shown a decreasing trend from 1992 to 2005 due to safety enhancements that have induced significant reductions.

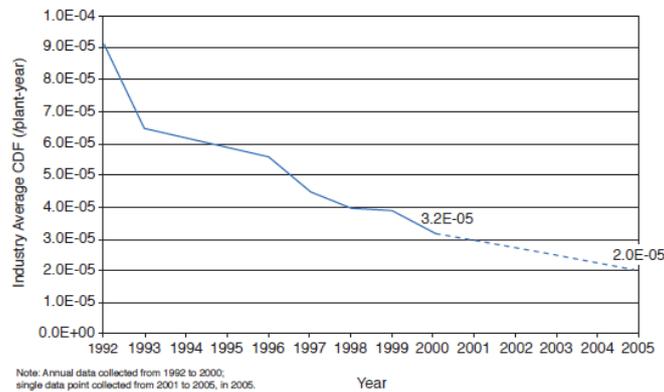


Figure 1: Core Damage Frequency Industry Average Trend (EPRI, 2008)

This paper belongs to the statistical approach of assessing the probability of nuclear accident based on historical observations. One problem is that as nuclear accidents are rare events, data is scarce.

Hofert and Wüthrich (2011) is the first attempt in this category. The authors used Poisson MLE to estimate the frequency of annual losses derived from a nuclear accident. The total number of accidents is computed as a Poisson random variable with constant arrival rate  $\lambda$ .

The authors recognized that  $\lambda$  estimates change significantly depending on the time period taken. This suggests that there are doubts about the assumption of a non-time dependent arrival rate.

If the events are correlated somehow, Poisson results are no longer valid. Hence it is not suitable to ignore the temporal nature of the data. Our paper tries to fill this gap in the literature. We propose to use a structural-time series approach that has been developed by Harvey and Fernandes (1989) and used in political science by Brandt and Williams (1998), but is yet to be used to assess nuclear risks.

## 3 How to model the probability of a nuclear accident?

### 3.1 Data

Our first task is to define the scope of our study: which events can be considered as nuclear accidents. In general, the criterion used to classify them is the magnitude of their consequences, which is closely linked with the amount of radioactive material that is released outside the unit.

For instance, the INES scale<sup>3</sup> ranks nuclear events into 7 categories. The first 3 are labeled as incidents and only the last 4 are considered as accidents. The *major nuclear accident* is rated as 7 and is characterized by a large amount of radioactive material release. Accidents like Chernobyl and Fukushima Dai-ichi are classified within this category, whereas Three Mile Island event is classified as *accident with wider consequences* (Level 5) because it had very limited release.

The definition of nuclear accident is an important empirical issue because as it gets narrower, the number of observed events is reduced. According to Cochran (2011) only 9 nuclear commercial reactors<sup>4</sup> have experienced partial core meltdown, including fuel damage, from 1955 to 2010; while Sovacool (2008) accounts 67 nuclear accidents from 1959 to 2011. However his list includes other types of accident in addition to core meltdowns<sup>5</sup>

This implies a trade-off between estimation reliability and the significance of the results. If we take a broader definition of accidents, we have more information in our sample; therefore, we can be more confident of the results, but what we can infer about the probability of a major nuclear accident is much more limited. On the contrary, if we restrict our attention only to the most dramatic accidents, we only have 2 cases (i.e., Chernobyl and Fukushima Dai-ichi) which undermines the degrees of freedom of any statistical model.

In order to avoid the sparseness issue that arises when we consider only major nuclear accidents and in order to be able to compare our results with the CDF computed in the PRA studies, we take core meltdowns with or without releases as the definition of nuclear accident.

Another reason to choose this scope is that the magnitude of internal consequences of a core meltdown (i.e., the loss of the reactor and its clean-up costs) do not differ much between a major and less severe accidents<sup>6</sup>. Then knowing the probability of a core meltdown as we have defined, could be useful to design insurance contracts to hedge nuclear operators against the internal losses that will face in case of any of these events.

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<sup>3</sup> This scale was designed in 1989 by a group of international experts in order to assure coherent reporting of nuclear accidents. In the scale each increasing level represents an accident approximately ten times more severe than the previous level. See <http://www-ns.iaea.org/tech-areas/emergency/ines.asp>

<sup>4</sup> See Table 7 in the Appendix

<sup>5</sup> This list was set to compare major accidents within the energy sector. This goal explains why the nuclear accidents sub-list is far to be comprehensive, representative (e.g., nuclear incidents in the USA are over-represented) and consistent.

<sup>6</sup> By contrast, the damages in case of radioactive elements releases in the environment could vary in several orders of magnitude depending on the meteorological conditions and the population density in the areas affected by the nuclear cloud

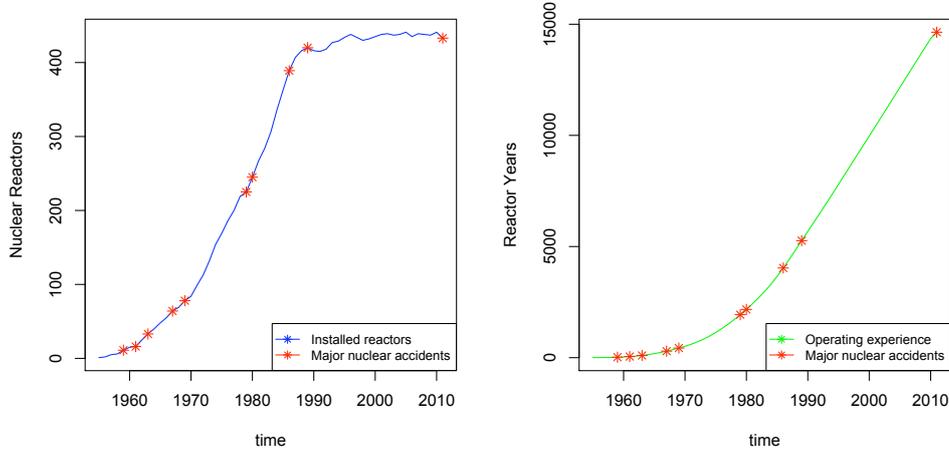


Figure 2: Major nuclear accidents, reactors and operating experience 1955-2011

Figure 2 plots the number of events collected by Cochran (2011) as well as the number of installed reactors and cumulated nuclear operating experience (in reactor years) published in the PRIS<sup>7</sup> database. These variables will be used in our estimation.<sup>8</sup>

At first glance, we can conclude that sparseness remains an issue even if the scope of accidents that we have taken is broader. It also seems that PRA results are not completely wrong, since most of the accidents were recorded at the early stages of nuclear power history. It is important to highlight that during the period in which nuclear experience grew exponentially (i.e., over the last 10 years), only (and fortunately) the Fukushima Dai-ichi meltdowns have been observed, which intuitively suggests that there has been a safety improvement trend.

Our second issue is to parametrize our problem. We have to assume that accidents are realizations that follow some known stochastic process.

Within the discrete probability functions, Poisson process is the best fit for our problem because it models the probability of a given number of discrete *events* occurring in a continuous but fixed interval of time. In our case, the "events" correspond to core meltdowns and the interval time to one year.

**Assumption 1:** Let  $y_t$  denote the number of nuclear accidents observed at year  $t$ . Assume that  $y_t$  is a Poisson random variable with an arrival rate  $\lambda$  and density function given by:

$$f(y_t|\lambda) = \frac{(\lambda E_t)^{y_t} \exp(-\lambda E_t)}{y_t!} \quad (1)$$

<sup>7</sup> The Power Reactor Information System

<sup>8</sup> Is important to mention that we have considered Fukushima Dai-ichi as one rather than three accidents

Given one year is the time interval chosen, we have to offset the observations by the exposure time in within this interval.  $E_t$  corresponds to the number of operative reactors at year  $t$  and it is the exposure time variable in our model. It is important to note that it is not the same to observe one accident when there is only one reactor in operation, than with 433 reactors working.

Under Assumption 1 our problem is fully parametrized and we only need to know  $\lambda$  in order to compute the probability of a nuclear accident at time  $t$ . Before we proceed further, we want to reiterate the necessary conditions for a count variable to follow Poisson distribution:

- i. The probability of two simultaneous events is negligible
- ii. The probability of observing one event in a time interval is proportional to the length of the interval
- iii. The probability of an event within a certain interval does not change over different intervals
- iv. The probability of an event in one interval is independent of the probability of an event in any other non-overlapping interval

For the case of nuclear accidents, assumptions i and ii do not seem far from reality. Even if in Fukushima Dai-ichi the events occurred the same day, the continuous nature of time allows us to claim that they did not happen simultaneously; and it is not unreasonable to expect that as we reduce the time interval, the likelihood of an accident goes down.

On the contrary, conditions iii and iv might be more disputable. For the moment we will assume that our data satisfy these assumptions and we will deal with both of them in section 4.

## 3.2 Frequentist approach

As mentioned before, knowing  $\lambda$  is enough to assess the probability of a nuclear accident under Assumption 1. The frequentist approach consist in using the observations in the sample  $(\{y_t\}_{t=0}^T, \{E_t\}_{t=0}^T)$  to compute the MLE estimate for our parameter of interest  $\lambda$ . In our case  $\lambda$  equals the cumulative frequency.

Table 1: Poisson with constant arrival rate (1955-2011)

Database	$\hat{\lambda}$	Coefficients	Estimate Std.	Error z value	$\Pr(>  z )$
Corchan	0.000667022	-7.3127	0.3162	-23.12	<2e-16 ***

If we compute  $\hat{\lambda}$  using the information available up to year 2010, we can define  $\Delta$  as the change in the estimated arrival rate to measure how Fukushima-Daiichi affected the rate.

$$\Delta = \frac{\hat{\lambda}_{2011} - \hat{\lambda}_{2010}}{\hat{\lambda}_{2010}} \quad (2)$$

Using this model, the Fukushima accident represented an increase in the arrival rate equal to  $\Delta = 0.07904$ . In other words, after assuming a constant world fleet of 433 reactors on the planet from 2010 to 2012, this model predicts the probability of a core meltdown accident as  $6.175 \cdot 10^{-4}$  in the absence of Fukushima Dai ichi accident. On including this accident, the probability jumps to  $6,667 \cdot 10^{-4}$  (i.e., about a 8% increase) because of the new catastrophe in Japan.

It is important to recognize that the simplicity of this model comes with a high price. First, it only takes into account the information that we have in our sample; given that what we have is predominantly zeros, it would be suitable to incorporate other sources of knowledge about the drivers of the frequency of a core meltdown. In fact, it seems odd to compute a predictive probability of accident that is only based on a few historical observations whereas there exists a huge amount of knowledge brought by nuclear engineers and scientists over the past 40 years on the potential causes that may result in major accidents, especially core meltdowns. Second, the change in the number of operating commercial reactors is the only considered time-trend. Our computation assumes as if all the reactors built over the past 50 years are the same and their safety features invariant with time. This basic Poisson model does not allow us to measure if there has been a safety trend that reduces the probability of a nuclear accident progressively as the PRA studies shown. We will try to address these limitations in the following methods.

### 3.3 Bayesian approach

A second alternative to assess the estimation of  $\lambda$  is to consider it as a random variable. Under this approach observations are combined with a prior distribution, denoted by  $f_0(\lambda)$  that encodes all the knowledge that we have about our parameter, using Bayes' law.

$$f(\lambda|y_t) = \frac{f(y_t|\lambda)f_0(\lambda)}{\int f(y_t|\theta)f_0(\theta)d\theta} \quad (3)$$

Equation 3 shows the updating procedure for a continuous random variable. Note that once we have updated  $f_0(\lambda)$  using the available information in  $\{y_t\}_{t=0}^T$  we can use mean of the posterior distribution  $f(\lambda|y_t)$  in Equation 1 to compute the probability of an accident.

The choice the prior distribution has always been the central issue in Bayesian models<sup>9</sup>. Within the possible alternatives, the use of a conjugate prior has two important advantages: it has a more clear cut interpretation and it is easier to compute.

Since we have already assumed that the accidents come from a Poisson distribution, we will assume that  $\lambda$  follows the conjugate distribution for this likelihood that is Gamma with parameters  $(a, b)$ .

**Assumption 2:** The arrival rate  $\lambda$  is a Gamma random variable with parameters  $(a, b)$  and density function given by:

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<sup>9</sup> For an extended discussion on the prior's selection, see Carlin and Louis (2000) or Bernardo and Smith (1994)

$$f_0(\lambda) = \frac{\exp(-b\lambda)\lambda^{a-1}b^a}{\Gamma(a)} \quad (4)$$

Given the properties of the gamma distribution, it is possible to interpret the chosen parameters. The intuition behind this updating procedure is the following: Prior to collecting any observations, our knowledge indicates that we can expect to observe  $a$  number of accidents in  $b$  reactor years. This gives us an expected rate equal to  $a/b$  which is the mean of gamma distribution.

The parameter that reflects how confident we are about this previous knowledge is  $b$ . Given that it is closely related with the variance ( $V(\lambda) = a/b^2$ ), the greater  $b$  is the more certain we are of our prior.

Once we have collected observations,  $y_t$  accidents in  $E_t$  reactor years, we can update our prior following a simple formula:

$$a_u = a + y_t \quad (5)$$

$$b_u = b + E_t \quad (6)$$

Which gives a new expected rate given by  $a_u/b_u$ .

PRA estimates have valuable information to construct the prior. For instance, if we use the upper bound found in WASH-1400 report that is a CDF equal to 3E-04 it means approximately 1 accident over 3.500 years, which can be expressed in terms of prior parameters as ( $a = 1, b = 3.500$ ). Likewise we can take the 8.91E-05 found in the NUREG-1150, which can be expressed as a pair of prior parameters ( $a = 1, b = 10.000$ ).

If we consider the first PRA as a good starting point, this will reflect not only a higher expected arrival rate but also that we are highly uncertain about the value of  $\lambda$ . Thus the results will be mainly driven by the observations and will quickly converge towards the results of the frequentist approach. In fact, if we use the ( $a = 1, b = 3.500$ ) as a prior, the Poisson-gamma model predicts an expected arrival rate for 2011 equal to  $\hat{\lambda}_{2011} = 5.99\text{E-}0.4$  and when we compute  $\Delta$  we find that the Fukushima Dai-ichi accident represented an increase of 7.09% ; these figures are quite similar of those we have obtained with the frequentist model.

However, if we take the NUREG-1150 study as a source for the prior we stay more stuck to the prior and need a larger number of observations to move far from it. With this information we find an expected arrival rate equal to  $\hat{\lambda}_{2011} = 4.3\text{E-}4$  and  $\Delta = 8\%$ .

Results in Figure 3 seem to show that there has been a sustained decrease in the expected rate of nuclear accidents over the past 10 years. To test if this has been the case, we will define  $\lambda$  as a function of a time trend. Note that such an approach challenges what is stated in condition (iii) because we are allowing that the arrival rate changes in each time interval (i.e., one year in our case) .

### 3.4 Poisson regression models

In this section we define the arrival rate as function of time in order to try to capture the safety and technological enhancements in nuclear power industry.

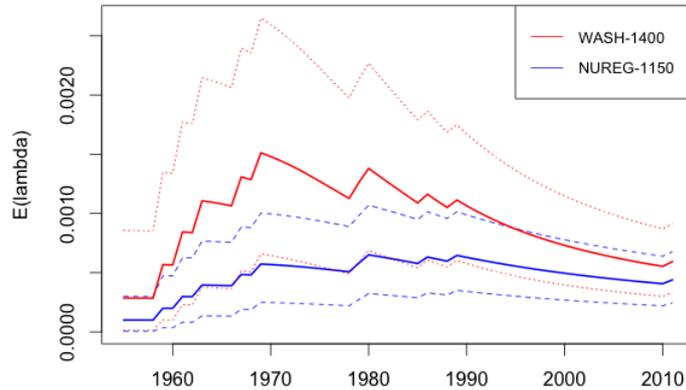


Figure 3: Bayesian updating with Poisson-Gamma model

**Assumption 3:** The systematic component (mean)  $\lambda_t$  is described by a logarithmic link function given by:

$$\lambda_t = \exp(\beta_0 + \beta_1 t) \quad (7)$$

Table 2: Poisson with deterministic time trend

	Coefficients	Estimate Std.	Error z value	Pr(>  z )	
Intercept	221.88694	55.78873	3.977	6.97e-05	***
time	-0.11548	0.02823	-4.091	4.29e-05	***

Results in Table 2 are MLE estimates for  $(\beta_0, \beta_1)$ . The time trend coefficient confirms the hypothesis that the probability of a nuclear accident has been reduced along this period. As expected, the estimate is negative and significant. Therefore, if the effect of regulatory strictness, technological improvements and safety investments can be summarized in this variable, we have found some evidence that supports the safety trend that PRA studies have shown.

In this case, the  $\hat{\lambda}_{2011}=3.20E-0.5$  is closer to the estimated CDF of last PRA studies. When we compute the Fukushima Dai-ichi effect on the arrival rate we find an increase of  $\Delta=2.3$ . Unlike in the previous two models this increase is significant.<sup>10</sup>

Summing up, we have found results supporting the idea of a decreasing arrival rate like in the PRA studies, however this challenges condition (iii).

<sup>10</sup>The Poisson regression can also be done under a Bayesian approach, nevertheless the prior is not as easy to construct as in the previous case, because in this setting the coefficients  $(\beta_0, \beta_1)$  are the random variables. Generally it is assumed that they come from a normal distribution  $\beta \sim N_k(b_0, B_0)$ . Where  $k = 2$  (in our case) is the number of explanatory variables;  $b_0$  is the vector of prior means and  $B_0$  is the variance-covariance matrix. Then to construct the prior will require a lot of assumptions not only in the level and precision of the coefficients, but also in how they are related. Given that precisely what we want is to relax as much assumptions as possible, we are not going to consider such a model.

Now we discuss the no correlation hypothesis (i.e., condition iv). At first glance one could say that nuclear accidents fulfill this condition. TMI is not related with what happened in Chernobyl, etc. Nevertheless we are using the sum of nuclear accidents in each year from 1959 to 2011 and from one year to another a significant part of the fleet is the same because nuclear reactors have a long life. Therefore, at some point our time series reflects how the state of technology and knowledge linked with nuclear safety has evolved during this period.

Given that we are using a time series, we have to check if there is a stochastic process that governs nuclear accidents' dynamics. This means to know if accidents are somehow correlated, or if instead we can assume that they are independent events (condition iv). To test this we propose to use a time series state-space approach.

Why conditions (iii) and (iv) are relevant? As in the classical regression analysis, the MLE Poisson estimation assigns an equal weight to each observation because under this assumption, all of them carry the same information. However, if observed events are somehow correlated with those in previous periods, some of them bring more information about the current state than others, therefore past observations have to be discounted accordingly.

## 4 Poisson Exponentially Weighted Average (PEWMA) model

Although Autoregressive Integrated Moving Average (ARIMA, hereafter) models are the usual statistical framework to study time series, there are two limitations to use them in the case of nuclear accidents.

First, it is inappropriate to use them with discrete variables because these models are based on normality assumptions. The second limitation (Harvey and Fernandes (1989)) is related to the fact that nuclear accidents are rare events. This implies that the mean tends to zero and that the ARIMA model could result in negative predictions, which is incompatible with Assumption 1 (Brandt and Williams (1998)).

To deal with these two limitations, Harvey and Fernandes (1989) and Brandt and Williams (1998) propose to use a structural time series model for count data. This framework has a time-varying mean as in the ARIMA models and allows for a Poisson or a negative binomial conditional distribution  $f(y_t|\lambda_t)$ .

The idea is to estimate the coefficients in the exponential link as in a Poisson regression, but each observation is weighted by a smoothing parameter; for this reason the model is called Poisson Exponentially Weighted Moving Average (PEWMA).

We briefly describe below the structural time series approach<sup>11</sup>, following what has been developed by Brandt and Williams (1998) and (2000).

The model is defined by three equations: measurement, transition and a conjugate prior. The first is the model's stochastic component, the second shows how the state  $\lambda_t$  changes over time and the third is the element that allows to identify the model.

Under Assumption 1, the measurement equation  $f(y_t|\lambda_t)$  is given by Equation 1. In this setting  $\lambda_t$  has two parts, the first shows the correlation across time and the

<sup>11</sup>Harvey and Shepard (1993) have elaborated a complete structural time series statistical analysis

second captures the effect of the explanatory variables realized in the same period, that corresponds to an exponential link  $\exp(X_t'\beta)$  like in the previous model.

**Assumption 3':** The mean  $\lambda_t$  has an unobserved component given by  $\lambda_{t-1}^*$  and an observed part denoted  $\mu_t$  described by link Equation 7.

$$\lambda_t = \lambda_{t-1}^* \mu_t \quad (8)$$

At its name implies, the transition equation describes the dynamics of the series. It shows how the state changes across time.

**Assumption 4:** Following Brandt and Williams (1998), we assume that the transition equation has a multiplicative form given by:

$$\lambda_t = \lambda_{t-1} \exp(r_t) \eta_t, \quad (9)$$

Where  $r_t$  is the rate of growth and  $\eta_t$  is a random shock that follows a Beta distribution.

$$\eta_t \sim \text{Beta}(\omega a_{t-1}; (1 - \omega) a_{t-1}) \quad (10)$$

As in a linear moving average model, we are interested in knowing for how long random shocks will persist in the series' systematic component. In PEWMA setting,  $\omega$  is precisely the parameter that captures persistence. It shows how fast the mean moves across time; in other words, how past observations should be discounted in future forecast. Smaller values of  $\omega$  means higher persistence, while values close to 1 suggests that observations are highly independent.

The procedure to compute  $f(\lambda_t|Y_{t-1})$  corresponds to an extended Kalman filter<sup>12</sup>. The computation is recursive and makes use of the advantages of the Poisson-Gamma Bayesian model that we discussed in the previous section.

The idea of the filter is the following. The first step consists in deriving the distribution of  $\lambda_t$  conditional on the information set up to  $t-1$ . We get it by combining an unconditional prior distribution  $\lambda_{t-1}^*$  that is assumed to be a Gamma, with the transition Equation in 9.

Using the properties of the Gamma distribution we obtain  $f(\lambda_t|Y_{t-1})$  with parameters  $(a_{t|t-1}, b_{t|t-1})$

$$\lambda_t|Y_{t-1} \sim \Gamma(a_{t|t-1}, b_{t|t-1}) \quad (11)$$

Where:

$$\begin{aligned} a_{t|t-1} &= \omega a_{t-1} \\ b_{t|t-1} &= \omega b_{t-1} \exp(-X_t'\beta - r_t) \end{aligned}$$

The second step consists in updating this prior distribution using the information set  $Y_t$  and Bayes' rule. This procedure give us a posterior distribution  $f(\lambda_t|Y_t)$ . As we have seen in the previous section, the use of the conjugate distribution results in a Gamma posterior distribution and the parameters are updated using a simple updating formula.

<sup>12</sup>The derivation of this density function is described in Brandt and Williams (1998)

$$\lambda_t|Y_t \sim \Gamma(a_t, b_t) \tag{12}$$

Where:

$$\begin{aligned} a_t &= a_{t|t-1} + y_t \\ b_t &= b_{t|t-1} + E_t \end{aligned}$$

This distribution becomes the prior distribution at  $t$  and we use again the transition equation and so on and so forth.

#### 4.1 PEWMA Results

Given that we are not going to use covariates, the only parameter to estimate is  $\omega$ . We use PEST<sup>13</sup> code, although some modifications have been made to include the number of reactors as an offset variable. This model needs prior parameters, and we choose ( $a = 1, b = 10.000$ ).

The estimates obtained with this model are summarized in Table 3.

Table 3: PEWMA model

	Model 1
$\hat{\omega}$	0.814
(s.e)	(0.035)
Z-score	23.139
LL	-37.948
AIC	75.896

Note that  $\hat{\omega}$  estimate is less than 1. If we use a Wald statistic to test independence, we reject the null hypothesis, thus Poisson regression and Bayesian updating method are not suitable to assess the probability of a nuclear accident because the independence assumption is violated. This means that past observations influence the current state, specially the more recent ones.

Table 4: Test for reduction to Poisson

	Model 1
Wald test for $\omega=1$	27.635
Wald p-value	1.46e-07

Our results mean that operational, technological and regulatory changes aimed to increase nuclear safety have taken their time and have impacted technical characteristics of the nuclear fleet progressively. It is undeniable that what nuclear operators, regulators and suppliers knew during the early 60's has evolved substantially, (i.e better operation records, new reactor designs) and the experience

<sup>13</sup>The code is available in <http://www.utdallas.edu/~pbrandt/pests/pests.htm>

gained from the observed accidents induced modifications that ended up reducing core meltdowns frequency.

If we use a narrower definition of nuclear accident (i.e nuclear accident with large releases), we will tend to accept independence assumption, because what the data reflect the particular conditions in which these accidents occurred. On the contrary, if we take a broader set we will tend to estimate a lower  $\omega$ , because data contain more information about nuclear safety as a whole. For instance, if we use Sovacool (2008) registered accidents as dependent variable, we find the results in Table 5.

Table 5: PEWMA model with broader definition of accident

	Model 1
$\hat{\omega}$	0.684
(s.e)	(0.039)
Z-score	17.299
LL	-69.626
AIC	139.253

Note that the  $\hat{\omega}$  obtained taking into account more accidents is smaller than the one that we estimated with our definition of nuclear accident.

## 4.2 Changes in $\hat{\omega}$

The predictions at each  $t$  in the filter for  $\lambda_t$  vary substantially depending on  $\omega$ . If all the events are independent  $\omega \rightarrow 1$ , they are considered equally important in the estimation, past accidents are as important as recent ones. As  $\omega$  gets smaller, recent events become more important than those observed in the past. The latter are therefore discounted.

Figure 4 plots the mean of  $f(\lambda_t|Y_t)$  for different values of  $\omega$ . Note that for higher values, the changes are more subtle because the effect of a new accident is diluted over the cumulated experience. When we discount past events, the observation in the latest periods are the most important, that is the reason why the changes in the expected arrival rate are more drastic than in the Poisson model.

The model estimates a  $\hat{\lambda}_{2010}= 4.42\text{e-}05$  and  $\hat{\lambda}_{2011}=1.954\text{e-}3$  which implies a  $\Delta=43.21$ .

As in figure 2, the evolution in the number of accidents is broadly featured with two periods. Until 1970 the arrival rate grows quickly to a very high peak. This is because there were several accidents between 1960 and 1970 and also the worldwide nuclear fleet stayed small. After 1970, one accounts several accidents but the operating experience has considerably increased. Between 1970 and 2010, one observes a decreasing trend for the arrival rate even though this period is featured with the most well known accidents of Three Mile Island and Chernobyl.

The Fukushima Dai-ichi results in a huge increase in the probability of an accident. The arrival rate in 2011 is again similar to the arrival rate computed in 1980 after the Three Mile Islands and the Saint Laurent des Eaux core meltdown. In other terms, Fukushima Dai-ichi accident has increased the nuclear risk for the near future in the

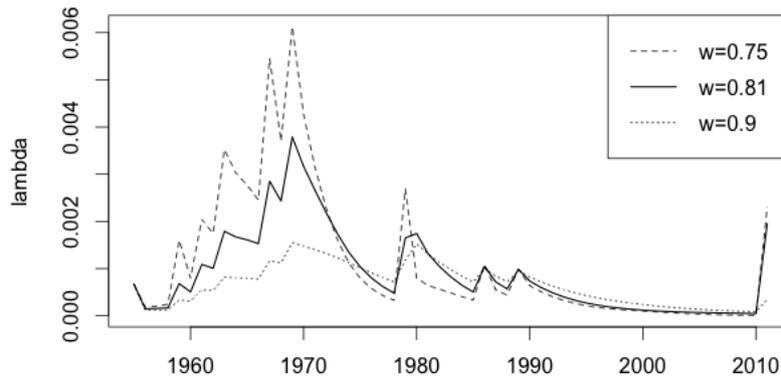


Figure 4: PEWMA with different  $\omega$

same extent it has decreased over the past 30 years owing to safety improvements. In informational terms, the adding of a new observation (i.e., new accident) is equivalent to the offsetting of decades of safety progress.

The Fukushima Dai-ichi effect of  $\Delta 43$  could appear as not realistic. In fact, at first glance the triple meltdown seems very specific and caused by a series of exceptional events (i.e., a seism of magnitude 9 and a tsunami higher than 10 meters). For most observers, however, the Fuskushima Dai-ichi accident is not a black swan<sup>14</sup>. Seisms followed by a wave higher than 10 meters have been previously documented in the area. It is true that they were not documented when the Fukushima Dai-ichi power plant was built in 1966. The problem is that this new knowledge appeared in the 80s and has then been ignored, deliberately or not, by the operator. It has also been ignored by the nuclear safety agency NISA because as well-demonstrated now the Nippon agency was captured by the nuclear operators (Gundersen (2012)).

Unfortunately, it is likely that several NPPs in the world have been built in hazardous areas, have not been retrofitted to take into account better information on natural risks collected after their construction, and are under-regulated by a non-independent and poorly equipped safety agency as NISA. The Fukushima Dai-ichi accident entails general lessons regarding the under-estimation of some natural hazardous initiating factors of a core meltdown and the over confidence on the actual control exerted by safety agency on operators. Note also that such an increase seems more aligned with a comment made after the Fukushima Dai-ichi accident: a nuclear catastrophe can even take place in a technologically advanced democracy. It means that a massive release of radioactive elements from a nuclear power plant into the environment is no longer a risk limited to a few unstable countries where scientific knowledge and technological capabilities are still scarce

The 10 times order of magnitude of the Fuskushima Dai-ichi effect computed by us could therefore be realistic.

<sup>14</sup>See FukushimaDai-ichi NPS Inspection Tour (2012)

## 5 Conclusion

Results in Table 6 recapitulates the different numbers for the arrival rate and the Fuskushima Dai-ichi effect we obtained in running the 4 models.

Table 6: Summary of results

Model	$\hat{\lambda}_{2010}$	$\hat{\lambda}_{2011}$	$\Delta$
MLE Poisson	6.175e-04	6.66e-04	0.0790
Bayesian Poisson-Gamma	4.069e-04	4.39e-04	0.0809
Poisson with time trend	9.691e-06	3.20e-05	2.303
PEWMA Model	4.420e-05	1.95e-03	43.216

The basic Poisson model and the Bayesian Poisson Gamma model provide similar figures. According to both models, the Fuskushima Dai-ichi effect is negligible. The reason is that both models do not take any safety progress into account. The arrival rate is computed as if all the reactors have been the same and identically operated from the first application of nuclear technology to energy to this day. This is a strong limitation because it is obvious that over the past 50 years the design of reactors and their operation have improved. The current worldwide fleet has a higher proportion of safer reactors than the 1960's fleet.

By contrast, the introduction of a time-trend into the Poisson model has led to a high decrease in the 2010 arrival rate. The time trend captures the effect of safety and technological improvements in the arrival rate. Being significant this variable allows to predict a lower probability of nuclear accident. If we use a Poisson regression, we find a tripling of the arrival rate owing to the Fukushima Dai-ichi accident. Note that both findings seem more aligned with common expertise. In fact, the probability of an accident by reactor-year as derived by the 2010 arrival rate is closer to PRAs' studies. Moreover, a significant increase in the arrival rate because of Fukushima Dai-ichi meltdowns fits better with the features of this accident and its context.

Nevertheless, the Poisson regression is based on independent observations. The PEWMA model has allowed us to test the validity of this hypothesis and we find that it is not the case. This seems to be more suitable because nuclear reactors have a long lifetime and the construction of new reactors, investments in safety equipment, learning process and innovations take time to be installed and adopted by the whole industry. Of course, the dependence hypothesis leads to a higher arrival rate because it introduces some inertia in the safety performances of the fleet. A new accident reveals some risks that have some systemic dimension.

In PEWMA models the Fukushima Dai-ichi event represents a very substantial, but not unrealistic increase in the estimated rate. By construction each observation in this model does not have the same weight. Past accidents are discounted more than new accidents. This captures the idea that lessons from past accidents have been learnt in modifying operations and design and that a new accident reveals new safety gaps that will be progressively addressed later in order to avoid a similar episode.

An issue that remains unresolved is the heterogeneity among the accidents. Although TMI, Chernobyl and Fukushima were core melt downs, both the causes and the consequences differed from one to another. A step further in addressing correctly

the probability of nuclear accidents, would consist in using a weighting index that reflects each event magnitude.

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## Appendix A List of nuclear accidents involving core meltdown

Table 7: Partial core melt accidents in the nuclear power industry

Year	Location	Unit	Reactor type
1959	California, USA	Sodium reactor experiment	Sodium-cooled power reactor
1961	Idaho, USA	Stationary Low-Power Reactor	Experimental gas-cooled, water moderated
1966	Michigan, USA	Enrico Fermi Unit 1	Liquid metal fast breeder reactor
1967	Dumfrieshire, Scotland	Chapelcross Unit 2	Gas-cooled, graphite moderated
1969	Loir-et-Chaire, France	Saint-Laurent A-1	Gas-cooled, graphite moderated
1979	Pennsylvania, USA	Three Mile Island	Pressurized Water Reactor (PWR)
1980	Loir-et-Chaire, France	Saint-Laurent A-1	Gas-cooled, graphite moderated
1986	Pripyat, Ukraine	Chernobyl Unit 4	RBKM-1000
1989	Lubmin, Germany	Greifswald Unit 5	Pressurized Water Reactor (PWR)
2011	Fukushima, Japan	Fukushima Daiichi Unit 1,2,3	Boiling Water Reactor (BWR)

Cochran (2011)