A Real-Time Robust SLAM for Large-Scale Outdoor Environments

Jianping Xie, Fawzi Nashashibi, Michel Null Parent, Olivier Garcia-Favrot

A REAL-TIME ROBUST SLAM for LARGE-SCALE OUTDOOR ENVIRONMENTS

Jianping Xie, Fawzi Nashashibi, and Michel Parent

INRIA Rocquencourt

78153 Le Chesnay Cedex, France

TEL: +33 139635789, FAX: +33 139635995, E-mail: {jianping.xie, fawzi.nashashibi, michel.parent}@inria.fr

Olivier Garcia Favrot

INDUCT Recherche And Développement

135 rue du Fossé Blanc 92230 Gennevilliers

TEL : +33 147949360 FAX : +33 147601633, E-mail: garciafavrot@hotmail.com

ABSTRACT

The problem of simultaneous localization and mapping (SLAM) is still a challenging issue in large-scale unstructured dynamic environments. In this paper, we introduce a real-time reliable SLAM solution with the capability of closing the loop using exclusive laser data. In our algorithm, a universal motion model is presented for initial pose estimation. To further refine robot pose, we propose a novel progressive refining strategy using a pyramid grid-map based on Maximum Likelihood mapping framework. We demonstrate the success of our algorithm in experimental result by building a consistent map along a 1.2 km loop trajectory (an area about 100,000 m^2) in an increasingly unstructured outdoor environment, with people and other clutter in real time.

INTRODUCTION

The Simultaneous Localization and Mapping (SLAM) problem is one of the fundamental issues in mobile robotics. SLAM requires a mobile robot to increasingly build a consistent map of an unknown environment using on-board sensors while concurrently localizing itself relative to this map. A solution to the SLAM problem has been regarded as an important prerequisite for autonomous robots as it would provide the means to make a robot truly autonomous under unknown environments. The SLAM problem has been intensively studied by researchers over past decades and many successful results are presented in the existed literature (1), (2).

SLAM approaches can be roughly classified according to the map representation and estimation algorithm. Popular methods for representing map of the environments include:

1

feature-based approach (3), grid-based approach (4), and topological approach (5). Considering the estimation algorithm for SLAM, The most popular approach is the Extended Kalman Filter (EKF-SLAM) (1). The effectiveness of the EKF approach comes from the fact that it estimates a fully correlated posterior over feature maps and robot poses. However, the EKF-SLAM algorithm suffers from its computational complexity and incorrect data association problem. It is clear that EKF-SLAM has quadratic complexity with respect to the size of the map. The solution of the EKF-SLAM is inconsistent due to its linearization approximation which induces inaccurate maps with filter divergence (6). Many approaches have been developed to overcome these shortcomings: Martinez-Cantin and Castellanos (7) introduced the Unscented Kalman Filter (UKF) (8)-(10) to the SLAM problem for outdoor environments. This approach avoids the analytical Taylor expansion based linearization of the nonlinear models and improves the consistency over the EKF-based approach. Rao-Blackwellized Particle Filter (RBPF) proposed by Murphy (11) is an effective approach for learning grid maps by decoupling pose state and the map. Based on the RBPF framework, FastSLAM (12), (13) uses particle filtering to address non-linearity and factorization to avoid large state vectors. Extended Information Filter (EIF) (14) has been used as a recursion for the inverse of the covariance matrix which has been shown to be approximately sparse. However, enabling real-time SLAM implementation in an increasingly unstructured large-scale outdoor environment is still a great challenge. The loop-closure problem, when a robot returns back to the same place after a large traverse, is especially difficult (1).

In this paper, we present a high-efficient robust SLAM for large-scale dynamic outdoor environments in real time using only laser sensor data. To estimate robot's movement, we introduce a universal motion model without any kinematics or dynamics knowledge of the robot. Thus enables us to perform our SLAM algorithm on different mobile platforms easily. Instead of using the ICP-based method like in (15) for robot localization, we propose a novel progressive scan matching strategy based on a pyramid grid-map which does not need to establish correspondence between feature and landmark in the map. More importantly, our matching approach greatly improves the localization accuracy so as to keep pose error away from growing without bound.

The rest of the paper is organized as follows. First we briefly review the SLAM problem in general probabilistic terms. Then we discuss more detailed information of motion estimation and scan matching approach for our SLAM. In turn, the real experimental results in large-scale outdoor scenarios are given to validate the SLAM algorithm. Finally, we present the conclusion and future work.

GENERAL FORM OF PROBABILISTIC SLAM

SLAM is a process in which a mobile robot can build the map M of an unknown environment and at the same time uses this map to recover its pose x_t. The sensor measu rement at time t is denoted by z_t and $z_{1:t} = \{z_1, z_2, \dots, z_t\}$ is the set of all measurements up to time t. Furthermore, the control is denoted by u_t which determines the changes of state in the time interval $(t, t+1)$. The set $u_{1:t}$ is the sequence of robot actions. The SLAM algorithm calculates the joint posterior over the past observations and controls:

$$P(x_t, M \mid z_{1:t}, u_{1:t}, x_0) \tag{1}$$

To compute the joint posterior, it requires a motion model and a measurement model which are respectively describing the effect of the control input and sensor observation. The robot motion model is defined as $P(x_t, \mid x_{t-1}, u_t)$. The pose x_t is a probabilistic distribution of the current robot control u_t and the previous pose x_{t-1} under the Markov assumption. Moreover, the sensor measurement model $P(z_t, \mid M, x_t)$ describes the probability of observing z_t when the robot pose x_t and the map M are known. In general, the recursive Bayesian formulation of the SLAM algorithm can be written as follows in two steps (1).

1) Time-update:

$$P(x_t, M \mid z_{1:t-1}, u_{1:t}, x_0) = \int P(x_t \mid x_{t-1}, u_t) \times P(x_{t-1}, M \mid z_{1:t-1}, u_{1:t-1}, x_0) dx_{t-1} \tag{2}$$

2) Measurement Update:

$$P(x_t, M \mid z_{1:t}, u_{1:t}, x_0) = \frac{P(z_t \mid x_t, M) P(x_t, M \mid z_{1:t-1}, u_{1:t}, x_0)}{P(z_t \mid z_{1:t-1}, u_{1:t}, x_0)} \tag{3}$$

$$P(x_t, M \mid z_{1:t}, u_{1:t}, x_0) = \eta P(z_t \mid x_t, M) P(x_t, M \mid z_{1:t-1}, u_{1:t}, x_0) \tag{4}$$

Finally, the joint posterior $P(x_t, M \mid z_{1:t}, u_{1:t}, x_0)$ can be obtained through a recursive procedure of Eq. (2) and Eq. (4) wherein η is a normalizer.

LOCALIZATION AND MAPPING

The SLAM problem stated in previous section could be treated as a Maximum Likelihood estimation problem (16). We employ a maximum likelihood SLAM approach for the matching process. As a good robot pose is obtained, we are able to incrementally build a consistent environment map when new measurements arrive. First, we introduce the motion estimation approach and then discuss the map representation method. After that, we present a matching approach based on the Maximum Likelihood framework using the grid map. Finally, a pyramid grid-map based progressive refining strategy is proposed to further improve the localization accuracy.

MOTION ESTIMATION

To model robot motion, we simply assume that the relative movement at time t-1 equals the one at time t, i.e., movement continuity property, $\triangle x_t = \triangle x_{t-1}$. This is because in most cases the relative robot movements will not suddenly suffer a ridiculous enormous change especially when the system update rate is high. We also have $v_t = v_{t-1}$, $w_t = w_{t-1}$, where v_t is the translational velocity and w_t is the rotational velocity. In order to avoid confusion, the pose state x_t is denoted by vector $(xp, yp, \theta p)^T$. The pose estimation is presented as:

$$x_t^* = x_{t-1} + \triangle x_t \tag{5}$$

By applying the continuity property to Eq. (5) which leads to:

$$x_t^* = x_{t-1} + \triangle x_{t-1} \tag{6}$$

Simply we calculate the motion model as follows: (η' is a normalizer)

$$P(x_t \mid x_{t-1}, u_t) = \eta' P(xp_t - xp_t^*) \cdot P(yp_t - yp_t^*) \cdot P(\theta p_t - \theta p_t^*) \tag{7}$$

where

$$P(xp_t - xp_t^*) = \frac{1}{\sqrt{2\pi\delta_{xp}^2}} e^{-\frac{(xp_t - xp_t^*)^2}{2\delta_{xp}^2}} \tag{8}$$

$$P(yp_t - yp_t^*) = \frac{1}{\sqrt{2\pi\delta_{yp}^2}} e^{-\frac{(yp_t - yp_t^*)^2}{2\delta_{yp}^2}} \tag{9}$$

$$P(\theta p_t - \theta p_t^*) = \frac{1}{\sqrt{2\pi\delta_{\theta p}^2}} e^{-\frac{(\theta p_t' - \theta p_t^*)^2}{2\delta_{\theta p}^2}} \tag{10}$$

The variances δ_{xp}, δ_{yp} and $\delta_{\theta p}$ are given by $\delta_{xp} = \lambda_1 v$, $\delta_{yp} = \lambda_2 v$ and $\delta_{\theta p} = \lambda_3 w$, λ_i ($i = 1,2\ 3$) is just the parameter. Yet, the rough estimated pose x_t^* is inaccurate. To refine the robot pose, we generate the candidate pose space by sampling the motion model $P(x_t, \mid x_{t-1}, u_t)$ with the sampling algorithm similar to (17), (18). The motion model and its sampling version are depicted in Fig. 1. Note that Fig. 1 is projected into x-y-space which lacks a dimension corresponding to robot's orientation. The refined robot pose will be found among candidate poses during the matching process.
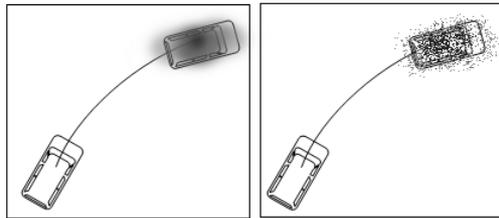

Fig. 1. Motion model $P(x_t, \mid x_{t-1}, u_t)$ of the robot (left) and its sampling version (right).

PERCEPTION REPRESENTATION

For sensor perception, we use occupancy grid map to represent the environment. Compared with the previous developed feature-based approach in (3), the grid map can be used to represent any environment. In addition, it is particularly suitable to deal with the uncertainty of sensor data collected from outdoor environments. The grid-based approach also allows integrating different sensors in the same framework with the consideration of the inherent sensor errors (18). This feature enables us to integrate other sensor data in future although we only utilize the laser data for the moment.

In this representation, the robot environment map M is discretized into two-dimensional square cells and each cell is associated with a value in [0, 1] indicating the probability of the cell. Fig. 2 shows an example of occupancy grid map. The grey-level in the occupancy map indicates the posterior of occupancy: The higher value of the grid cell, the darker a grid cell is and the more likely it is occupied.
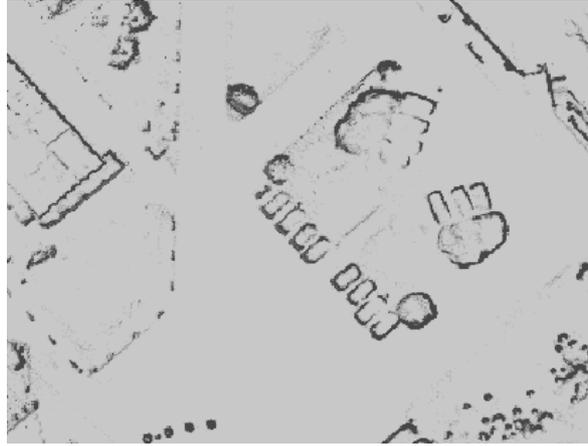

Fig. 2. Occupancy grid map.

Assuming these grid cells are independent and the poses $x_{0:t}$ are known. Given observations $z_{1:t}$, the posterior probability $P(m \mid x_{0:t}, z_{1:t})$ for each grid cell m is determined by using Bayes theorem:

$$P(m \mid x_{0:t}, z_{1:t}) = \frac{P(z_t \mid x_{0:t}, z_{1:t-1}, m) P(m \mid x_{0:t}, z_{1:t-1})}{P(z_t \mid x_{0:t}, z_{1:t-1})} \tag{11}$$

Assume that z_t is independent from $x_{0:t-1}$ and $z_{0:t-1}$ at given m:

$$P(m \mid x_{0:t}, z_{1:t}) = \frac{P(m \mid x_t, z_t) P(z_t \mid x_t) P(m \mid x_{0:t}, z_{1:t-1})}{P(m) P(z_t \mid x_{0:t}, z_{1:t-1})} \tag{12}$$

By analogy, we have the opposite event:

$$P(\overline{m} \mid x_{0:t}, z_{1:t}) = \frac{P(\overline{m} \mid x_t, z_t) P(z_t \mid x_t) P(\overline{m} \mid x_{0:t}, z_{1:t-1})}{P(\overline{m}) P(z_t \mid x_{0:t}, z_{1:t-1})} \tag{13}$$

Dividing Eq. (12) by Eq. (13) leads cancellation of various difficult-to-calculate probabilities:

$$\frac{P(m \mid x_{0:t}, z_{1:t})}{P(\overline{m} \mid x_{0:t}, z_{1:t})} = \frac{P(m \mid x_t, z_t)P(\overline{m})P(m \mid x_{0:t}, z_{1:t-1})}{P(\overline{m} \mid x_t, z_t)P(m)P(\overline{m} \mid x_{0:t}, z_{1:t-1})} \tag{14}$$

Log odds ratio term in Eq. (15) is a common technique to solve above calculation.

$$\log(Odds(x)) = \log(\frac{P(x)}{P(\overline{x})}) = \log(\frac{P(x)}{1 - P(x)}) \tag{15}$$

Implementing log odds ratio for Eq. (14) leads to an elegant recursive formula in log-odds term:

$$\log(Odds(m \mid x_{0:t}, z_{1:t}))$$
$$= \log(Odds(m \mid x_t, z_t)) - \log(Odds(m)) + \log(Odds(m \mid x_{0:t-1}, z_{1:t-1})) \tag{16}$$

where $P(m)$ is the prior occupancy probability of a grid cell which normally is set to 0.4-0.6. The remaining probability $P(m \mid x_t, z_t)$ is called the inverse sensor model. We adopt a similar model according to (18). Posterior probability $P(m \mid z_{1:t}, x_{0:t})$ can be recovered easily since the Odds($m \mid z_{1:t}, x_{0:t}$) can be computed recursively by Eqs. (16), (17) and (18).

$$Odds(m \mid x_{0:t}, z_{1:t}) = \frac{P(m \mid x_{0:t}, z_{1:t})}{1 - P(m \mid x_{0:t}, z_{1:t})} \tag{17}$$

$$P(m \mid x_{0:t}, z_{1:t}) = (1 + \frac{1}{Odds(m \mid x_{0:t}, z_{1:t})})^{-1} \tag{18}$$
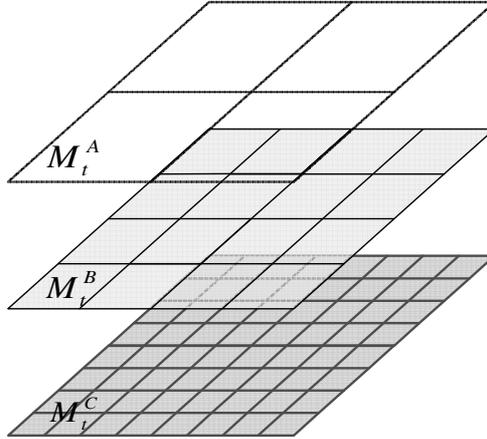

Fig. 3. The pyramid grid-map: multi-resolution grid maps for the same area.

In real implementation, instead of using one fixed resolution grid map, we perform a pyramid grid-map which contains multi-resolution grid maps for representing the identical physical area. The pyramid map representation is depicted in Fig. 3.

MAXIMUM LIKELIHOOD MAPPING
Generating the good localization is crucial for building a consistent map of the environment. The rough pose estimation from previous discussion is inaccuracy due to its simple motion

model. If the drift of robot pose could not be properly corrected, the pose error would increase without bound. We introduce a grid-based scan matching approach under the maximum likelihood mapping framework to correctly refine the robot pose. The matching problem can be considered as a maximum likelihood problem (16), (18). Rather than matching only two consecutive scans, we refine the robot pose by comparing the current laser scan with the existed grid map. The t–th pose is now obtained as the maximum likelihood estimation:

$$\hat{x}_t = \arg \max_{x_t} \left\{ P(z_t \mid x_t, M_{t-1}) \cdot P(x_t \mid \hat{x}_{t-1}, u_t) \right\} \tag{19}$$

The resulting pose $\hat{x}_t$ is used to update map M_t according to Eqs. (16) and (18):

$$M_t = M_{t-1} \cup \left\{ z_t, \hat{x}_t \right\} \tag{20}$$

By alternating the process of pose refining and map updating, the robot simultaneously improves its localization and environment map. In Eq. (19), the term $P(z_t \mid x_t, M_{t-1})$ is the measurement model which is the probability of the current measurement z_t given the pose x_t and the map M_{t-1}. The left term is motion model which is already known from previous description. To compute $P(z_t \mid x_t, M_{t-1})$, we adopt a high-efficient method (18). It only focuses on the grid cells which are hit by the end-points of laser beams. A voting scheme is used to calculate the probability. First, we denote the current laser scan z_t which contains N individual measurements corresponding to N laser beams by $z_t = \{z_t^1, z_t^2, ..., z_t^N\}$. Each z_t^K ($K = 1, ..., N$) is projected into the global coordinate space in the map. The grid cells corresponding to the projected end-points is called hit(k). If this cell is occupied, a sum proportional to the occupancy value of the cell is voted. The posterior probability of occupancy of the grid cell m_i at time t is denoted by $P(m_t^i)$. Then a final voted score represents the likelihood of the whole scan measurements:

$$P(z_t \mid x_t, M_{t-1}) \propto \sum_{k=1}^{N} \left\{ P(m_{t-1}^{hit_t(k)}), \ m_{t-1}^{hit_t(k)} \text{ is occupied} \right\} \tag{21}$$

We implement the maximization of Eq. (19) by performing an extensive search over candidate pose space which generated from motion model part. The resulting pose is the pose at which the measurement probability achieves a maximum value. The most likely robot pose is recovered when the current laser scan correctly aligned with existing map.

PROGRESSIVE REFINING STRATEGY

In order to further improve the mapping result, we apply a progressive refining strategy using the pyramid grid-map based on previous mapping framework: As Fig. 4 shows, the mapping process first takes place in low-resolution grid map M_t^A (20×20cm/pixel). The first rough estimated pose $x_t^{*(1)}$ and candidate pose space $x_t^{'(1)}$ are obtained by using motion model and the sampling algorithm. The refined pose $x_t^{(1)}$ is found by performing maximum likelihood mapping algorithm. So far, the pose refining process is finished in the first-level map, and then we enter a higher resolution map M_t^B (10×10cm/pixel). The same mapping procedures are carried out in the second map starting as $x_t^{*(2)} = x_t^{(1)}$. It means that we regard the previous refined pose as the raw estimation in current level map. Again, after the mapping process is completed, we come into a higher resolution grid map to continue our mapping algorithm. Finally the best robot pose is recovered as $x_t^{(3)}$ in the highest resolution map M_t^C (5×5cm/pixel).
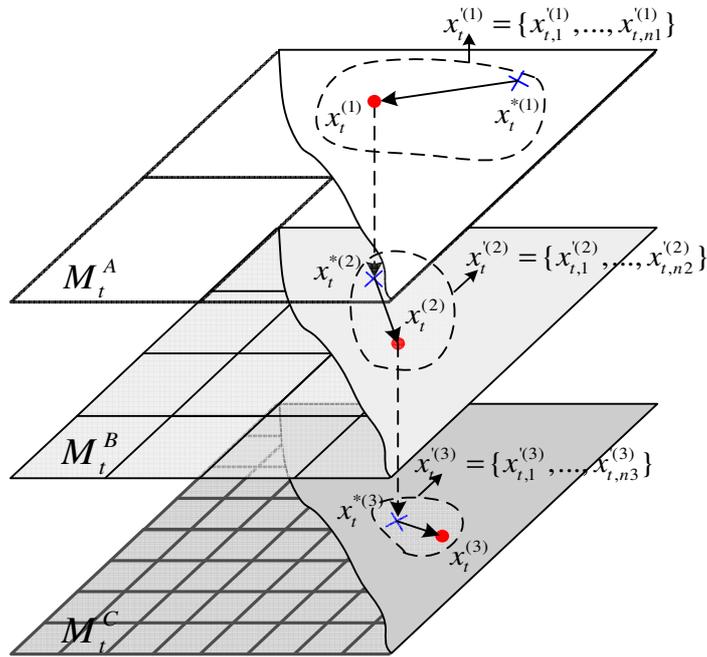

Fig. 4. Progressive refining strategy using the pyramid grid-map.

Note that the sampling candidate pose space area is reduced during the progressive mapping process (see Fig. 4). To prevent the mapping process from unexpected mismatching, we examine the corrections of the poses and set the restriction as $|\triangle x_t^{(j)}| < |\triangle x_t^{(k)}| \, (j > k)$, where $\triangle x_t^{(i)} = x_t^{(i)} - x_t^{*(i)} \, (i = 1, 2, 3)$ denotes the pose correction. The matching process will stop

if the relationship of corrections goes against the restriction, for instance: $|\triangle x_t^{(3)}| > |\triangle x_t^{(2)}|$, the matching process ends and $x_t^{(2)}$ returns as the finally result.

PERFORMANCE EVALUATION

The proposed SLAM algorithm has been implemented and applied to different scenarios and demos, one of which is described in detail in this section.

EXPERIMENT CONFIGURATION

The experiment vehicle shown as Fig. 5 is a fully autonomous vehicle equipped with two Alasca IBEO laser scanners. Each laser has 4 layer scan planes, a 240° field of view and a maximum range of 200 m. Both of them are mounted at the left and right front corners respectively. The frequency of the dual laser system is set to 12.5 Hz.

Fig. 5. Dual laser system of AGV.

EXPERIMENT RESULTS

In order to validate the SLAM algorithm, a number of experiments have been carried out. In the experiment as shown in Fig. 6, the vehicle travels from the initial position A and loops back to the position B. Note that the experiment area covers a range about 100,000 m^2 and the traveling distance is over 1.2 km along with the perimeter of a cluster of buildings, car parking slots, large playground, and tennis court.

Fig. 7 shows the environment map constructed by the conventional maximum likelihood mapping approach using 20×20cm/pixel resolution grid map which suffers from cyclic environments (16). For instance, the pose drift could not be revised correctly during the mapping process thus the estimation errors could be gradually increased without bound. As presented in Fig. 7, the conventional method is unable to build a consistent map, e.g., especially, for the loop closure problem. The loop part in Fig. 7 is zoomed up as shown in Fig. 9(a) which indicates that the map is inconsistent and completely distorted when the vehicle

revisits the same place.

Fig 6. Aerial photo of INRIA Rocquencourt.
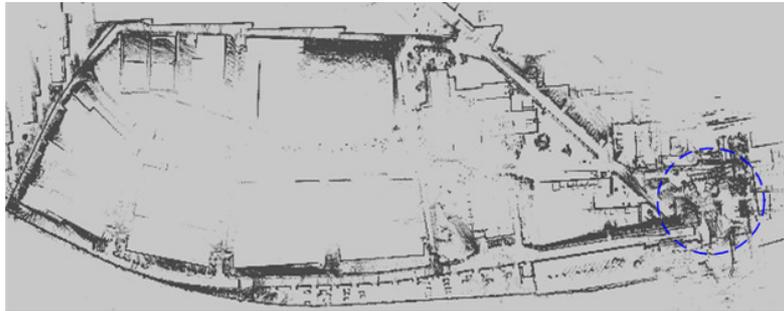
Fig 7. Result of SLAM generated by conventional method
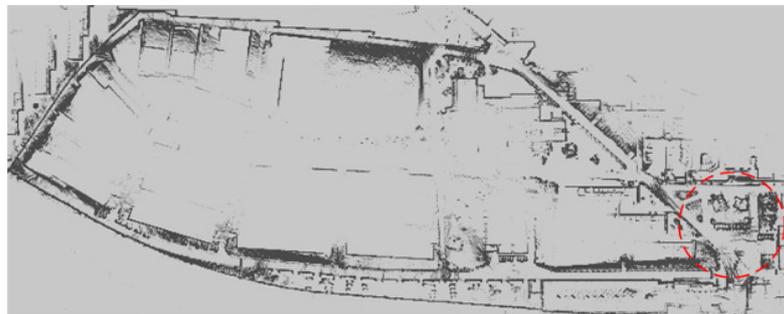

Fig 8. Result of SLAM generated by proposed method

However, as shown in Fig. 8 and Fig. 9(b), the environment map built by the proposed algorithm in which the map is consistently generated and the loop is almost seamlessly closed. Comparing to the conventional maximum likelihood mapping method, the progressive refining strategy based SLAM algorithm achieves higher localization accuracy and improved map consistency. This is because that 1) the proposed algorithm provides a flexible mechanism to describe the uncertainty of sensor perception using the pyramid grid-map presented in Fig. 3; 2) the proposed one is also capable to control the estimation errors within a certain range thanks to the progressive refining strategy presented in Fig. 4. For instance, the conventional maximum likelihood mapping method fails to reduce robot pose drift in cyclic environments, whereas the robot pose is gradually refined and recovered in the proposed one as shown in Fig. 9(b).
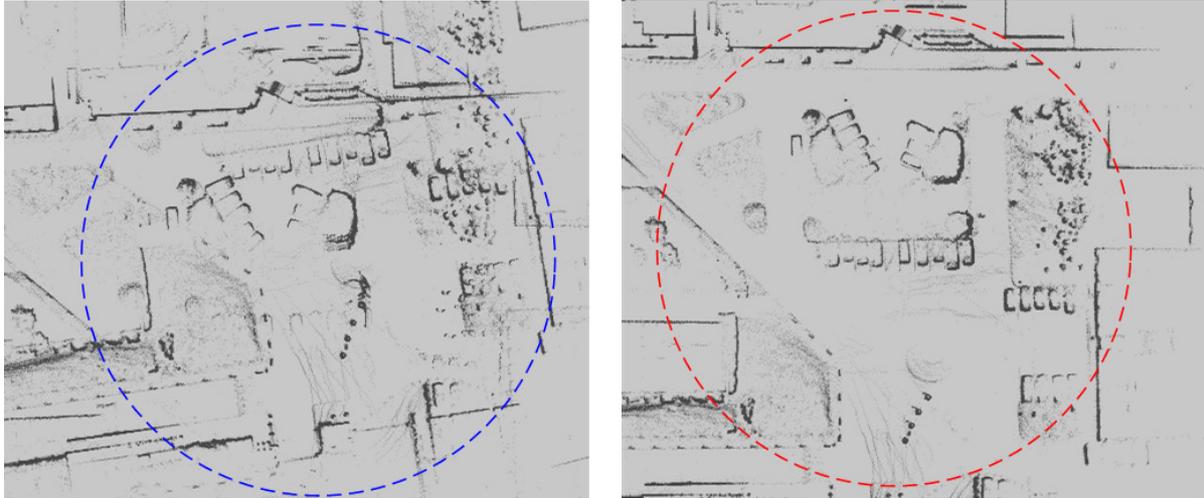
| (a) The conventional method | (b) The proposed method |

Fig 9. Zoomed area of the loop parts

CONCLUSIONS AND FUTURE WORKS

In this paper we have presented a high-efficient reliable SLAM for outdoor application and shown it working on a challenging large-scale dynamic environment in real time. For perception representation, we proposed a pyramid grid-based map with different resolution scales. By applying a novel progressive refining strategy, our SLAM algorithm is able to incrementally build a consistent map for large-scale outdoor environment and close the loop. However, the pyramid grid-map representation and progressive refining process require more computational and memory resources comparing to traditional method. Improving computational efficiency and reducing memory cost are still very challenging. Moreover, future works including moving objects detection and tracking algorithm are needed to achieve further improvements.

ACKNOWLEDGMENT

REFERENCES

(1) Durrant-Whyte, H., and Bailey, T., "Simultaneous Localization and Mapping: Part I," IEEE Robotics & Automation Magazine, Jun. 2006, vol. 13, no. 2, pp. 99–110.

(2) Bailey, T., and Durrant-Whyte, H., "Simultaneous Localization and Mapping (SLAM): Part II," IEEE Robotics & Automation Magazine, Sept. 2006, vol. 13, no. 3, pp. 108–117.

(3) Leonard, J.J., and Durrant-Whyte, H., "Simultaneous Map Building and Localization for an Autonomous Mobile Robot," In Proc. of IEEE/RSJ International Workshop on Intelligent

Robots and Systems, Nov. 1991, pp. 1442-1447.

(4) Elfes, A., "Occupancy grids: a probabilistic framework for robot perception and navigation," PhD thesis, Carnegie Mellon University, 1989.

(5) Choset, H., and Nagatani, K., "Topological Simultaneous Localization and Mapping (SLAM): Toward Exact Localization Without Explicit Localization," IEEE Transactions on Robotics and Automation, Apr. 2001, vol. 17, no. 2, pp. 125–137.

(6) Kim, C., Sakthivel, R., and Chung, W. K., "Unscented FastSLAM: A Robust and Efficient Solution to the SLAM Problem," IEEE Transactions on Robotics, Aug. 2008, vol. 24, no. 4, pp. 808–820.

(7) Martinez-Cantin, R., and Castellanos, J. A., "Unscented SLAM for Large-Scale Outdoor Environments," In Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems, Aug. 2005, pp. 3427-3432.

(8) Julier, S., Uhlmann, J., and Durrant-Whyte, H., "A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators," IEEE Transactions on Automatic Control, Mar. 2000, vol. 45, no. 3, pp. 477–482.

(9) Julier, S., and Uhlmann, J., "A Counter Example to the Theory of Simultaneous Localization and Map Building," In Proc. of IEEE International Conference on Robotics and Automation, May 2001, pp. 4238-4243.

(10) Julier, S., and Uhlmann, J., "Unscented Filtering and Nonlinear Estimation," In Proc. of the IEEE, Mar. 2004, vol. 92, no. 3, pp. 401–422.

(11) Murphy, K., "Bayesian Map Learning in Dynamic Environments," Advances in Neural Information Processing Systems, 1999, vol. 12, pp. 1015-1021.

(12) Montemerlo, M., Thrun, S., Koller, D., and Wegbreit, B., "FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem", In Proc. of the AAAI National Conference on Artificial Intelligence, Oct. 2002, pp. 593–598.

(13) Montemerlo, M., "FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem With Unknown Data Association," PhD thesis, Carnegie Mellon University, 2003.

(14) Thrun, S., and Lui, Y., "Multi-Robot SLAM With Sparse Extended Information Filters," In Proc. of International Symposium of Robotics Research, Oct. 2003, pp. 254-266.

(15) Wang, C.C., "Simultaneous Localization, Mapping and Moving Object Tracking," PhD thesis, Carnegie Mellon University, 2004.

(16) Thrun, S., Burgard, W., and Fox, D., "A Real-Time Algorithm for Mobile Robot Mapping With Applications to Multi-Robot and 3D Mapping," In Proc. of IEEE International Conference on Robotics & Automation, Apr. 2000, pp. 321-328.

(17) Thrun, S., Burgard, W., and Fox, D., "Probabilistic Robotics," The MIT Press, Sept. 2005.

(18) Vu, T.D., "Vehicle Perception: Localization, Mapping with Detection, Classification and Tracking of Moving Objects," PhD thesis, Grenoble Institute of Technology, 2009.