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NUMERICAL AND EXPERIMENTAL STUDIES OF SUSPENSIONS OF FIBER AND SPHERICAL SOLID PARTICLES

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Abstract. This paper is devoted the behavior of suspension of fiber-spheres particles. We pointed out by experimental and numerical observations that the Jefery approximation is no more valid as the sphere particle concentration increases.

1 Introduction

The orientation of long bodies (one dimension is much prevailing upon the other two) in liquids of different nature is a fundamental issue in many problems of practical interest. In particular for composite material, the addition of spherical particles, short or long fibers to polymer matrix is well known to enhance the mechanical properties of composite material. The degree of enhancement depends strongly on the orientation of the fibers and the distribution or aggregation of various particles in the final product. Then, a better knowledge of the motion of solid particles in polymer liquids is important for a better understanding of the rheology of such suspensions. We have developed a method to simulate fiber motions in flow by using finite element method with a multi-domain approach of two phases (namely a viscous fluid and rigid bodies) [4]. The present formulation is quite well suited for suspensions of micro/nano particles in simple shear flows and can be easily extended to viscoelastic flow problems. Computations are made on an elementary volume subjected to a constant shear rate which is imposed by using sliding bi-periodic frames. We focus our computations on fiber-spheres system. The aim is to analyze the influence of sphere concentration on the fiber rotation. As the motion of a single fiber is rather well known (Jeffery’s theory [3]), the behaviour of a fiber interacting with spherical
particles has been less studied. On the other hand, experimental observations are made by using a transparent counter-rotating plate-and-plate shear cell coupled with an optical microscope [1]. One finds that the increasing of spherical particle concentration decreases the fiber period of rotation. Moreover the fiber changes alternatively its plane of rotation: it can stay in the shear plane and suddenly it comes to perpendicular plane whatever the initial conditions. These changes of rotation plane increase with sphere concentrations.

2 The Jeffery’s motion

It is well known that the motion of an axisymmetric particle, suspended in a Newtonian matrix under simple shear flow, consists of a spin around its axis of symmetry and the precession of this axis around the vorticity axis of the undisturbed flow [3]. The Jeffery’s equation [3] gives the evolution of the vector \( \mathbf{p} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) aligned along the main direction of fiber. For the aspect ratio \( \beta \) and a simple shear flow \( \mathbf{v} = (\dot{\gamma} y, 0, 0) \), one gets an analytical solution for the two angles

\[
\tan \phi = \frac{1}{\beta} \tan \left( C_\phi - \frac{\beta}{\beta^2 + 1} \dot{\gamma} t \right)
\]

\[
\tan \theta = \left( \frac{C_\theta}{(\beta^2 \sin^2 \phi + \cos^2 \phi)^{\frac{1}{2}}} \right)
\]

where the two coefficients \( C_\phi \) and \( C_\theta \) depend on the initial condition. The macroscopic applied shear rate \( \dot{\gamma} \) can be directly related to the period of the precession rotation \( T \)

\[
T = \frac{2 \pi}{\dot{\gamma}} \left( \frac{\beta^2 + 1}{\beta} \right)
\]

Moreover, the fiber follows an orbit around the \( 0z \) axis. The amplitude of this orbit depends on the initial position and the fiber has a "kayaking" like motion. Unfortunately, the experimental apparatus only records the fiber motion int the \((x, z)\) which correspond to the angle \( \alpha \) for which the Jeffery’s solution reads (see Figure ??)

\[
\tan \left( \frac{\pi}{2} - \alpha \right) = C_\theta \cos \left( -\frac{2\pi t}{T} + C_\phi \right)
\]

Note that the Jeffery’s solution for an ellipsoidal particle can be applied to our cylindrical particle provided that an equivalent aspect ratio is used.

3 Experimental observations

3.1 Rheo-optical set-ups

The suspensions were observed while subjected to a simple shear flow at room temperature (21 °C) with a transparent counter-rotating rheometer. The counter-rotating
rheometer consists of two 40 mm diameter glass plates rotating in opposite directions [5]. Each plate is independently driven by a motor. The advantage of this geometry is that the relative rotation velocities of the two plates can be adjusted so that the velocity of a given particle is zero in the reference framework of the laboratory [5]. This enables one to observe the behaviour of a particle suspended in a matrix under shear for a long time and to measure, for example, its period of rotation. The observations were made in the plane formed by the vorticity axis and the flow direction with a Wild Leitz optical microscope. Images were captured by a video system and stored on video tapes.

3.2 Effect of spherical particule concentration on the rotation of fiber

Our experiments are made with glass spherical and cylindrical particles in a Newtonian silicon fluid of hight viscosity ($\eta = 200 \text{Pa.s}$). The sphere radii are between 100 and 150 $\mu m$ whereas the fibers have a radii between 20 et 14 $\mu m$ and an aspect ratio from 5 to 30. Measurements of the evolution of angle $\alpha$ are made for an isolated fiber and three spherical particule concentrations : 5, 8 and 10 %.

The isolated fiber and the less concentrated case are well fitted by analytical formula (2). As shown in figure 1, the fiber motion is more complicated for the highest sphere concentrations. In fact the angle $\alpha$ is sometime equals to 0 or $\pi$. That means that the fiber which is initially in the vorticity plane moves int he shear plane. Therefore, its motion can not be approached by Jeffery’s formula.

![Figure 1: Measured angle $\alpha$ for the two concentrations $\varphi_s = 5$ and 10 %; the red line plots the Jeffery’solution.](image1.png)

4 Numerical computations

A finite element method has been developed for the simulation of particulate Stokes flows [4]. This formulation is based on the fictitious domain method [2], which consists in treating the entire fluid-particle domain as a fluid. One splits the whole domain in two sub-domains and which are associated respectively to fluid and solid areas. The fluid in...
the solid domain is ensured to move accurately by adding the rigidity constraint in the weak formulation with a distributed Lagrange multiplier [4].

![Figure 2: Computed angle $\theta$ for $\varphi_s = 17.5\%$; the green line plots the Jefery'solution.](image)

The computations are made on an unitary cubic domain with periodic boundary conditions. The suspension is composed by one fiber of aspect ratio $\beta = 8.6$ surrounded by spherical particle of radius $0.075$ with a concentration of $17.5\%$. The figure 2 shows that the Jeffery solution is not valid as the interactions between spheres and fiber increase. That implies that the macroscopic equation as the Folger Tucker equation which describes the behavior of fiber suspensions has to used with caution for more complex suspensions.

**REFERENCES**


