

## An optimal approach dedicated to energy efficiency of electrical systems

Vincent Mazauric and Nadia Maïzi

vincent.mazauric@schneider-electric.com

Schneider Electric, Innovation Department, 38 TEC, 38050, Grenoble cedex 9, France

nadia.maizi@cma.ensmp.fr

Mines ParisTech, Center for Applied Mathematics, BP 207, 06904, Sophia-Antipolis cedex, France

### 1. Abstract

In order to provide a vision of power electrical engineering for future energy-supply crossroads and challenges, variational principles are used to express local laws of electromagnetism : they are derived from thermodynamic principles and by assuming a reversible transfer of the global power exchanged throughout the electrical network. This approach is suitable for consolidating energy processes involved in electromagnetic and electromechanical conversions, from deep within the structure of the materials to the power network operations. Two kinds of devices are identified: (i) those dedicated to power conversion; and (ii) their couplings involved in transmission lines, distribution busways, cablings and connections. Furthermore, the approach is shown to provide a unique description framework for power management, energy efficiency and sustainable long-term planning exercises.

**2. Keywords:** Electromagnetic fields, Energy conversion, Variational principles, Power systems, Long-term planning

### 3. Introduction

Future investments in the electricity-sectors over the next three decades are estimated at about US\$10Trillion [1], which is equivalent to the 2/3 of the total energy investment needs, worldwide, in all areas of the energy-supply chain and is three times higher in real terms than investments in the electricity-sectors over the past three decades.

Such a growth in the electricity-sector is mainly driven by the following assumptions: an increase of domestic energy demand in emerging countries and the refurbishment of ageing facilities in developing countries; moreover, a move toward electricity is expected to promote technologies with a lower environmental impact to combat global warming due to greenhouse gases emissions and to counterbalance fossil energy depletion.

While most scenarios on the future of the electricity sector assess a mix through available energy sources (hydro, nuclear, renewables, fossil, etc.), they usually neglect the actual energy efficiency of the electrical supply chain as a whole.

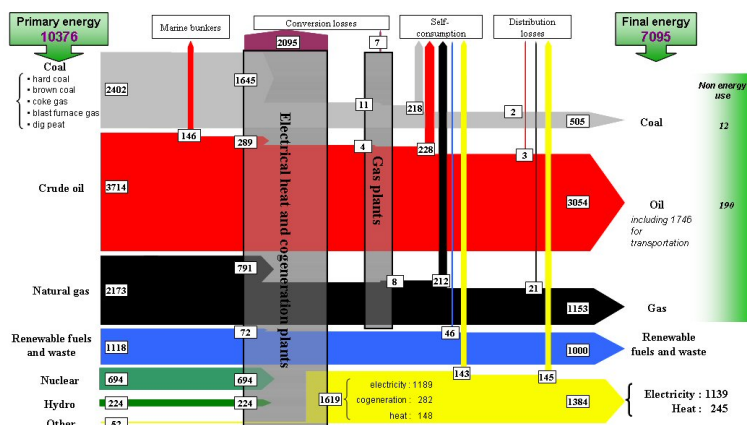


Figure 1: Energy supply-chain (compiled from [2])

Indeed, electrical energy is severely disadvantaged by the efficiency of the Carnot cycle, transmission losses and the low performance of applications. If we analyze the chain linking the producer to the consumer, shown

in figure 1 via the conversion of the primary energy source to the final (commercial) energy supply and to useful energy (the energy actually needed according to available technologies), we find that electricity drains around 32% of the primary energy source and releases 27% of useful electricity. And, in the meantime, the overall “efficiency” of the “all-fuel” energy chain is around 37% [3].

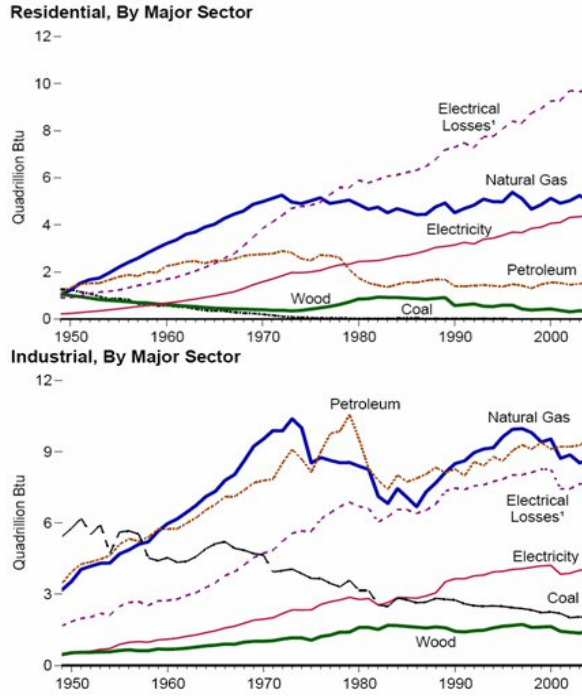


Figure 2: US energy consumption in the residential and commercial sector (up) and industry sector (down) from 1950 to 2005 [5].

Moreover, this abysmal lack of efficiency may be exacerbated by the fact that the optimization of the electrical power system is far from being achieved, as illustrated by observing electrical losses in the USA given in figure 2: for both sectors represented, the level of losses induced by the electrical generation follows a growth in electricity demand.

Ultimately, electricity generation contributes to 40% of global CO<sub>2</sub> emissions, before deforestation and transportation [4].

This could infer that the state-of-the-art of electrical engineering would not allow energy efficiency of the whole electrical supply chain to be achieved.

Conversely, this paper depicts an energy-efficient description of electromagnetism based on a *reversible* interpretation of the Faraday’s Law (section 4). This type of physics-oriented framework appears suitable for the consolidation (i) in space, of all the scales involved in the conversion process of the electromagnetic energy; (ii) in time, over the whole life cycle of the electrical supply-chain. Whereas the latter appears to be related to network management (section 5.1), the former addresses some issues on developments in designing devices (section 5.2). Finally, the interplay between energy efficient issues should be discussed within a long-term planning exercise dedicated to sustainable development (section 6).

#### 4. Variational principles in electromagnetism

To circumvent the impossibility of providing any deterministic evolution, thermodynamics assumes that the steady-state of any system is obtained from the maximum-entropy principle [6][7] maintaining the macroscopic information on the system including the positions of the moving bodies  $\mathbf{X}$  and the internal energy  $U$ ; in the specific context of electromagnetism (figure 3) this refers to: the magnetic flux  $\phi$  excited by the current generator

and the electric charge  $Q$  squeezed from the earth.

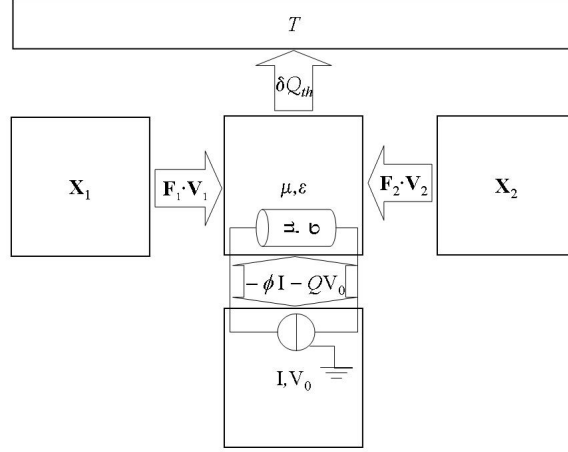


Figure 3: Chart of the energy exchanges between the various subsystems involved in the thermodynamic framework.  $(\mu, \sigma)$  and  $(\mu, \epsilon)$  denote respectively the behavior laws of conductors region  $C$  and dielectrics region  $D$ . While current generator and moving part can only exchange work with the electromagnetic field (respectively through a current variation and a modification of the boundary conditions on the magnetic moving parts), the thermostat may only receive heat from the other sub-systems. In the case of an adiabatic evolution, the heat received  $\delta Q_{th} = T dS_{th}$  by the thermostat vanishes.

Whereas thermodynamic assignment should also include an explanation of non-equilibrium conditions and irreversible processes, since equilibrium is merely an ideal limiting case of the behavior of matter, classical thermodynamic approaches of electromagnetism do not consider any extensions toward time-varying regimes [9][10][11][12]. While some improvements are summarized in [13] for steady current flow regimes, no general contribution is available for transients. In the following, a thermodynamic interpretation of the Faraday's Law within the quasi-static approximation is given, firstly within an overall description of the electrical system, and then by adopting a local viewpoint.

Through the enforcement of an isothermal evolution thanks to a contact of the system with a thermostat at temperature  $T$ , two cases – summarized in the *second principle* – may be considered in order to be coherent with the maximum-entropy principle checked by any isolated system:

1. the evolution does not modify the entropy  $S + S_{th}$  of the isolated system. Such an evolution is said to be *reversible* and involves mainly work exchanges, through sufficiently smooth variations in the energy couplings, namely:
  - the mechanical power exchanged between actuators  $P_{mec} = \sum \mathbf{F} \cdot \mathbf{V}$  where  $\mathbf{V} = \frac{d\mathbf{X}}{dt}$  is the velocity of the moving body  $\mathbf{X}$  experiencing the external force  $\mathbf{F}$ ; and in the context of electromagnetism
  - the electromagnetic coupling  $\frac{d(\phi I + QV_0)}{dt}$  where  $I$  is the net current exciting the electromagnetic field and the  $V_0$  earth voltage.
2. the evolution increases the entropy of the isolated system so that some amount of the work exchanged between actuators has been lowered in heat during the process: such an evolution is said to be *irreversible*.

The *first principle* conveys the energy conservation and reads

$$\frac{dU}{dt} = P_{mec} - T \frac{dS_{th}}{dt} \quad (1)$$

where  $\mathbb{T} \frac{dS_{\text{th}}}{dt}$  denotes the heat produced by the field, *i.e.* received by the thermostat. Hence, the irreversibility experienced by the system coupled with its thermostat may be discussed from the Helmholtz free-energy  $F = U - \mathbb{T}S$  by expressing

$$P_{\text{mec}} - \frac{dF}{dt} = \mathbb{T} \left( \frac{dS}{dt} + \frac{dS_{\text{th}}}{dt} \right) \quad (2)$$

where the RHS matches the power lowered in heat by the whole system, commonly known as the Joule losses  $P_{\text{Joule}}$ . According to the second principle, this term is always positive and the lower the Joule losses, the more reversible the evolution. In order to explicitly take into account the inertial behavior of the electromagnetic coupling (Lenz's Law), it is convenient to use the Gibbs free-energy  $G = F - \phi\mathbb{I} - QV_0$  on which another reversible assignment may be expressed

$$P_{\text{mec}} - \frac{dG}{dt} = \min \left( P_{\text{Joule}} + \frac{d(\phi\mathbb{I})}{dt} + \frac{d(QV_0)}{dt} \right) \quad (3)$$

Within the quasi-static approximation, the Faraday's Law of induction is achieved by forcing the condition of reversibility expressed by the Gibbs free-energy, whereas the condition obtained from the Helmholtz free-energy only restores the flow behavior of the direct current. In order to derive the Maxwell-Faraday equation from (3), it is convenient to describe electromagnetic effects through macroscopic fields linked to the sources.

#### 4.1. Source fields

After spatial averaging on the microscopic distribution of charge to discard short time- and space-variations with respect to excitation provided by generators, the conservation of the electric charge, reads [8]

$$\text{div} \mathbf{J} + \partial_t \rho = 0 \quad (4)$$

where  $\rho$  is the average charge density and  $\mathbf{J}$  denotes the free current density, *i.e.* involving charges able to move on a large scale with respect to atomic structure. Conductors may provide such non-vanishing sources. To replace, in an unbound way, the span of the electromagnetic interaction, it is convenient to define the so-called electric displacement field  $\mathbf{D}$  and magnetic field  $\mathbf{H}$ . They satisfy respectively the Maxwell-Gauss equation

$$\text{div} \mathbf{D} = \rho \quad (5)$$

and the Maxwell-Ampere equation

$$\text{curl} \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D} \quad (6)$$

Among the admissible couple of fields  $(\mathbf{D}, \mathbf{H})$ , the thermodynamic principles will enforce those matching the complementary Maxwell equations.

The quasi-static approximation assumes no coexistence between the free current density and the displacement terms in the Maxwell-Ampere equation (6). Thus, the current density  $\mathbf{J}$  remains divergence-free and only a static averaged charge density  $\rho$  may exist in conductors. Hence, considering invariance of Joule losses under Galilean transformations, such a charge density does vanish in the *bulk* of conductors (*i.e.*  $\mathbf{D} \equiv 0$  therein). As a result only free currents carried by conductors may yield heat losses and free charges may only exist on the surface of conductors where they polarize the dielectric region.

#### 4.2. Statics

For magnetostatics, the  $\mathbf{H}$  field is simply created by a static current density  $\mathbf{J}$ , which is actually given by the minimization of the Joule losses (see below or [13]). Hence, equilibrium is obtained by considering variations  $\delta \mathbf{H}$  of the field  $\mathbf{H}$  which do not change either the Gibbs free-energy, or the current density. Introducing  $(-\mathbf{B})$  as a conjugate field of  $\mathbf{H}$  related to the variation of the magnetostatic contribution to Gibbs' free-energy density with respect to  $\mathbf{H}$ , the stationary condition of

$$G(\mathbb{T}, \mathbb{I}, \mathbf{X}) = \int \left( \int_0^{\mathbf{H}} (-\mathbf{B}) \cdot \delta \mathbf{h} \right) d^3 r \quad (7)$$

under admissible variations  $\delta\mathbf{H} = -\mathbf{grad}\delta\phi$  keeping  $\mathbf{J}$  constant in the conductors, yields, after an integration by part throughout the whole space

$$\operatorname{div}\mathbf{B} = 0 \quad (8)$$

along with a condition that falls on  $\mathbf{B}$ .

Thus,  $\mathbf{B}$  appears as the magnetic flux density. Note that the numerical resolution of (7) requires a magnetic behavior field-to-field relationship  $\mathbf{B}(\mathbf{H})$ . For permanent, local, non-dispersive and homogeneous media, this behavior law has to check  $\delta\mathbf{B} \cdot \delta\mathbf{H} > 0$  to enforce the stationary point of the Gibbs free-energy to its minimum.

The same kind of thermodynamic procedure describes the electrostatic behavior of a dielectric region  $D$  polarized by a surface charge distribution spread over the surface of conductors connected to voltage generators. The electrostatic Gibbs free-energy minimum enforces a behavior law  $\mathbf{D}(\mathbf{E})$  of the dielectric media for which  $\delta\mathbf{D} \cdot \delta\mathbf{E} > 0$ .

#### 4.3. Time-stepping evolution within the quasi-static approximation

Expressing the weak reversibility condition (3) from continuous fields, the conservation of the mechanical power supplied by the actuators checks

$$P_{\text{mec}} - \frac{dG}{dt} = \min_{\mathbf{H}, \mathbf{E}} \left( \int_C \sigma^{-1} (\mathbf{curl}\mathbf{H})^2 d^3r + \frac{d}{dt} \int (\mathbf{B}(\mathbf{H}) \cdot \mathbf{H} + \mathbf{D}(\mathbf{E}) \cdot \mathbf{E}) d^3r \right) \quad (9)$$

where:

1. the first term in the RHS corresponds to Joule losses monitored in the conductors  $C$  ( $\sigma > 0$ ). This term is even to respect the invariance of losses with inversion of time;
2. the second term in the RHS is related to the variation with time of the energy coupling the current generators and the electromagnetic field.

After calculations on the convective derivative of the coupling energy [14], the contribution of conducting regions  $C$  to the minimization of (9) reads

$$\min_{\mathbf{H}} \int_C \left( \sigma^{-1} (\mathbf{curl}\mathbf{H})^2 + \partial_t (\mathbf{B} \cdot \mathbf{H}) \right) d^3r \quad (10)$$

Hence, some tedious variational calculations and an integration by part over the entire space yield a Maxwell-Faraday equation and Ohm's Law in the stationary frame of each conductor [15]. Then, considering invariance over time of any magnetic flux under Galilean transformation, it follows that the electric field  $\mathbf{E}$  in *conductors* [16]:

- is given through the so-called Ohm's law with motion

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad (11)$$

where  $\mathbf{V}$  is the *local velocity* of the conductor in the stationary frame and  $\sigma$  its conductivity; and

- checks the Maxwell-Faraday equation

$$\mathbf{curl}\mathbf{E} = -\partial_t\mathbf{B} \quad (12)$$

Hence, the contribution of the dielectric region  $D$  to the minimization of the power supplied by actuators (9) reads

$$\min_{\mathbf{H}, \mathbf{E}} \int_D \frac{\partial}{\partial t} (\mathbf{B} \cdot \mathbf{H} + \mathbf{D} \cdot \mathbf{E}) d^3r \quad (13)$$

and enforces a reversible exchange between magnetic and electrostatic energies. Some calculations yield:

- the Maxwell-Faraday equation (12) in the *dielectric region* too;
- a matching condition on the tangential component of the electric field  $\mathbf{E}$  across the dielectric/conductor interface  $\partial C$

$$(\mathbf{E} + \mathbf{V} \times \mathbf{B} - \sigma^{-1}\mathbf{curl}\mathbf{H}) \times \mathbf{n} = 0 \quad (14)$$

### 4.3.1 Quasi-static field sketch

To summarize, stationary condition (7) is related to the determination of the magnetic field according to the current flow density whereas (10) yields the eddy current distribution according to the magnetic flux variations. At each time-step, the surface charges spread out on the conductors to minimize the electromagnetic power supplied to the dielectrics (13) but with respect to the current flowing on  $\partial C$  according to (14). Such an approach:

- enforces the best *reversible* evolution of the electromagnetic field;
- corresponds the best just-in-time workflow between actuators over a time-period;
- sketches the quasi-static evolution of the electromagnetic field, providing a thermodynamic-oriented interpretation of the variational theory of electromagnetism [17].

Taking advantage of their quadratic property, thermodynamic functionals (7),(9) support spatial filtering operations and mean field assumptions to provide a fully multi-scale framework.

## 5. Multi-scale modeling

So far, the electrical network has been considered within a global description. For modeling purposes or network management, it is convenient to derive the voltage drop  $U_{\Omega_i}$  of a dipolar component from the Poynting's vector  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ . If  $I_i$  denotes the net current flowing therein, and  $\Omega_i$  a surrounding volume of the component, the voltage drop  $U_{\Omega_i}$  is defined by

$$U_{\Omega_i} = -\frac{1}{I_i} \oint_{\partial\Omega_i} (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} \, d^2r \quad (15)$$

As the Maxwell-Faraday equation and Ohm's Law with motion are satisfied in each subspace  $\Omega_i$ , the electrical power  $P_{\text{elec}} = \sum_i U_{\Omega_i} I_i$  and the mechanical power within the partition  $\cup_i \Omega_i$  surrounding all the conductors  $C_i \subset \Omega_i$ , check the energy conservation equation

$$P_{\text{mec}} + P_{\text{elec}} = \int_C \sigma^{-1} (\mathbf{curl} \mathbf{H})^2 \, d^3r + \frac{d}{dt} \int \left( \int_0^{\mathbf{B}} \mathbf{H} \cdot \delta \mathbf{b} + \int_0^{\mathbf{D}} \mathbf{E} \cdot \delta \mathbf{d} \right) \, d^3r \quad (16)$$

Note that the last term in the RHS denotes the electromagnetic power over the whole space, corresponding to the reactive power in time-harmonic regime. In order to highlight the multi-scale ability of the former approach, we consider successively: the management of a power network, to underline the role of the energy coupling the electromagnetic field and the excitation in the reliability of the installation; the design of devices, to introduce the relevant methods.

### 5.1. Power network management

The conservation equation (16) addresses two distinct behavior patterns in the network.

The cancellation of the electrical power in (16) over the partition  $\cup_i \Omega_i$  expresses the conservation of the energy supplied by the field. The fields obtained in each subspace  $\Omega_i$  may be composed to check the crossing conditions at the interfaces  $\partial\Omega_i$ , and the RHS of (16) is simply the mechanical power (9) received by the field from the actuators. Furthermore, the definition (15) becomes intrinsic in the sense that:

- two parallel-connected branches  $i$  and  $j$  have the same voltage drop as a third one

$$U_{\Omega_i} = U_{\Omega_j} = U_{\Omega_i \cup \Omega_j} \quad (17)$$

which consequently does not depend on  $\Omega_i \supset C_i$  so that

$$U_{\Omega_i} \equiv U_i = \frac{1}{I_i} \int_{C_i} (\mathbf{J} \cdot \mathbf{E} + \mathbf{H} \cdot \partial_t \mathbf{B}) \, d^3r \quad (18)$$

- it matches the usual expression of the voltage drop obtained from  $\mathbf{AV}$  formulation [18], and

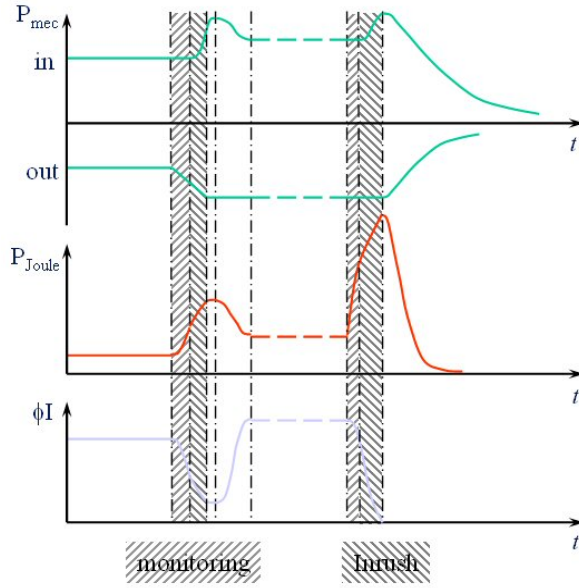


Figure 4: Two events experienced by the network

- it enforces a *reversible* – *i.e.* *adapted* – electrical coupling between the electromagnetic field in each subspace  $\Omega_i$  and the remaining electrical network.

Conversely, exhibiting a difference in the voltage drop (15) between two parallel-connected branches of the network means that the power network experiences an *irreversible* process at the time scale  $dt$  and the length scale  $\sim \sqrt[3]{|\Omega_i|}$  on which it is monitored. In other words, sudden change occurs either outside the partition (*i.e.* extra connection such as switching, earth leakage or lightning) or within it (*i.e.* transition in behavior laws due to ageing) leading to unbalanced behavior in the network. Only a rush to adapt the (mechanical) power production or a disconnection of loads (black-out) may lead to its recovery, as illustrated in figure 4 where two events experienced by the network are represented: it is shown that while an admissible load fluctuation is lifted by the coupling energy ( $\phi I$ ) and the generation realignment (left in figure 4), the heat of the coupling energy is completely reduced by a short circuit, leading to a collapse in power transmission (right in figure 4).

## 5.2. Electrical design

The previous approach addresses a fuller justification of the Finite Element Method (FEM) which consists in building an approximation of the variational formulations given in equations (7) and (10) but with a finite number of degrees of freedom chosen on a mesh (figure 5) [19]. Assuming a balanced network, the design can be partitioned into each device, the behavioral composition of which provides the global design. Following (9), the contribution of  $\Omega_i$  to the global mechanical power checks

$$P_{\text{mec}}(\Omega_i) - \frac{dG(\Omega_i)}{dt} + \sum_{j(i)} U_j I_j = \min_{(\mathbf{H}, \mathbf{E})|_{\Omega_i}} \left( \int_{C_i} \sigma^{-1} (\mathbf{curl} \mathbf{H})^2 d^3 r + \frac{d}{dt} \int_{\Omega_i} (\mathbf{B} \cdot \mathbf{H} + \mathbf{D} \cdot \mathbf{E}) d^3 r \right) \quad (19)$$

where  $j(i)$  indexes the electrical circuits connected to  $\Omega_i$ ,  $P_{\text{mec}}(\Omega_i)$  gathers the Laplace's force ( $\mathbf{J} \times \mathbf{B}$ ) experienced by the conductors and the switching reluctance effects acting on materials included in  $\Omega_i$ , and  $G(\Omega_i)$  is the restriction of the Gibbs free energy to  $\Omega_i$ .

Several kinds of devices may be pointed out.

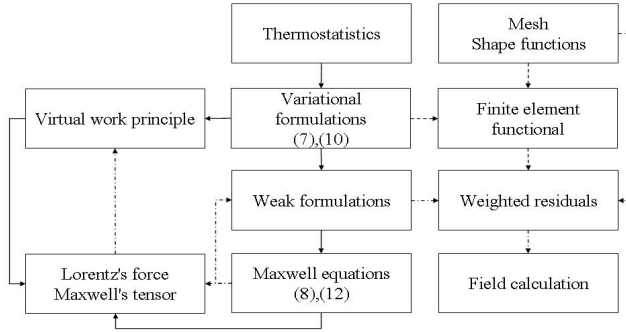


Figure 5: Flowchart of the thermodynamic approach of electromagnetism. The full lines show the main steps of the approach, and the dashed lines indicate its numerical description. The mixed lines provide the usual introduction of Finite Element Method.

### 5.2.1 Motors, generators and transformers

While the optimal behavior of electromechanical devices achieves over any thermodynamic cycle

$$\min \oint \left( P_{\text{mec}}(\Omega_i) + \sum_{j(i)} U_j I_j \right) dt \quad (20)$$

to ensure the highest efficiency, the case of transformer requires  $P_{\text{mec}}(\Omega_i) \sim 0$  as an additional constraint to avoid mechanical power trapping within it. As efficiency is expressed directly from the energy conversion balance (19), the FEM remains the obvious method for designing such devices. To achieve a sufficiently low level of *integration*, vanishing boundary conditions on the fields are prescribed on  $\partial\Omega_i$ , taken far away enough from the device. Thus coupling between devices is only performed thanks to electrical circuits, the ideal behavior of which is achieved when the voltage does not drop. For higher levels of integration, a *robust* design should ensure the unsensitivity of the device's lack of sensitivity to the outer electromagnetic field acting on  $\partial\Omega_i$ .

### 5.2.2 Couplings

Transmission lines, distribution busways, cablings, PCBs, etc. belong to this category of components. Their optimal behavior achieves

$$\forall j \in \Omega_i : \min (U_j I_j) \quad \text{with: } P_{\text{mec}}(\Omega_i) \sim 0 \quad (21)$$

to express that neither Joule losses nor time-varying-induced-reactive power occur therein. While such a condition enforces signal integrity flowing through any electrical coupling, it involves:

- opened domains where power transmission prevents vanishing fields;
- extended domains where the air volume could lead to huge problem;
- and ill-known boundary conditions in between the conductors of the same circuit loop.

As a result, the FEM appears ill-suited to designing electrical couplings. For time-harmonic design, the Partial Element Equivalent Circuit (PEEC) method [20][21][22] takes advantage of the knowledge of the current flow orientation and the localization of the electromagnetic energy in the conductors to provide an alternative to the FEM.



## 6. Conclusions

The previous point of view has consisted in starting from variational principles obtained from thermodynamics and showing that they yield functionals containing the Maxwell's equations. Thus, electromagnetic conversion only achieves the best just-in-time workflow with the minimum loss of value, due to the dispersive properties of the materials. Therefore, optimization leads to applying FEM for power conversion purposes while applying PEEC to couplings. Both methods will be available in the same CAD environment [23].

Unfortunately, in spite of this ideal design, a real network undergoes irreversible processes for the length and time scales during which it is monitored. Usually, this lack of information is counteracted by the reactive energy (stock) for admissible load fluctuations. Hence, for a given level of reliability, network management results from a compromise between: (i) stock of reactive energy, and subsequent oversizing, extra-losses and sensitivity to time-varying fields; and (ii) investment in the monitoring and rush production capabilities.

In long-cycle industries, scientific uncertainties must be reduced in order to develop the most sustainable technologies and anticipate market trends. In order to center eco-design methodology on medium and long-term prospective thinking, Life Cycle Assessment should be sufficiently consolidated to match the technological breakdown required by technical and economic energy long-term planning tools such as the MARKAL model [24]. Hence, the method will provide a complete energy-related description of the energy management chain, from deep within the structure of materials to the acknowledgment of demand fluctuations, based on an optimal description of energy transfers, therefore conducive to:

- optimization of architectures subject to operating and environmental constraints;
- discrimination of technologies and research initiatives benefiting from favorable environmental leverage;
- analysis of the sensitivity of prospective scenarios to determining operating factors, in particular the flexibility of demand;
- arbitration between the availability of electrical power and the investment to be made between the production mix and transport, monitoring and reserve capacities, as well as the sensitivity of such arbitration.

Such an approach should improve the Life Cycle Assessment of the electrical supply-chain, give an insight into a power electrical engineering dedicated to energy efficiency and ultimately provide key features of the electricity production sector to achieve sustainable long-term planning [25].

## 7. References

- [1] World Energy Investment Outlook 2003, vol. 11 of *World energy outlook*, Paris, France: Organisation for Economic Co-operation and Development/International Energy Agency, 2003.
- [2] Key world energy statistics from the IEA, *Organisation for Economic Co-operation and Development/International Energy Agency*, Paris, France, 2004.
- [3] C. Gibowski, Les atouts de la conversion électromagnétique et électromécanique pour l'optimisation de l'efficacité énergétique, *Master of Science OSE <http://www.ose.cma.fr>*, Ecole des Mines de Paris, 2005.
- [4] H.-H. Rogner, Nuclear power and climate change (invited), in *United Nations Framework Convention on Climate Change Conference (COP-12)*, Nairobi, Kenya, 2006.
- [5] Annual Energy Review 2005, *Energy Information Administration*, Department of Energy, Washington DC, USA, 2006.
- [6] E. Jaynes, Information theory and statistical mechanics, *Physical Review*, vol. 106, no. 4, pp. 620–630, 1957.
- [7] A. Garrett, Macroirreversibility and microreversibility reconciled: The second law, in *Maximum entropy in action* (B. Buck and V. Macaulay, eds.), pp. 139–170, Oxford, UK: Oxford University Press, 1991.

- [8] G. Russakoff, A derivation of the macroscopic maxwell equations, *American Journal of Physics*, vol. 38, no. 10, pp. 1188–1195, 1970.
- [9] S. Bobbio, The use of energy and co-energy for the evaluation of forces in non-linear, anisotropic dielectric solids, *The European Physical Journal B*, vol. 16, pp. 43–48, 2000.
- [10] F. Delfino, R. Procopio, and M. Rossi, Evaluation of forces in magnetic materials by means of energy and co-energy methods, *The European Physical Journal B*, vol. 25, pp. 31–38, 2002.
- [11] L. Landau and E. Lifschitz, *Electrodynamics of continuous media*, New York, NY, USA: Pergamon, 1960.
- [12] B. Nogarede, Machines tournantes: conversion électromécanique de l'énergie, *Techniques de l'ingénieur: Traité de génie électrique*, vol. D3 410, pp. 1–19.
- [13] T. Christen, Application of the maximum entropy production principle to electrical system, *Journal of Physics D: Applied Physics*, vol. 39, pp. 4497–4503, 2006.
- [14] I. Gradshteyn and I. Ryzhik, Integral rate of change theorems, in *Table of integrals, series and products* A. Jeffrey ed., pp. 1123–1124, San Diego, USA: Academic Press, 5th ed., 1994.
- [15] V. Mazauric, From thermostatics to maxwell's equations: A variational approach of electromagnetism, *IEEE Transactions on Magnetism*, vol. 40, no. 2, pp. 945–948, 2004.
- [16] J. Jackson, Time-varying fields, maxwell equations, conservation laws, in *Classical electrodynamics*, pp. 209–268, New York, USA: John Wiley and sons, 1975.
- [17] P. Hammond, Energy methods in electromagnetism, *Monographs in electrical and electronic engineering*, Clarendon, Oxford University Press, New York, USA, 1981.
- [18] O. Biro, K. Preis, W. Renhart, G. Vrisk, and F. Richter, Computation of 3d current driven skin effect problems using a current vector potential, *IEEE Transaction on Magnetism*, vol. 29, no. 2, pp. 1325–1332, 1993.
- [19] M. Chari and S. Salon, Numerical Methods in Electromagnetism, *Electromagnetism*, vol. 7, San Diego: Academic Press, 2000.
- [20] C. Hoer and C. Love, Exact inductance equations for rectangular conductors with applications to more complicated geometries, *Journal of Research of the National Bureau of Standards C*, vol. 69C, no. 2, pp. 127–137, 1965.
- [21] A. Ruehli, Inductance calculations in a complex integrated circuit environment, *IBM - Journal of Research and Development*, pp. 470–481, 1972.
- [22] J.-P. Gonnet, V. Mazauric, J.-M. Guichon, and P. Wendling, Peec method for power busbar design (invited), in *American Society of Naval Engineers Annual Meeting and Symposium* S. Surko ed., American Association of Naval Engineers, Arlington, VA, USA, 2004.
- [23] <http://www.magsoft-flux.com/>, Flux/inca: Cad package for electromagnetic and thermal analysis, 2006.
- [24] N. Maïzi, E. Assoumou, M. Bordier, and V. Mazauric, Key features of the electricity production sector through long-term planning: The french case, in *IEEE PES Power Systems Conference and Exhibition*, vol. 1, (Atlanta, GA, USA), p. 54, 2006.
- [25] V. Mazauric, N. Maïzi, G. Meunier, P. Wendling, and M. Vilcot, Energy efficiency and eco-design: Major issues for schneider electric (invited), in *United Nations Framework Convention on Climate Change Conference (COP-12)* (F. Broadbent, ed.), (Nairobi, Kenya), pp. 46–48, Responding to Climate Change, 2006.